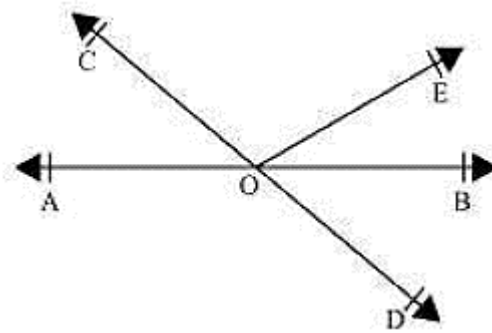


Exercise (6.1)**Question 1:**

In the given figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$ find $\angle BOE$ and reflex $\angle COE$.

**Solution 1:**

AB is a straight line, rays OC and OE stand on it.

$$\therefore \angle AOC + \angle COE + \angle BOE = 180^\circ$$

$$\Rightarrow (\angle AOC + \angle BOE) + \angle COE = 180^\circ$$

$$\Rightarrow 70^\circ + \angle COE = 180^\circ$$

$$\Rightarrow \angle COE = 180^\circ - 70^\circ = 110^\circ$$

$$\text{Reflex } \angle COE = 360^\circ - 110^\circ = 250^\circ$$

CD is a straight line, rays OE and OB stand on it.

$$\therefore \angle COE + \angle BOE + \angle BOD = 180^\circ$$

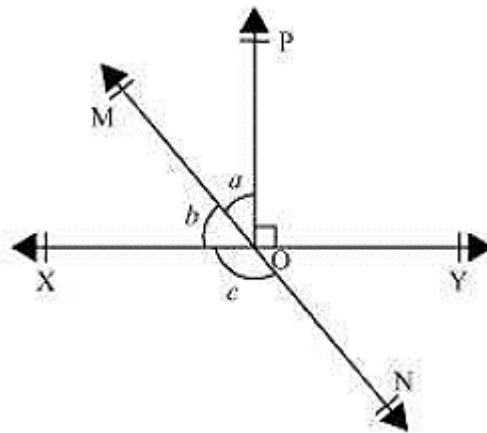
$$\Rightarrow 110^\circ + \angle BOE + 40^\circ = 180^\circ$$

$$\Rightarrow \angle BOE = 180^\circ - 150^\circ = 30^\circ$$

Hence, $\angle BOE = 30^\circ$ and Reflex $\angle COE = 250^\circ$

Question 2:

In the given figure, lines XY and MN intersect at O. If $\angle POY = 90^\circ$ and $a : b = 2 : 3$, find c.

**Solution 2:**

Let the common ratio between a and b be x.

$$\therefore a = 2x, \text{ and } b = 3x$$

XY is a straight line, rays OM and OP stand on it.

$$\therefore \angle XOM + \angle MOP + \angle POY = 180^\circ$$

$$b + a + \angle POY = 180^\circ$$

$$3x + 2x + 90^\circ = 180^\circ$$

$$5x = 90^\circ$$

$$x = 18^\circ$$

$$a = 2x = 2 \times 18 = 36^\circ$$

$$b = 3x = 3 \times 18 = 54^\circ$$

MN is a straight line. Ray OX stands on it.

$$\therefore b + c = 180^\circ \text{ (Linear Pair)}$$

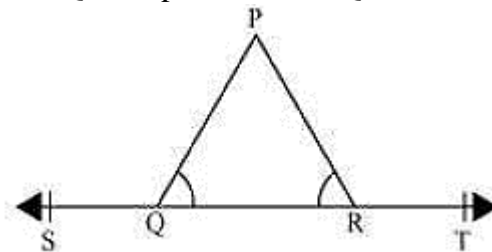
$$54^\circ + c = 180^\circ$$

$$c = 180^\circ - 54^\circ = 126^\circ$$

$$\therefore c = 126^\circ$$

Question 3:

In the given figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Solution 3:

In the given figure, ST is a straight line and ray QP stands on it.

$$\therefore \angle PQS + \angle PQR = 180^\circ \text{ (Linear Pair)}$$

$$\angle PQR = 180^\circ - \angle PQS \quad \dots (1)$$

$$\angle PRT + \angle PRQ = 180^\circ \text{ (Linear Pair)}$$

$$\angle PRQ = 180^\circ - \angle PRT \quad \dots (2)$$

It is given that $\angle PQR = \angle PRQ$.

