

EX 30.1

Q1

We have,

$$f(x) = 3x$$

$$\therefore f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(a) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(2+h) - 6}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3h}{h}$$

$$= \lim_{h \rightarrow 0} 3$$

$$\therefore f'(2) = 3$$

Q2

We have,

$$f(x) = x^2 - 2$$

$$\therefore f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(10) = \lim_{h \rightarrow 0} \frac{f(10+h) - f(10)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(10+h)^2 - 2 - 98}{h}$$

$$= \lim_{h \rightarrow 0} \frac{100 + 20h + h^2 - 100}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(20+h)}{h}$$

$$= \lim_{h \rightarrow 0} (20+h)$$

$$\therefore f'(10) = 20$$

Q3

We have,

$$f(x) = 99x$$

$$\therefore f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\begin{aligned} f'(100) &= \lim_{h \rightarrow 0} \frac{f(100+h) - f(100)}{h} \\ &= \lim_{h \rightarrow 0} \frac{99(100+h) - 9900}{h} \\ &= \lim_{h \rightarrow 0} \frac{9900 + 99h - 9900}{h} \\ &= \lim_{h \rightarrow 0} 99 \end{aligned}$$

$$\therefore f'(100) = 99$$

Q4

We have,

$$f(x) = x$$

$$\therefore f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\begin{aligned} f'(1) &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(1+h) - 1}{h} \\ &= \lim_{h \rightarrow 0} 1 \end{aligned}$$

$$\therefore f'(1) = 1$$

Q5

We have,

$$f(x) = \cos x$$

$$\therefore f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cos(0+h) - \cos 0}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cos h - \cos 0}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(1 - \frac{h^2}{2!} + \frac{h^4}{4!} - \dots) - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} - \dots)}{h}$$

$$= \lim_{h \rightarrow 0} h(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} - \dots)$$

$$= 0$$

$$\therefore f'(0) = 0$$

$$\left[\because \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \right]$$

Q6

We have,

$$f(x) = \tan x$$

$$\therefore f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\tan h - \tan 0}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\tan h}{h}$$

$$= 1$$

$$\left[\because \lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1 \right]$$

$$\therefore f'(0) = 1$$

Q7(i)

We have,

$$f(x) = \sin x$$

$$\therefore f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'\left(\frac{\pi}{2}\right) = \lim_{h \rightarrow 0} \frac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{2}+h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cos h - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\left(1 - \frac{h^2}{2!} + \frac{h^4}{4!} - \dots\right) - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h\left(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} + \dots\right)}{h}$$

$$= \lim_{h \rightarrow 0} h\left(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} + \dots\right)$$

$$= 0$$

$$\therefore f'\left(\frac{\pi}{2}\right) = 0$$

$$\left[\because \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \right]$$

Q7(ii)

We have,

$$f(x) = x$$

$$\therefore f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1+h-1}{h}$$

$$= \lim_{h \rightarrow 0} 1$$

$$\therefore f'(1) = 1$$

Q7(iii)

We have,

$$\therefore f(x) = 2 \cos x$$

$$\begin{aligned}\therefore f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2 \cos\left(\frac{\pi}{2} + h\right) - 2 \cos \frac{\pi}{2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2 \sin h - 0}{h} \\ &= -2\end{aligned}$$

$$\left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$\therefore f'\left(\frac{\pi}{2}\right) = -2$$

Q7(iv)

We have, $f(x) = \sin 2x$

Therefore,

$$\begin{aligned}f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin 2\left(\frac{\pi}{2} + h\right) - \sin 2\left(\frac{\pi}{2}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} \times 2 + 2h\right) - \sin(\pi)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-\cos 2h - 0}{h} \\ &= -2\end{aligned}$$

$$\text{Therefore } f'\left(\frac{\pi}{2}\right) = -2$$

Ex 30.2

Q1(i)

We have,

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{f\left(\frac{2}{x+h}\right) - f\left(\frac{2}{x}\right)}{h} \\&= \lim_{h \rightarrow 0} \frac{\left(\frac{2x - 2x - 2h}{hx(x+h)}\right)}{h} \\&= \lim_{h \rightarrow 0} \frac{2(x - x - h)}{hx(x+h)} \\&= \lim_{h \rightarrow 0} \frac{-2h}{hx(x+h)} \\&= \lim_{h \rightarrow 0} \frac{-2}{x(x+h)} \\&= \frac{-2}{x^2}\end{aligned}$$

