

Exercise 12.1

Question 1:

Evaluate

$$(i) 3^{-2} \quad (ii) (-4)^{-2} \quad (iii) \left(\frac{1}{2}\right)^{-5}$$

Answer:

$$(i) 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$(ii) (-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$(iii) \left(\frac{1}{2}\right)^{-5} = \frac{1}{(2)^{-5}} = (2)^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

Question 2:

Simplify and express the result in power notation with positive exponent.

$$(i) (-4)^5 \div (-4)^8 \quad (ii) \left(\frac{1}{2^3}\right)^2$$

$$(iii) (-3)^4 \times \left(\frac{5}{3}\right)^4 \quad (iv) (3^{-7} \div 3^{-10}) \times 3^{-5}$$

$$(v) 2^{-3} \times (-7)^{-3}$$

Answer:

$$(i) (-4)^5 \div (-4)^8 = (-4)^{5-8} \quad (a^m \div a^n = a^{m-n})$$

$$= (-4)^{-3}$$

$$= \frac{1}{(-4)^3} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$(ii) \left(\frac{1}{2^3}\right)^2 = \frac{1}{(2^3)^2} = \frac{1}{2^6} \quad \left((a^m)^n = a^{mn}\right)$$

$$(iii) (-3)^4 \times \left(\frac{5}{3}\right)^4 = (-1 \times 3)^4 \times \frac{5^4}{3^4}$$

$$= (-1)^4 \times 3^4 \times \frac{5^4}{3^4} \quad \left[(ab)^m = a^m \times b^m\right]$$

$$= (-1)^4 \times 5^4$$

$$= 5^4 \quad \left[(-1)^4 = 1\right]$$

$$(iv) (3^{-7} \div 3^{-10}) \times 3^{-5} = (3^{-7 - (-10)}) \times 3^{-5} \quad (a^m \div a^n = a^{m-n})$$

$$= 3^3 \times 3^{-5}$$

$$= 3^{3 + (-5)} \quad (a^m \times a^n = a^{m+n})$$

$$= 3^{-2}$$

$$= \frac{1}{3^2} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$(v) 2^{-3} \times (-7)^{-3} = \frac{1}{2^3} \times \frac{1}{(-7)^3} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$= \frac{1}{[2 \times (-7)]^3} \quad \left[a^m \times b^m = (ab)^m\right]$$

$$= \frac{1}{(-14)^3}$$

Question 3:

Find the value of.

$$(i) (3^0 + 4^{-1}) \times 2^2 \quad (ii) (2^{-1} \times 4^{-1}) \div 2^{-2}$$

$$(iii) \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} \quad (iv) (3^{-1} + 4^{-1} + 5^{-1})^0$$

$$(v) \left\{ \left(\frac{-2}{3} \right)^{-2} \right\}^2$$

Answer:

$$(i) (3^0 + 4^{-1}) \times 2^2 = \left(1 + \frac{1}{4} \right) \times 2^2 \quad \left(a^0 = 1 \text{ and } a^{-m} = \frac{1}{a^m} \right)$$

$$= \frac{5}{4} \times 4 = 5$$

$$(ii) (2^{-1} \times 4^{-1}) \div 2^{-2} = [2^{-1} \times \{(2)^2\}^{-1}] \div 2^{-2}$$

$$= (2^{-1} \times 2^{-2}) \div 2^{-2} \quad \left((a^m)^n = a^{mn} \right)$$

$$= 2^{-1+(-2)} \div 2^{-2} \quad (a^m \times a^n = a^{m+n})$$

$$= 2^{-3} \div 2^{-2}$$

$$= 2^{-3 - (-2)} \quad (a^m \div a^n = a^{m-n})$$

$$= 2^{-3+2} = 2^{-1}$$

$$= \frac{1}{2} \quad \left(a^{-m} = \frac{1}{a^m} \right)$$

$$(iii) \left(\frac{1}{2} \right)^{-2} + \left(\frac{1}{3} \right)^{-2} + \left(\frac{1}{4} \right)^{-2} = \left(\frac{2}{1} \right)^2 + \left(\frac{3}{1} \right)^2 + \left(\frac{4}{1} \right)^2 \quad \left(\therefore a^{-m} = \frac{1}{a^m} \right)$$

$$= 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29$$

$$(iv) (3^{-1} + 4^{-1} + 5^{-1})^0 = \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} \right)^0 \quad \left(a^{-m} = \frac{1}{a^m} \right)$$

$$= 1 \quad (a^0 = 1)$$

$$(v) \left\{ \left(\frac{-2}{3} \right)^{-2} \right\}^2 = \left\{ \left(\frac{3}{-2} \right)^2 \right\}^2 \quad \left(a^{-m} = \frac{1}{a^m} \right)$$

$$= \left\{ \frac{3^2}{(-2)^2} \right\}^2 \quad \left[\left(\frac{a}{b} \right)^m = \frac{a^m}{b^m} \right]$$

$$= \left(\frac{9}{4} \right)^2 = \frac{81}{16}$$

Question 4:

Evaluate (i) $\frac{8^{-1} \times 5^3}{2^{-4}}$ (ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