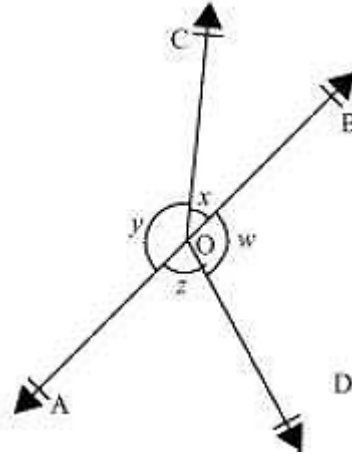
Equating Equations (1) and (2), we obtain

$$180^\circ - \angle PQS = 180^\circ - \angle PRT$$

$$\angle PQS = \angle PRT \text{ (Proved)}$$

Question 4:

In the given figure, if $x + y = w + z$ then prove that AOB is a line.



Solution 4:

It can be observed that,

$$x + y + z + w = 360^\circ \text{ (Complete angle)}$$

It is given that,

$$x + y = z + w$$

$$\therefore x + y + x + y = 360^\circ$$

$$2(x + y) = 360^\circ$$

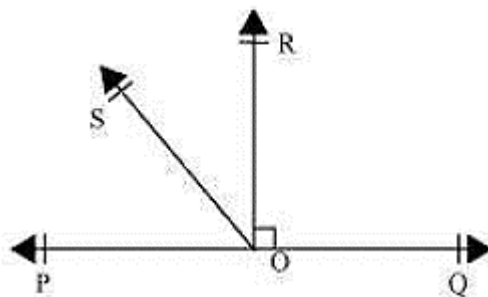
$$x + y = 180^\circ$$

Since x and y form a linear pair, therefore, AOB is a line. (Proved)

Question 5:

In the given figure, POQ is a line. Ray OR is perpendicular to line PQ . OS is another ray lying between rays OP and OR . Prove that

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$



Solution 5:

It is given that $OR \perp PQ$

$$\angle POR = 90^\circ$$

$$\angle POS + \angle SOR = 90^\circ$$

$$\angle ROS = 90^\circ - \angle POS \dots (1)$$

$$\angle QOR = 90^\circ \text{ (As } OR \perp PQ)$$

$$\angle QOS - \angle ROS = 90^\circ$$

$$\angle ROS = \angle QOS - 90^\circ \dots (2)$$

On adding Equations (1) and (2), we obtain

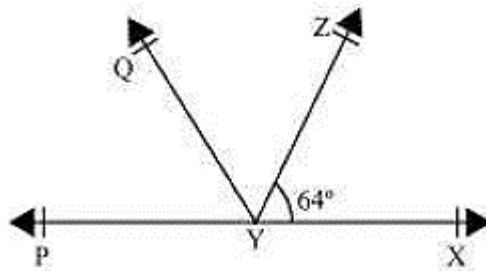
$$2 \angle ROS = \angle QOS - \angle POS$$

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

Question 6:

It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Solution 6:



It is given that line YQ bisects $\angle PYZ$.

