Ex 24.3

Q1

We will use equation in diameter form.

$(x-2)(x+2)+(y+3)(y-4)=0$

$x^2-4+y^2-y-12=0$

$x^2+y^2-y-16=0$..........(1)

2g=0, 2f=-1 ....[From (1)]

$g=0, f=-\frac{1}{2}$

Centre of the circle is $(0, \frac{1}{2})$

$r=\sqrt{0+\frac{1}{4}+16}$

$r=\frac{\sqrt{65}}{2}$

Q2

Centre of the circles

$x^2+y^2+6x-14y-1=0$

is $(-3,7)$

and $x^2+y^2-4x+10y-2=0$

is $(2,-5)$

Equation of the circle is

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$\Rightarrow (x+3)(x-2)+(y-7)(y+5)=0$

$\Rightarrow x^2+3x-2x-6+y^2-7y+5y-35=0$

$\Rightarrow x^2+y^2+x-2y-41=0$

Q3

Let the sides AB, BC, CD and DA of the square $ABCD$ be represented by the equations
$y = 3$, $x = 6$, $y = 6$ and $x = 9$ respectively.

Then, coordinates are
$A(6,3)$, $B(9,3)$, $C(9,6)$ and $D(6,6)$.

The equation of the circle with diagonal AC

$$(x-6)(x-9)+(4-3)(4-6)=0$$
$$\Rightarrow \quad x^2-6x-9x+54+y^2-3y-6y+18=0$$
$$\Rightarrow \quad x^2+y^2-15x-9y+72=0$$

The equation of the circle with diagonal BD as diameter is

$$(x-9)(x-6)+(y-3)(y-6)=0$$
$$\Rightarrow \quad x^2-9x-6x+54+y^2-3y-6y+18=0$$
$$\Rightarrow \quad x^2+y^2-15x-9y+72=0$$

$$x^2+y^2-15x-9y+72=0$$

Q4

The given ecuation are

$$x-3y=4 \ldots\ldots\ldots(i)$$
$$3x+y=22 \ldots\ldots(ii)$$
$$x-3y=14 \ldots\ldots(iii)$$
$$3x+y=62 \ldots\ldots(iv)$$

Let A, B, C & D are the points of intersection of the lines (i)&(ii), (i)&(iii), (iii)&(iv) and (iv)&(i)

$\therefore \quad A=(7,1)$, $B=(8,-2)$, $C=(20,2)$ & $D=(19,5)$

AC will be the diameter of the circle

so,

the equation of circle is

$$(x-7)(x-20)-(y-1)(y-2)=0$$

$$\Rightarrow \quad x^2+y^2-27x-3y+142=0$$

Q5

The line $3x + 4y = 12$
will meet the axis at $A(0, 3)$ & $B(4, 0)$

Since the circle passes through origin & A and B
$\therefore$ AB is a diameter

Thus the equation of circle is

$$(x - 0)(x - 4) + (y - 3)(y - 0) = 0$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 3y = 0$$

Q6

Since the circle is passes through origin and cut intercept a and b on $x - axis$ & $y - axis$

$\therefore$ Coordinate of circle $A = (0, b)$ and $B = (a, 0)$
AB is the diameter of circle

$\therefore$ The equation of circle is

$$(x - a)(x - 0) + (y - 0)(y - b) = 0$$

$$\Rightarrow \quad x^2 + y^2 \pm ax \pm by = 0$$

Q7

Equation of circle in diameter form is,

(x+4) (x-12) + (y-3) (y+1) =0

$x^2-8x-48+y^2-2y-3=0$

$x^2-8x-2y+y^2-51=0$................(1)

To find y-intercept, put x=0 in (1),

$y^2-2y-51=0$

$$y=\frac{2\pm\sqrt{4+204}}{2}$$

$$y=\frac{2\pm\sqrt{208}}{2}$$

y intercepts are $1\pm4\sqrt{13}$

Q8

The given equations are

$x^2+2ax-b^2=0$....... (i)

$x^2+2px-q^2=0$...... (ii)

We have, the roots of (i) will give the absissae and the roots of (ii) will give the ordinate of $A\&B$ respectively.

Now roote of (i)

$$x=\frac{-2a\pm\sqrt{4a^2+4b^2}}{2}=-9\pm\sqrt{a^2+b^2}$$

Roots of (ii)

$$x=\frac{-2P\pm\sqrt{4P^2+4q^2}}{2}=-P\pm\sqrt{P^2+q^2}$$

Coordinates of $A=\left(-a+\sqrt{a^2+b^2},-P\sqrt{P^2+q^2}\right)$

$B=\left(-a-\sqrt{a^2+b^2},-P-\sqrt{P^2+q^2}\right)$

so, the equation of circle is

$$\left(x+a-\sqrt{a^2+b^2}\right)\left(x+a+\sqrt{a^2+b^2}\right)+\left(y+P-\sqrt{P^2+q^2}\right)\left(y+P+\sqrt{P^2+q^2}\right)=0$$

$\Rightarrow \quad x^2+y^2+2ax+2Py-\left(a^2+b^2+P^2+q^2\right)+a^2+P^2=0$

$\Rightarrow \quad x^2+y^2+2ax+2Py-\left(b^2+q^2\right)=0$

The radius is

$$r=\sqrt{g^2+f^2-c}$$

$$=\sqrt{a^2+p^2-\left(b^2+q^2\right)}$$

$$=\sqrt{a^2+b^2+p^2+q^2}$$

Q9

Here AB and AD are taken as x – axis and y – axis respectively.

