Exercise 10.1: Circles

1. Fill in the blanks:

(i) The common point of tangent and the circle is called _________.

Soln: point of contact.

(ii) A circle may have _____ parallel tangent.

Soln : two

(iii) A tangent to a circle intersects it in ______ point.

Soln: one

(iv) A line intersecting a circle in two points is called a _______

Soln: secant.

(v) The angle between tangent at a point P on circle and radius through the point is _______

Soln: 90°.

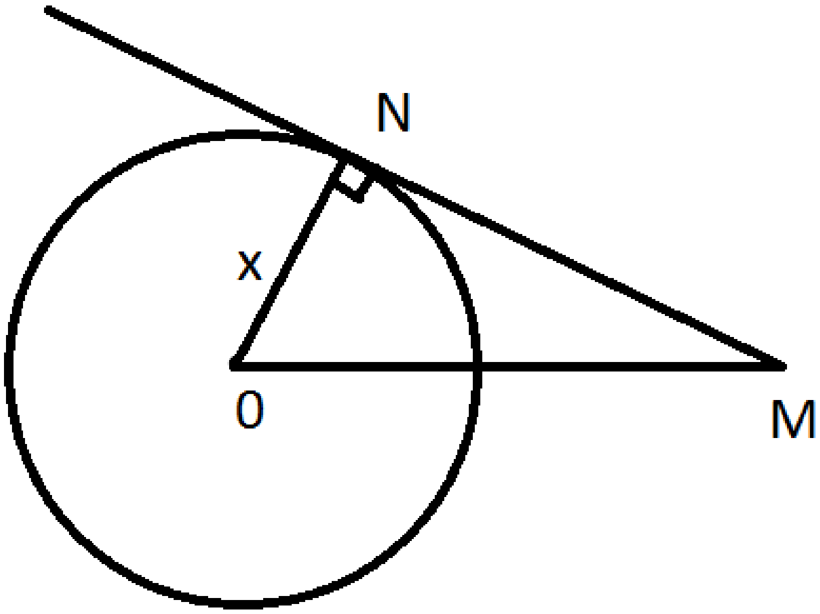
Q 2. How many tangents can a circle have?

Ans:

Tangent: a line intersecting circle in one point is called a tangent

As there are infinite number of points on the circle , a circle has many (infinite) tangents.

Q 3. 'O' is the centre the circle shown below with a radius of 8 cm. The circle cuts the tangent AB through O at B such that AB = 15 cm. Find OB.



Ans:

Given data : AB = 15 cm

OA = 8 cm (radius of the circle)

We know that : the tangent cuts the circle at 90 degrees. Therefore, OA is the hypotenuse of the triangle OAB . Hence, the longest side can be found by using pythagoras Theorem.

We have,

OB = 17 cm

Therefore, OB = 17 cm

Q 4. If the tangent at point P to the circle with centre O cuts a line through O at Q such that PQ = 24 cm and OQ = 25 cm . find the radius of the circle.

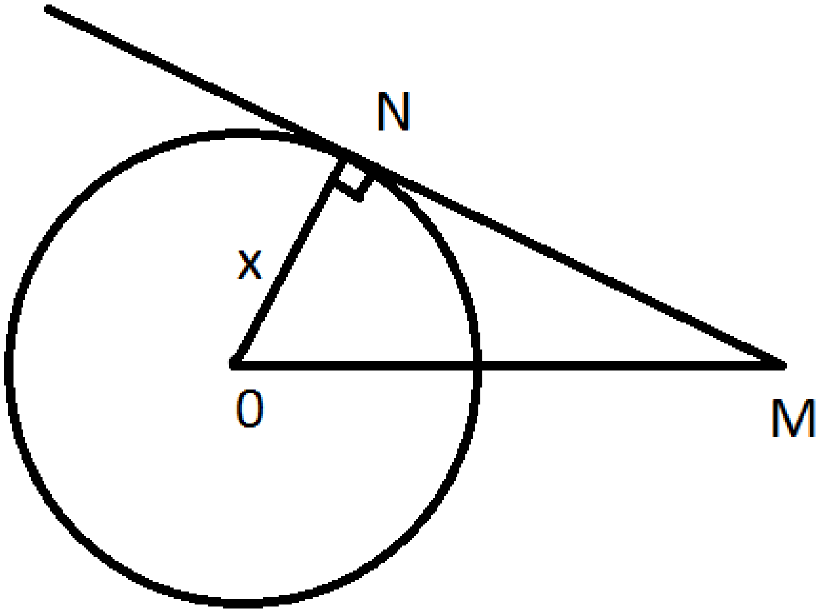
Ans :

given data:

PQ = 24 cm

OQ = 25 cm

OP = radius = ?



P is a point of contact , at point of contact , tangent and radius are perpendicular to each other.