Hence, $\angle QYP = \angle ZYQ$

It can be observed that PX is a line. Rays YQ and YZ stand on it.

$$\angle XYZ + \angle ZYQ + \angle QYP = 180^\circ$$

$$64^\circ + 2\angle QYP = 180^\circ$$

$$2\angle QYP = 180^\circ - 64^\circ = 116^\circ$$

$$\angle QYP = 58^\circ$$

Also, $\angle ZYQ = \angle QYP = 58^\circ$

$$\text{Reflex } \angle QYP = 360^\circ - 58^\circ = 302^\circ$$

$$\angle XYQ = \angle XYZ + \angle ZYQ$$

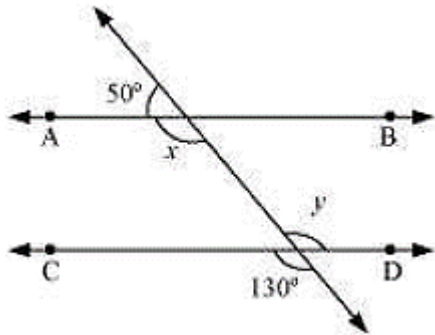
$$= 64^\circ + 58^\circ = 122^\circ$$

Hence, $\angle XYQ = 122^\circ$, Reflex $\angle QYP = 302^\circ$

Exercise (6.2)

Question 1:

In the given figure, find the values of x and y and then show that $AB \parallel CD$.



Solution 1:

It can be observed that,

$$50^\circ + x = 180^\circ \text{ (Linear pair)}$$

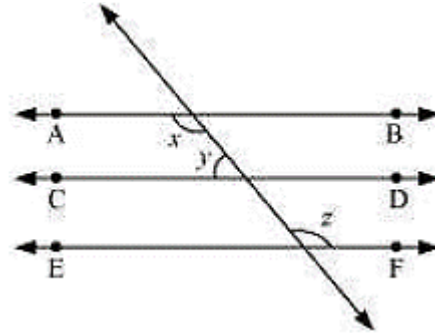
$$x = 130^\circ \quad \dots (1)$$

Also, $y = 130^\circ$ (Vertically opposite angles)

As x and y are alternate interior angles for lines AB and CD and also measures of these angles are equal to each other, therefore, line $AB \parallel CD$.

Question 2:

In the given figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x .



Solution 2:

It is given that $AB \parallel CD$ and $CD \parallel EF$

$\therefore AB \parallel CD \parallel EF$ (Lines parallel to the same line are parallel to each other)

It can be observed that

$$x = z \text{ (Alternate interior angles)} \quad \dots (1)$$

It is given that $y : z = 3 : 7$

Let the common ratio between y and z be a .

$$\therefore y = 3a \text{ and } z = 7a$$

Also, $x + y = 180^\circ$ (Co-interior angles on the same side of the transversal)

$$z + y = 180^\circ \text{ [Using Equation (1)]}$$

$$7a + 3a = 180^\circ$$

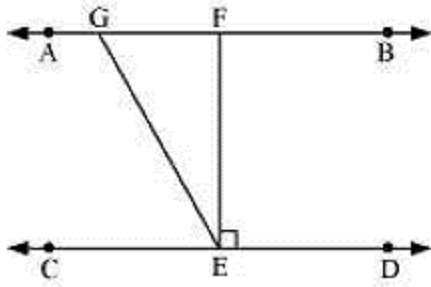
$$10a = 180^\circ$$

$$a = 18^\circ$$

$$\therefore x = 7a = 7 \times 18^\circ = 126^\circ$$

Question 3:

In the given figure, If $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126^\circ$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.



Solution 3:

It is given that,

$AB \parallel CD$ and $EF \perp CD$

$$\angle GED = 126^\circ$$

$$\angle GEF + \angle FED = 126^\circ$$

$$\angle GEF + 90^\circ = 126^\circ$$

$$\angle GEF = 36^\circ$$

As $\angle AGE$ and $\angle GED$ are alternate interior angles.