Q1(ii)

We have,

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \\&= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h} \\&= \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}h} \times \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \\&= \lim_{h \rightarrow 0} \frac{x - (x+h)}{\sqrt{x}\sqrt{x+h}h(\sqrt{x} + \sqrt{x+h})} \\&= \lim_{h \rightarrow 0} \frac{-h}{\sqrt{x}\sqrt{x+h}h(\sqrt{x} + \sqrt{x+h})} \\&= \lim_{h \rightarrow 0} \frac{-1}{x\sqrt{x+h} + \sqrt{x}(\sqrt{x+h})} \\&= \lim_{h \rightarrow 0} \frac{-1}{2x\sqrt{x}} \\&= \frac{-1}{2} x^{-\frac{3}{2}}\end{aligned}$$

Q1(iii)

We have,

$$f(x) = \frac{1}{x^3}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h} \\&= \lim_{h \rightarrow 0} \frac{x^3 - (x+h)^3}{x^3 h (x+h)^3} \\&= \lim_{h \rightarrow 0} \frac{x^3 - (x^3 + 3x^2h + 3xh^2 + h^3)}{x^3 h (x+h)^3} \\&= \lim_{h \rightarrow 0} \frac{x^3 - x^3 - 3x^2h - 3xh^2 - h^3}{x^3 (x+h)^3} \\&= \lim_{h \rightarrow 0} \frac{-3x^2h - 3xh^2 - h^3}{x^3 (x+h)^3} \\&= \frac{-3x^2}{x^6} \\&= \frac{-3}{x^4}\end{aligned}$$

Q1(iv)

We have,

$$f(x) = \frac{x^2 + 1}{x}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\frac{(x+h)^2 + 1}{(x+h)} - \frac{x^2 + 1}{x}}{h} \\&= \lim_{h \rightarrow 0} \frac{x[x^2 + h^2 + 2xh + 1] - (x^2 + 1)(x+h)}{hx(x+h)} \\&= \lim_{h \rightarrow 0} \frac{x^3 + xh^2 + 2x^2h + x - x^3 - x - x^2h - h}{hx(x+h)} \\&= \lim_{h \rightarrow 0} \frac{xh + 2x^2 - x^2 - 1}{x(x+h)} \\&= \frac{x^2 - 1}{x^2} \\&= 1 - \frac{1}{x^2}\end{aligned}$$

We have,

$$f(x) = \frac{x^2 + 1}{x}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\frac{(x+h)^2 + 1}{(x+h)} - \frac{x^2 + 1}{x}}{h} \\&= \lim_{h \rightarrow 0} \frac{x[x^2 + h^2 + 2xh + 1] - (x^2 + 1)(x+h)}{hx(x+h)} \\&= \lim_{h \rightarrow 0} \frac{x^3 + xh^2 + 2x^2h + x - x^3 - x - x^2h - h}{hx(x+h)} \\&= \lim_{h \rightarrow 0} \frac{xh + 2x^2 - x^2 - 1}{x(x+h)} \\&= \frac{x^2 - 1}{x^2} \\&= 1 - \frac{1}{x^2}\end{aligned}$$

Q1(v)

We have,

$$f(x) = \frac{x^2 - 1}{x}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\frac{(x+h)^2 - 1}{(x+h)} - \frac{x^2 - 1}{x}}{h} \\&= \lim_{h \rightarrow 0} \frac{x(x^2 + h^2 + 2xh - 1) - (x+h)(x^2 - 1)}{x(x+h)h} \\&= \lim_{h \rightarrow 0} \frac{xh + 2x^2 - x^2 + 1}{x(x+h)} \\&= \frac{x^2 + 1}{x^2} \\&= 1 + \frac{1}{x^2}\end{aligned}$$

Q1(vi)

We have,

$$f(x) = \frac{x+1}{x+2}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\frac{(x+h)+1}{(x+h)+2} - \frac{x+1}{x+2}}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+2)(x+h+1) - (x+1)(x+h+2)}{(x+h+2)(x+2)h} \\&= \lim_{h \rightarrow 0} \frac{(x^2 - 2x + xh + 2h + 2 + x) - (x^2 + xh + 2x + x + h + 2)}{(x+h+2)(x+2)h} \\&= \lim_{h \rightarrow 0} \frac{h}{(x+h+2)(x+2)h} \\&= \frac{1}{(x+2)^2}\end{aligned}$$