Answer:

$$(i) \quad \frac{8^{-1} \times 5^3}{2^{-4}} = \frac{2^4 \times 5^3}{8^1} \quad \left(a^{-m} = \frac{1}{a^m} \right)$$

$$= \frac{2^4 \times 5^3}{2^3} = 2^{4-3} \times 5^3 \quad (a^m \div a^n = a^{m-n})$$

$$= 2 \times 125 = 250$$

$$(ii) \quad (5^{-1} \times 2^{-1}) \times 6^{-1} = \left(\frac{1}{5} \times \frac{1}{2} \right) \times \frac{1}{6} \quad \left(a^{-m} = \frac{1}{a^m} \right)$$

$$= \frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$$

Question 5:

Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Answer:

$$5^m \div 5^{-3} = 5^5$$

$$5^{m - (-3)} = 5^5 \quad (a^m \div a^n = a^{m-n})$$

$$5^{m+3} = 5^5$$

Since the powers have same bases on both sides, their respective exponents must be equal.

$$m + 3 = 5$$

$$m = 5 - 3$$

$$m = 2$$

Question 6:

Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

Answer:

$$(i) \left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} = \left\{\left(\frac{3}{1}\right)^1 - \left(\frac{4}{1}\right)^1\right\}^{-1} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$= \{3 - 4\}^{-1} = (-1)^{-1} = \frac{1}{-1} = -1$$

$$(ii) \left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}} \quad \left[\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$$

$$= \frac{8^7}{5^7} \times \frac{5^4}{8^4} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$= \frac{8^{7-4}}{5^{7-4}} \quad \left(a^m \div a^n = a^{m-n}\right)$$

$$= \frac{8^3}{5^3} = \frac{512}{125}$$

Question 7:

Simplify. (i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} (t \neq 0)$ (ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Answer:

$$(i) \frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} = \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$$

$$= \frac{5^2 \times t^{-4}}{5^{-3+1} \times 2 \times t^{-8}} \quad (a^m \times a^n = a^{m+n})$$

$$= \frac{5^2 \times t^{-4}}{5^{-2} \times 2 \times t^{-8}}$$

$$= \frac{5^{2-(-2)} t^{-4-(-8)}}{2} \quad (a^m \div a^n = a^{m-n})$$

$$= \frac{5^4 t^4}{2} = \frac{625 t^4}{2}$$

$$(ii) \quad \frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}} \quad [(a \times b)^m = a^m \times b^m]$$

$$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-5+3-(-7)} \quad (a^m \div a^n = a^{m-n})$$

$$= 3^0 \times 2^0 \times 5^5 \quad (a^0 = 1)$$

$$= 5^5$$

Exercise 12.2**Question 1:**

Express the following numbers in standard form.

- (i) 0.00000000000085 (ii) 0.000000000000942
(iii) 6020000000000000 (iv) 0.00000000837
(v) 31860000000

Answer:

- (i) $0.00000000000085 = 8.5 \times 10^{-12}$
(ii) $0.000000000000942 = 9.42 \times 10^{-12}$
(iii) $6020000000000000 = 6.02 \times 10^{15}$
(iv) $0.00000000837 = 8.37 \times 10^{-9}$
(v) $31860000000 = 3.186 \times 10^{10}$

Question 2:

Express the following numbers in usual form.

- (i) 3.02×10^{-6} (ii) 4.5×10^4
(iii) 3×10^{-8} (iv) 1.0001×10^9
(v) 5.8×10^{12} (vi) 3.61492×10^6

Answer:

- (i) $3.02 \times 10^{-6} = 0.00000302$
(ii) $4.5 \times 10^4 = 45000$
(iii) $3 \times 10^{-8} = 0.00000003$
(iv) $1.0001 \times 10^9 = 1000100000$
(v) $5.8 \times 10^{12} = 5800000000000$
(vi) $3.61492 \times 10^6 = 3614920$

Question 3:

Express the number appearing in the following statements in standard form.

- (i) 1 micron is equal to $\frac{1}{1000000}$ m.
(ii) Charge of an electron is 0.000, 000, 000, 000, 000, 000, 16 coulomb.
(iii) Size of a bacteria is 0.0000005 m

(iv) Size of a plant cell is 0.00001275 m

(v) Thickness of a thick paper is 0.07 mm

Answer:

$$(i) \frac{1}{1000000} = 1 \times 10^{-6}$$

$$(ii) 0.000, 000, 000, 000, 000, 000, 16 = 1.6 \times 10^{-19}$$

$$(iii) 0.0000005 = 5 \times 10^{-7}$$

$$(iv) 0.00001275 = 1.275 \times 10^{-5}$$

$$(v) 0.07 = 7 \times 10^{-2}$$

Question 4:

In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Answer:

Thickness of each book = 20 mm

Hence, thickness of 5 books = (5×20) mm = 100 mm

Thickness of each paper sheet = 0.016 mm

Hence, thickness of 5 paper sheets = (5×0.016) mm = 0.080 mm

Total thickness of the stack = Thickness of 5 books + Thickness of 5 paper sheets

$$= (100 + 0.080) \text{ mm}$$

$$= 100.08 \text{ mm}$$

$$= 1.0008 \times 10^2 \text{ mm}$$