Since $ABCD$ is a square thus the coordinate of

$A = (0, 0)$

$B = (a, 0)$ C=(a,a)

$D = (0, a)$

$\because$ BD is a diameter

so, the equation of circle is

$(x - a)(x - 0) + (y - 0)(y - a) = 0$

$\Rightarrow \quad x^2 + y^2 - ax - ay = 0$

so,

$x^2 + y^2 - a(x + y) = 0$

Q10

The given equation of line & circle in

$2x - y + 6 = 0 \ldots\ldots\ldots (i)$

$x^2 + y^2 - 2y - 9 = 0 \ldots\ldots (ii)$

The point of intersection of $(i) \& (ii)$ is

$x^2 + (2x + 6)^2 - 2(2x + 6) - 9 = 0$

$\Rightarrow \quad x^2 + 4x^2 + 24x + 36 - 4x - 12 - 9 = 0$

$\Rightarrow \quad 5x^2 + 20x + 15 = 0$

$\Rightarrow \quad x^2 + 4x + 3 = 0$

$\Rightarrow \quad (x + 3)(x + 1) = 0 \quad \Rightarrow \quad x = (-3, -1)$

$\therefore \quad y = (0, 4)$

$\therefore \quad A = (-3, 0) \& \quad B = (-1, 4)$

$\because$ AB is a diameter, so the equation of circle is

$(x + 3)(x + 1) + (y - 0)(y - 4) = 0$

$\Rightarrow \quad x^2 + y^2 + 4x - 4y + 3 = 0$

Q11

The triangle is formed by

$x = 0 \ldots\ldots\ldots (i)$

$y = 0 \ldots\ldots\ldots (ii) \&$

$lx + my = 1 \ldots\ldots\ldots (iii)$

The line (iii) cuts the axis at

$A = \left(0, \frac{1}{m}\right)$ and at $B = \left(\frac{1}{l}, 0\right)$

Now, AB will be the diameter of circle.

$\therefore$ equation of circle will be,

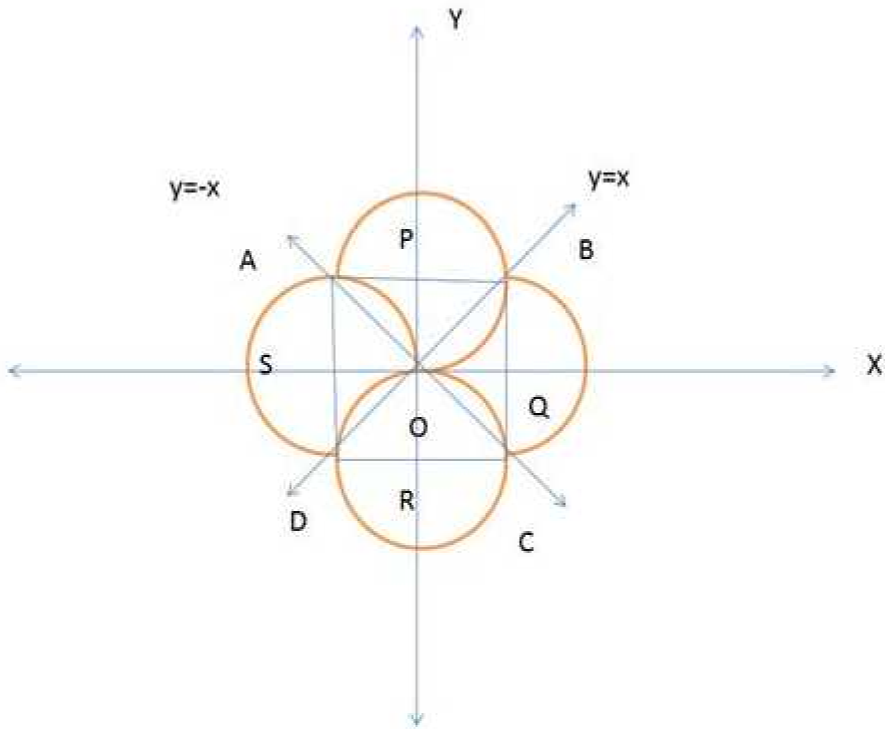
$$\left(x - \frac{1}{l}\right)(x - 0) + (y - 0)\left(y - \frac{1}{m}\right) = 0$$

$$\Rightarrow \quad x^2 + y^2 - \frac{x}{l} - \frac{y}{m} = 0$$

$$\Rightarrow \quad x^2 + y^2 - \frac{x}{l} - \frac{y}{m} = 0$$

Q12

Figure shows four such circles.



Angle between y=x and y=-x is $\frac{\pi}{2}$.

$\therefore$ Angle between OB and OA $=\frac{\pi}{2}$

Hence, AB, BC, CD and AD are diameters of circles.

$\angle BOQ=\frac{\pi}{4}$

$\text{Sin } \angle BOQ=\frac{BQ}{OB}$