Q1(vii)

We have,

$$f(x) = \frac{x+2}{3x+5}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(x+h+2)}{3(x+h)+5} - \frac{x+2}{3x+5}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(3x+5)(x+h+2) - (x+2)(3x+3h+5)}{(3x+5)(3x+3h+5)h} \\ &= \lim_{h \rightarrow 0} \frac{(3x^2 + 5x + 3xh + 5h + 6x + 10) - (3x^2 + 3xh + 5x + 6x + 6h + 10)}{(3x+5)(3x+3h+5)h} \\ &= \lim_{h \rightarrow 0} \frac{-h}{(3x+5)(3x+3h+5)h} \\ &= \lim_{h \rightarrow 0} \frac{-1}{(3x+5)(3x+3h+5)} \\ &= \frac{-1}{(3x+5)^2}\end{aligned}$$

Q1(viii)

We have,

$$f(x) = kx^n$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{k(x+h)^n - kx^n}{h} \\ &= k \lim_{h \rightarrow 0} \frac{\left(x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \dots\right) - x^n}{h} \quad \left[\because (x+y)^n = x^n + nx^{n-1}y + \dots\right] \\ &= k \lim_{h \rightarrow 0} \left(nx^{n-1} + \frac{n(n-1)}{2!}x^{n-2}h + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^2 + \dots\right) \\ &= k nx^{n-1} + 0 + 0 + \dots \\ &= k nx^{n-1}\end{aligned}$$

Q1(ix)

We have,

$$f(x) = \frac{1}{\sqrt{3-x}}$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{3-(x+h)}} - \frac{1}{\sqrt{3-x}}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{3-x} - \sqrt{3-(x+h)}}{\sqrt{3-x} \sqrt{3-(x+h)} \times h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{3-x} - \sqrt{3-(x+h)}}{\sqrt{3-x} \sqrt{3-(x+h)} \times h} \times \frac{\sqrt{3-x} + \sqrt{3-(x+h)}}{\sqrt{3-x} + \sqrt{3-(x+h)}} \quad [\text{Rationalising the numerator by } \sqrt{3-x} + \sqrt{3-(x+h)}] \\ &= \lim_{h \rightarrow 0} \frac{(3-x) - (3-(x+h))}{\sqrt{3-x} \sqrt{3-(x+h)} \times h (\sqrt{3-x} + \sqrt{3-(x+h)})} \\ &= \lim_{h \rightarrow 0} \frac{h}{\sqrt{3-x} \sqrt{3-(x+h)} \times h (\sqrt{3-x} + \sqrt{3-(x+h)})} \\ &= \frac{1}{(3-x) \times 2\sqrt{3-x}} \\ &= \frac{1}{2(3-x)^{\frac{3}{2}}} \end{aligned}$$

Q1(x)

We have,

$$f(x) = x^2 + x + 3$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{(x+h)^2 + (x+h) + 3\} - x^2 - x - 3}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + h^2 + 2xh + x + h + 3 - x^2 - x - 3}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2 + h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2x + h + 1)}{h} \\ &= 2x + 0 + 1 \\ &= 2x + 1 \end{aligned}$$

Q1(xi)

We have,

$$f(x) = (x+2)^3$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h+2)^3 - (x+2)^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{(x+2)+h\}^3 - (x+2)^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+2)^3 + h^3 + 3h(x+2)^2 + 3(x+2)h^2 - (x+2)^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x+2)^2 + 3(x+2)h + h^2}{h} \\ &= 3(x+2)^2 \end{aligned}$$

Q1(xii)

We have,

$$f(x) = x^3 + 4x^2 + 3x + 2$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 + 4(x+h)^2 + 3(x+h) + 2 - (x^3 + 4x^2 + 3x + 2)}{h} \end{aligned}$$

On solving we get,

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{x^3 + h^3 + 3x^2h + 3h^2x + 4x^2 + 4h^2 + 8hx + 3x + 3h + 2 - x^3 - 4x^2 - 3x - 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3 + 4h^2 + 8hx + 3h}{h} \\ &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 + 4h + 8x + 3 \\ &= 3x^2 + 8x + 3 \end{aligned}$$

Q1(xiii)