Therefore triangle is right angled triangle angle OPQ = 90°

BY pythagoras theorem,

OP = 7 cm

Therefore , OP = radius = 7 cm

Exercise 10.2: Circles

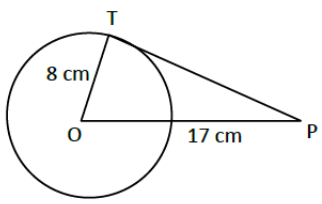
1. If PT is a tangent at T to a circle whose center is O and OP = 17 cm, OT = 8 cm. Find the length of tangent segment PT.

Sol:

OT = radius = 8cm

OP = 17cm

PT = length of tangent = ?



T is point of contact. We know that at point of contact tangent and radius are perpendicular.

∴ OTP is right angled triangle ∠OTP = 90°, from Pythagoras theorem $OT^2 + PT^2 = OP^2$

$8^2 + PT^2 = 17^2$

PT $\sqrt{17^2 - 8^2} = \sqrt{289 - 64}$

$= \sqrt{225} = 15cm$

∴ PT = length of tangent = 15 cm.

2. Find the length of a tangent drawn to a circle with radius 5cm, from a point 13 cm from the center of the circle.

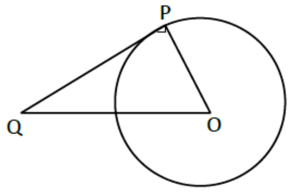
Sol:

Consider a circle with center O.

OP = radius = 5 cm.

A tangent is drawn at point P, such that line through O intersects it at Q, OB = 13cm.

Length of tangent PQ = ?



A + P, we know that tangent and radius are perpendicular.

ΔOPQ is right angled triangle, ∠OPQ = 90°

$By\ pythagoras\ theorem, OQ^2 = OP^2 + PQ^2$

$\Rightarrow 13^2 = 5^2 + PQ^2$

$\Rightarrow PQ^2 = 169 - 25 = 144$

⇒ PQ = $\sqrt{144} = 12cm$

Length of tangent = 12 cm

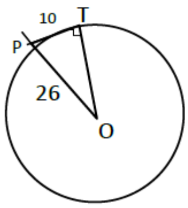
3. A point P is 26 cm away from O of circle and the length PT of the tangent drawn from P to the circle is 10 cm. Find the radius of the circle.

Sol:

Given OP = 26 cm

PT = length of tangent = 10cm

radius = OT = ?



At point of contact, radius and tangent are perpendicular ∠OTP = 90°, ΔOTP is right angled triangle.

By Pythagoras theorem, $OP^2 = OT^2 + PT^2$

$26^2 = OT^2 + 10^2$

$OT^k = \left(\sqrt{676 - 100}\right)^k$

$OT = \sqrt{576}$

= 24 cm

OT = length of tangent = 24 cm

4. If from any point on the common chord of two intersecting circles, tangents be drawn to circles, prove that they are equal.

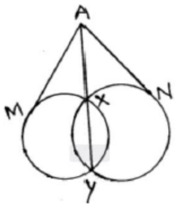
Sol:

Let the two circles intersect at points X and Y.

XY is the common chord.

Suppose 'A' is a point on the common chord and AM and AN be the tangents drawn A to the circle

We need to show that AM = AN.



In order to prove the above relation, following property will be used.

"Let PT be a tangent to the circle from an external point P and a secant to the circle through P intersects the circle at points A and B, then $PT^2 = PA \times PB$"

Now AM is the tangent and AXY is a secant $\therefore AM^2 = AX \times AY \ldots..(i)$

AN is a tangent and AXY is a secant $\therefore AN^2 = AX \times AY \ldots..(ii)$

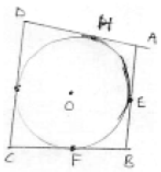
From (i) & (ii), we have $AM^2 = AN^2$

∴ AM = AN

5. If the quadrilateral sides touch the circle prove that sum of pair of opposite sides is equal to the sum of other pair.

Sol:

Consider a quadrilateral ABCD touching circle with center O at points E, F, G and H as in figure.



We know that

The tangents drawn from same external points to the circle are equal in length.

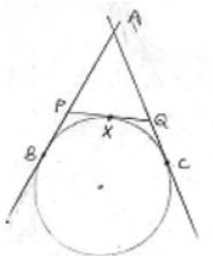
1. Consider tangents from point A [AM ⊥ AE]
   AH = AE …. (i)
2. From point B [EB & BF]
   BF = EB …. (ii)
3. From point C [CF & GC]
   FC = CG …. (iii)
4. From point D [DG & DH]
   DH = DG …. (iv)
   Adding (i), (ii), (iii), & (iv)
   (AH + BF + FC + DH) = [(AC + CB) + (CG + DG)]
   ⇒ (AH + DH) + (BF + FC) = (AE + EB) + (CG + DG)
   ⇒ AD + BC = AB + DC [from fig.]
   Sum of one pair of opposite sides is equal to other.