$$\angle AGE = \angle GED = 126^\circ$$

However, $\angle AGE + \angle FGE = 180^\circ$ (Linear pair)

$$126^\circ + \angle FGE = 180^\circ$$

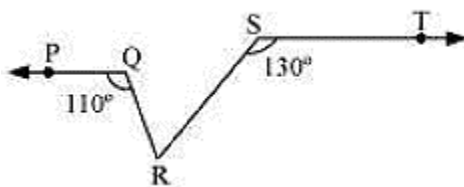
$$\angle FGE = 180^\circ - 126^\circ = 54^\circ$$

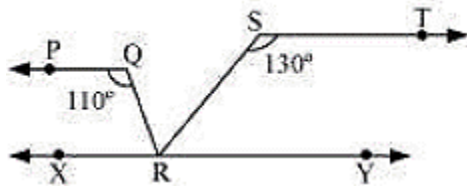
Hence, $\angle AGE = 126^\circ$, $\angle GEF = 36^\circ$, $\angle FGE = 54^\circ$

Question 4:

In the given figure, if $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, find $\angle QRS$.

[Hint: Draw a line parallel to ST through point R .]



Solution 4:

Let us draw a line XY parallel to ST and passing through point R.

$\angle PQR + \angle QRX = 180^\circ$ (Co-interior angles on the same side of transversal QR)

$$110^\circ + \angle QRX = 180^\circ$$

$$\angle QRX = 70^\circ$$

Also,

$\angle RST + \angle SRY = 180^\circ$ (Co-interior angles on the same side of transversal SR)

$$130^\circ + \angle SRY = 180^\circ$$

$$\angle SRY = 50^\circ$$

XY is a straight line. RQ and RS stand on it.

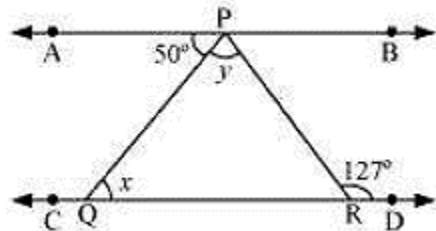
$$\angle QRX + \angle QRS + \angle SRY = 180^\circ$$

$$70^\circ + \angle QRS + 50^\circ = 180^\circ$$

$$\angle QRS = 180^\circ - 120^\circ = 60^\circ$$

Question 5:

In the given figure, if $AB \parallel CD$, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y.

**Solution 5:**

$\angle APR = \angle PRD$ (Alternate interior angles)

$$50^\circ + y = 127^\circ$$

$$y = 127^\circ - 50^\circ$$

$$y = 77^\circ$$

Also,

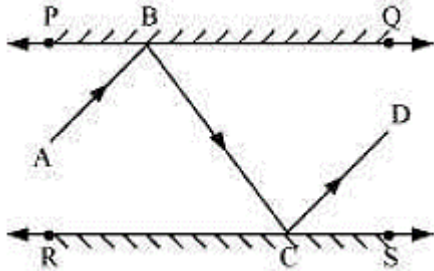
$\angle APQ = \angle PQR$ (Alternate interior angles)

$$50^\circ = x$$

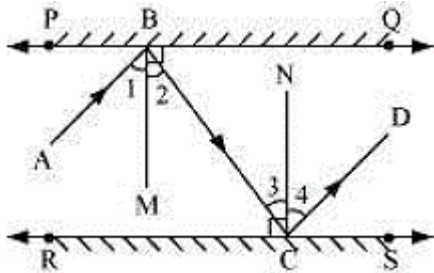
Therefore, $x = 50^\circ$ and $y = 77^\circ$

Question 6:

In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.



Solution 6:



Let us draw $BM \perp PQ$ and $CN \perp RS$.

As $PQ \parallel RS$,

Therefore, $BM \parallel CN$

Thus, BM and CN are two parallel lines and a transversal line BC cuts them at B and C respectively.