We have,

$$f(x) = x^3 - 5x^2 + x - 5$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{(x+h)^3 + (x+h) - 5(x+h)^2 - 5\} - \{x^3 - 5x^2 + x - 5\}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{x^3 + h^3 + 3x^2h + 3h^2x + x + h - 5x^2 - 5h^2 - 10xh - 5\} - \{x^3 - 5x^2 + x - 5\}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{3x^2h + 3h^2x + h^3 + h - 5h^2 - 10xh\}}{h} \\ &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 + 1 - 5h - 10x \\ &= 3x^2 - 10x + 1\end{aligned}$$

Q1(xiv)

We have,

$$f(x) = \sqrt{2x^2 + 1}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)^2 + 1} - \sqrt{2x^2 + 1}}{h}\end{aligned}$$

Multiplying Numerator and Denominator by $\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1}$

$$\begin{aligned}&= \lim_{h \rightarrow 0} \frac{\{2(x+h)^2 + 1 - (2x^2 + 1)\}}{h(\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1})} \\ &= \lim_{h \rightarrow 0} \frac{2x^2 + 2h^2 + 4xh + 1 - 2x^2 - 1}{h(\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1})} \\ &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h(\sqrt{2(x+h)^2 + 1} + \sqrt{2x^2 + 1})} \\ &= \frac{4x}{2\sqrt{2x^2 + 1}} \\ &= \frac{2x}{\sqrt{2x^2 + 1}}\end{aligned}$$

Q1(xv)

We have, $f(x) = \frac{2x+3}{x-2}$

Therefore,

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left(\frac{2x+2h+3}{x+h-2} \right) - \left(\frac{2x+3}{x-2} \right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2x^2 + 2hx + 3x - 4x - 4h - 6 - 2x^2 - 2hx + 4x - 3x - 3h + 6}{h(x+h-2)(x-2)} \\ &= \lim_{h \rightarrow 0} \frac{-7}{(x+h-2)(x-2)} \\ &= \boxed{\frac{-7}{(x-2)^2}} \end{aligned}$$

Q2(i)

We have,

$$f(x) = e^{-x}$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{-(x+h)} - e^{-x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{-x} (e^{-h} - 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-e^{-x} (e^{-h} - 1)}{-h} \\ &= -e^{-x} \end{aligned} \quad \left[\because \lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} = 1 \right]$$

Q2(ii)

We have,

$$f(x) = e^{3x}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{3(x+h)} - e^{3x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{3(x)} \cdot e^{3h} - e^{3x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{3(x)} (e^{3h} - 1)}{h}\end{aligned}$$

Multiplying Numerator and Denominator by 3

$$\begin{aligned}&= \lim_{h \rightarrow 0} e^{3(x)} \frac{(e^{3h} - 1)}{3h} \quad \left[\lim_{h \rightarrow 0} \frac{e^{3h} - 1}{3} = 1 \right] \\ &= 3e^{3x}\end{aligned}$$

Q2(iii)

We have,

$$f(x) = e^{ax+b}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{a(x+h)+b} - e^{ax+b}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{ax} \times e^{ah} \times e^b - e^{ax} \times e^b}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^b \times e^{ax} (e^{ah} - 1)}{h} \\ &= \lim_{h \rightarrow 0} e^{ax+b} \times \frac{a(e^{ah} - 1)}{a \cdot h}\end{aligned}$$

Multiplying Numerator and denominator by a

$$= ae^{ax+b}$$

$$\left[\lim_{h \rightarrow 0} \frac{(e^{ah} - 1)}{ah} = 1 \right]$$

Q2(iv)

We have,

$$f(x) = xe^x$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)e^{(x+h)} - xe^x}{h} \\ &= \lim_{h \rightarrow 0} \frac{xe^x \cdot e^h + he^x \cdot e^h - xe^x}{h} \\ &= \lim_{h \rightarrow 0} xe^x \left(\frac{e^h - 1}{h} \right) + \frac{he^{x+h}}{h} \\ &= xe^x + e^x \\ &= e^x (x+1)\end{aligned}$$

Q2(v)

Let $f(x) = -x$. Then, $f(x+h) = -(x+h)$

$$\begin{aligned}\therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{-(x+h) + (x)}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{-h}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} -1 \\ \Rightarrow \frac{d}{dx}(f(x)) &= -1\end{aligned}$$

Q2(vi)