6. If AB, AC, PQ are tangents in Fig. and AB = 5cm find the perimeter of ΔAPQ.

Sol:

Perimeter of ΔAPQ, (P) = AP + AQ + PQ

= AP + AQ + (PX + QX)



We know that

The two tangents drawn from external point to the circle are equal in length from point A,

AB = AC = 5 cm

From point P, PX = PB

From point Q, QX = QC

Perimeter (P) = AP + AQ + (PB + QC)

= (AP + PB) + (AQ + QC)

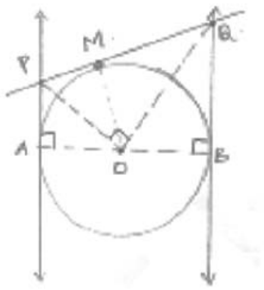
= AB + AC = 5 + 5

= 10 cms.

7. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at center.

Sol:

Consider circle with center 'O' and has two parallel tangents through A & B at ends of diameter.



Let tangents through M intersects the tangents parallel at P and Q required to prove is that $\angle POQ = 90°$.

From fig. it is clear that ABQP is a quadrilateral

$\angle A + \angle B = 90° + 90° = 180°$ [At point of contact tangent & radius are perpendicular]

$\angle A + \angle B + \angle P + \angle Q = 360°$ [Angle sum property]

$\angle P + \angle Q = 360° - 180° = 180°$ …..(i)

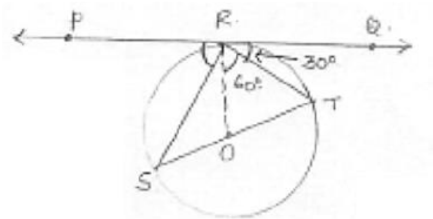
At P & Q $\angle APO = \angle OPQ = \frac{1}{2}\angle P$

$\angle BQO = \angle PQO = \frac{1}{2}\angle Q$ in (i)

$2\angle OPQ + 2\angle PQO = 180°$

$\angle OPQ + \angle PQO = 90°$ .... (ii)
In ΔOPQ, $\angle OPQ + \angle PQO + \angle POQ = 180°$ [Angle sum property]
$90° + \angle POQ = 180°$ [from (ii)]
$\angle POQ = 180° - 90° = 90°$
$\therefore \angle POQ = 90°$

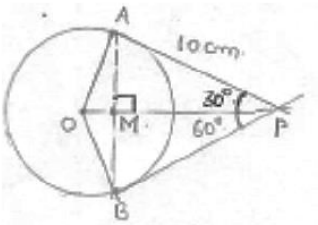
8. In Fig below, PQ is tangent at point R of the circle with center O. If $\angle TRQ = 30°$. Find $\angle PRS$.



Sol:
Given $\angle TRQ = 30°$.
At point R, $OR \perp RQ$.
$\angle ORQ = 90°$
$\Rightarrow \angle TRQ + \angle ORT = 90°$
$\Rightarrow \angle ORT = 90° - 30° = 60°$
ST is diameter, $\angle SRT = 90°$ [$\because$ Angle in semicircle = 90°]
$\angle ORT + \angle SRO = 90°$

$\angle SRO + \angle PRS = 90°$
$\angle PRS = 90° - 30° = 60°$

9. If PA and PB are tangents from an outside point P. such that PA = 10 cm and $\angle APB = 60°$. Find the length of chord AB.
Sol:
AP = 10 cm $\angle APB = 60°$
Represented in the figure
We know that



A line drawn from center to point from where external tangents are drawn divides or bisects the angle made by tangents at that point $\angle APO = \angle OPB = \frac{1}{2} \times 60° = 30°$

The chord AB will be bisected perpendicularly

$\therefore$ AB = 2AM

In ΔAMP,

$$\sin 30^\circ = \frac{opp.side}{hypotenuse} = \frac{AM}{AP}$$

$$AM = AP \sin 30^\circ$$

$$= \frac{AP}{2} = \frac{10}{2} = 5cm$$

AP = 2 AM = 10 cm ---- Method (i)

In ΔAMP, $\angle$AMP = 90°, $\angle$APM = 30°

$$\angle AMP + \angle APM + \angle MAP = 180^\circ$$

$$90^\circ + 30^\circ + \angle MAP = 180^\circ$$

$$\angle MAP = 180^\circ$$

In ΔPAB, $\angle$MAP = $\angle$BAP = 60°, $\angle$APB = 60°

We also get, $\angle$PBA = 60°

$\therefore\Delta$PAB is equilateral triangle

AB = AP = 10 cm. -----Method (ii)