$\angle 2 = \angle 3$ (Alternate interior angles)

However, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ (By laws of reflection)

$\angle 1 = \angle 2 = \angle 3 = \angle 4$

Also,

$\angle 1 + \angle 2 = \angle 3 + \angle 4$

$\angle ABC = \angle DCB$

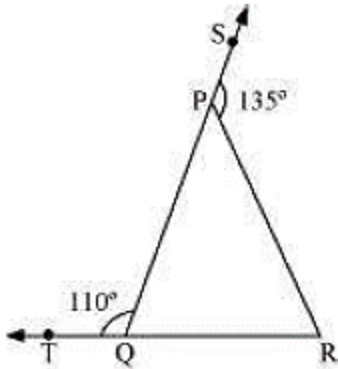
However, these are alternate interior angles.

$\therefore AB \parallel CD$

Exercise (6.3)

Question 1:

In the given figure, sides QP and RQ of $\triangle PQR$ are produced to points S and T respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$, find $\angle PRQ$.



Solution 1:

It is given that,

$$\angle SPR = 135^\circ \text{ and } \angle PQT = 110^\circ$$

$$\angle SPR + \angle QPR = 180^\circ \text{ (Linear pair angles)}$$

$$135^\circ + \angle QPR = 180^\circ$$

$$\angle QPR = 45^\circ$$

Also, $\angle PQT + \angle PQR = 180^\circ$ (Linear pair angles)

$$110^\circ + \angle PQR = 180^\circ$$

$$\angle PQR = 70^\circ$$

As the sum of all interior angles of a triangle is 180° , therefore, for $\triangle PQR$,

$$\angle QPR + \angle PQR + \angle PRQ = 180^\circ$$

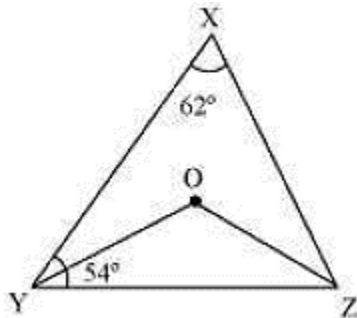
$$45^\circ + 70^\circ + \angle PRQ = 180^\circ$$

$$\angle PRQ = 180^\circ - 115^\circ$$

$$\angle PRQ = 65^\circ$$

Question 2:

In the given figure, $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of $\triangle XYZ$, find $\angle OZY$ and $\angle YOZ$.



Solution 2:

As the sum of all interior angles of a triangle is 180° , therefore, for $\triangle XYZ$,

$$\angle X + \angle XYZ + \angle XZY = 180^\circ$$

$$62^\circ + 54^\circ + \angle XZY = 180^\circ$$

$$\angle XZY = 180^\circ - 116^\circ$$

$$\angle XZY = 64^\circ$$

$$\angle OZY = \frac{64}{2} = 32^\circ \text{ (OZ is the angle bisector of } \angle XZY)$$

$$\text{Similarly, } \angle OYZ = \frac{54}{2} = 27^\circ$$

Using angle sum property for $\triangle OYZ$, we obtain

$$\angle OYZ + \angle YOZ + \angle OZY = 180^\circ$$

$$27^\circ + \angle YOZ + 32^\circ = 180^\circ$$

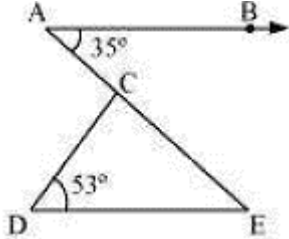
$$\angle YOZ = 180^\circ - 59^\circ$$

$$\angle YOZ = 121^\circ$$

Hence, $\angle OZY = 32^\circ$ and $\angle YOZ = 121^\circ$

Question 3:

In the given figure, if $AB \parallel DE$, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, find $\angle DCE$.