Let $f(x) = (-x)^{-1}$. Then, $f(x+h) = -(x+h)^{-1}$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{(-(x+h))^{-1} - (-x)^{-1}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{\frac{-1}{x+h} + \frac{1}{x}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{\frac{-x + x + h}{x(x+h)}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{h}{hx(x+h)}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{1}{x(x+h)}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \frac{1}{x(x+0)}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \frac{1}{x^2}$$

Q2(vii)

Let $f(x) = \sin(x+1)$. Then, $f(x+h) = \sin((x+h)+1)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{\sin((x+h)+1) - \sin(x+1)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{2 \sin \left[\frac{((x+h)+1) - (x+1)}{2} \right] \cos \left[\frac{((x+h)+1) + (x+1)}{2} \right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{2 \sin \left[\frac{h}{2} \right] \cos \left[\frac{2x+2+h}{2} \right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{\sin \left[\frac{h}{2} \right]}{\frac{h}{2}} \times \lim_{h \rightarrow 0} \cos \left[\frac{2x+2+h}{2} \right]$$

$$\Rightarrow \frac{d}{dx}(f(x)) = 1 \times \cos \left[\frac{2x+2+0}{2} \right]$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \cos(x+1)$$

Q2(viii)

Let $f(x) = \cos\left(x - \frac{\pi}{8}\right)$. Then, $f(x+h) = \cos\left(x+h - \frac{\pi}{8}\right)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{\cos\left(x+h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{-2 \sin\left[\frac{\left(x+h - \frac{\pi}{8}\right) + \left(x - \frac{\pi}{8}\right)}{2}\right] \sin\left[\frac{\left(x+h - \frac{\pi}{8}\right) - \left(x - \frac{\pi}{8}\right)}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \frac{-2 \sin\left[\frac{2x+h - \frac{2\pi}{8}}{2}\right] \sin\left[\frac{h}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h \rightarrow 0} \sin\left[\frac{2x+h - \frac{2\pi}{8}}{2}\right] \times \lim_{h \rightarrow 0} \frac{\sin\left[\frac{h}{2}\right]}{\frac{h}{2}}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = -\sin\left[\frac{2x+0 - \frac{2\pi}{8}}{2}\right] \times 1$$

$$\Rightarrow \frac{d}{dx}(f(x)) = -\sin\left(x - \frac{\pi}{8}\right)$$

Q2(ix)

We have,

$$f(x) = x \sin x$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+h) \sin(x+h) - x \sin x}{h} \\&= \lim_{h \rightarrow 0} \frac{x \{\sin(x+h) - \sin x\}}{h} + \sin(x+h) \\&= \lim_{h \rightarrow 0} \frac{x \times 2 \cos\left(x + \frac{h}{2}\right) \sin \frac{h}{2}}{h} + \sin(x+h) \\&= 2x \times \cos x \times \frac{1}{2} + \sin x \\&= x \times \cos x + \sin x \\&= \sin x + x \cos x\end{aligned}$$

$$\left[\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2} \right]$$

$$\left[\therefore \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right]$$

Q2(x)

We have,

$$f(x) = x \cos x$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+h) \cos(x+h) - x \cos x}{h} \\&= \lim_{h \rightarrow 0} \frac{x \cos(x+h) + h \cos(x+h) - x \cos x}{h} \\&= \lim_{h \rightarrow 0} \frac{x \{\cos(x+h) - \cos x\}}{h} + \cos(x+h) \\&= \lim_{h \rightarrow 0} x \cdot 2 \sin\left(x - x + \frac{h}{2}\right) \sin\left(x + \frac{h}{2}\right) + \cos(x+h) \\&= \lim_{h \rightarrow 0} 2x \cdot \sin\left(\frac{-h}{2}\right) \sin\left(x + \frac{h}{2}\right) + \cos(x+h) \\&= -x \sin x + \cos x\end{aligned}$$

$$\left[\therefore \cos A - \cos B = 2 \sin \frac{B-A}{2} \sin \frac{B+A}{2} \right]$$

Q2(xi)

We have,

$$f(x) = \sin(2x - 3)$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\sin(2(x+h) - 3) - \sin(2x - 3)}{h} \\&= \lim_{h \rightarrow 0} \frac{2 \cos \frac{(2x+2h-3)+2x-3}{2} \times \sin \frac{(2x+2h-3)-(2x-3)}{2}}{h} \left[\because \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2} \right] \\&= \lim_{h \rightarrow 0} 2 \cos(2x - 3 + h) \cdot \frac{\sin h}{2} \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\&= 2 \cos(2x - 3)\end{aligned}$$