**Solution 3:**

$AB \parallel DE$ and AE is a transversal.

$$\angle BAC = \angle CED \text{ (Alternate interior angles)}$$

$$\angle CED = 35^\circ$$

In $\triangle CDE$,

$$\angle CDE + \angle CED + \angle DCE = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$53^\circ + 35^\circ + \angle DCE = 180^\circ$$

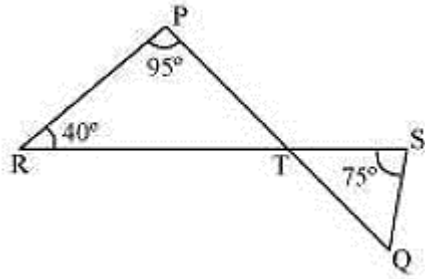
$$\angle DCE = 180^\circ - 88^\circ$$

$$\angle DCE = 92^\circ$$

Question 4:

In the given figure, if lines PQ and RS intersect at point T , such that $\angle PRT = 40^\circ$,

$\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$, find $\angle SQT$.



Solution 4:

Using angle sum property for $\triangle PRT$, we obtain

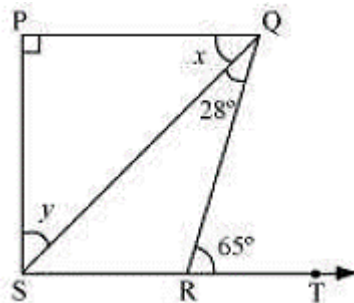
$$\begin{aligned} \angle PRT + \angle RPT + \angle PTR &= 180^\circ \\ 40^\circ + 95^\circ + \angle PTR &= 180^\circ \\ \angle PTR &= 180^\circ - 135^\circ \\ \angle PTR &= 45^\circ \\ \angle STQ &= \angle PTR = 45^\circ \text{ (Vertically opposite angles)} \\ \angle STQ &= 45^\circ \end{aligned}$$

By using angle sum property for $\triangle STQ$, we obtain

$$\begin{aligned} \angle STQ + \angle SQT + \angle QST &= 180^\circ \\ 45^\circ + \angle SQT + 75^\circ &= 180^\circ \\ \angle SQT &= 180^\circ - 120^\circ \\ \angle SQT &= 60^\circ \end{aligned}$$

Question 5:

In the given figure, if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y .



Solution 5:

It is given that $PQ \parallel SR$ and QR is a transversal line.

$$\begin{aligned} \angle PQR &= \angle QRT \text{ (Alternate interior angles)} \\ x + 28^\circ &= 65^\circ \end{aligned}$$

$$x = 65^\circ - 28^\circ$$

$$x = 37^\circ$$

By using the angle sum property for $\triangle SPQ$, we obtain

$$\angle SPQ + x + y = 180^\circ$$

$$90^\circ + 37^\circ + y = 180^\circ$$

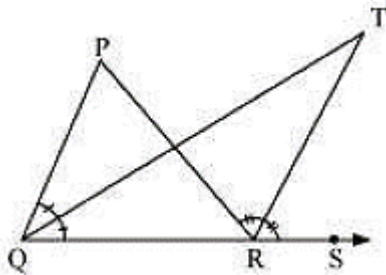
$$y = 180^\circ - 127^\circ$$

$$y = 53^\circ$$

$$x = 37^\circ \text{ and } y = 53^\circ$$

Question 6:

In the given figure, the side QR of $\triangle PQR$ is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2} \angle QPR$.



Solution 6:

In $\triangle QTR$, $\angle TRS$ is an exterior angle.

$$\therefore \angle QTR + \angle TQR = \angle TRS$$

$$\angle QTR = \angle TRS - \angle TQR \quad \dots (1)$$

For $\triangle PQR$, $\angle PRS$ is an external angle.

$$\therefore \angle QPR + \angle PQR = \angle PRS$$

$$\angle QPR + 2\angle TQR = 2\angle TRS \text{ (As QT and RT are angle bisectors)}$$

$$\angle QPR = 2(\angle TRS - \angle TQR)$$

$$\angle QPR = 2\angle QTR \text{ [By using Equation (1)]}$$

$$\angle QTR = \frac{1}{2} \angle QPR$$