Q3(i)

We have,

$$f(x) = \sqrt{\sin 2x}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\sqrt{\sin 2(x+h)} - \sqrt{\sin 2x}}{h}\end{aligned}$$

Multiplying Numerator and Denominator by $(\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x})$

$$\begin{aligned}&= \lim_{h \rightarrow 0} \frac{\sqrt{\sin 2(x+h)} - \sqrt{\sin 2x}}{h} \times \frac{\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}}{\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}} \\&= \lim_{h \rightarrow 0} \frac{\sin(2x+2h) - \sin 2x}{h(\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x})} \\&= \lim_{h \rightarrow 0} \frac{\sin(2x+2h) - \sin 2x}{h(\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x})} \left[\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2} \right] \\&= \lim_{h \rightarrow 0} \frac{2 \cos(2x+h) \times \sin h}{h} \times \frac{1}{\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}} \\&= \frac{2 \cos 2x}{2\sqrt{\sin 2x}} \\&= \frac{\cos 2x}{\sqrt{\sin 2x}}\end{aligned}$$

Q3(ii)

We have,

$$f(x) = \frac{\sin x}{x}$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h)}{x+h} - \frac{\sin x}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x \sin(x+h) - (x+h) \sin x}{xh(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{x(\sin x \cos h + \cos x \sin h) - x \sin x - h \sin x}{xh(x+h)} \quad \left[\because \sin(A+B) = \sin A \cos B + \cos A \sin B \right] \\ &= \lim_{h \rightarrow 0} \frac{x \sin x (\cos h - 1) + x \cos x \sin h - h \sin x}{xh(x+h)} \quad \left[\because 1 - \cos h = 2 \sin^2 \frac{h}{2} \right] \\ &= \frac{-x \sin x}{x(x+h)} \times \frac{2 \sin^2 \frac{h}{2}}{\frac{h^2}{4}} \times \frac{h}{4} + \frac{x \cos x}{x^2} - \frac{\sin x}{x^2} \end{aligned}$$

$$\therefore h \rightarrow 0 \Rightarrow \frac{h}{2} \rightarrow 0 \quad \text{and} \quad \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{-x \cos x - \sin x}{x^2} \\ &= \frac{-x \cos x - \sin x}{x^2} \end{aligned}$$

Q3(iii)

We have,

$$f(x) = \frac{\cos x}{x}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\cos(x+h)}{x+h} - \frac{\cos x}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x \cdot \cos(x+h) - (x+h) \cos x}{(x+h)xh} \quad [\because \cos(A+B) = \cos A \cos B - \sin A \sin B] \\ &= \lim_{h \rightarrow 0} \frac{x [\cos x \cos h - \sin x \sin h] - x \cos x - h \cos x}{(x+h)xh} \\ &= \lim_{h \rightarrow 0} \frac{x \cos x (\cos h - 1) - x \cos x \sin h - h \cos x}{(x+h)xh} \\ &= \lim_{h \rightarrow 0} \frac{-x \cos x \cdot 2 \sin^2 \frac{h}{2} \times \frac{h^2}{4} - x \sin x - h \cos x}{(x+h)x \frac{h^2}{4}} \\ &= \lim_{h \rightarrow 0} \frac{-x \sin x}{x^2} - \frac{\cos x}{x^2} \\ &= -\frac{x \sin x + \cos x}{x^2} \end{aligned}$$

Q3(iv)

We have,

$$f(x) = x^2 \sin x$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^2 \sin(x+h) - x^2 \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x^2 + h^2 + 2hx) (\sin x \cos h + \cos x \sin h) - x^2 \sin x}{h} \quad [\because \sin(A+B) = \sin A \cos B + \cos A \sin B] \\ &= \lim_{h \rightarrow 0} \frac{x^2 \sin x (\cos h - 1) + h(h+2x) \sin x \cos h + (x+h)^2 \cos x \sin h}{h} \\ &= \lim_{h \rightarrow 0} \left[x^2 \sin x \times \frac{2 \sin^2 \frac{h}{2} \times \frac{h^2}{4}}{\left(\frac{h}{2}\right)^2} + (1+2x) \sin x \cos h + (x+h)^2 \cos x \right] \\ &= 0 + (2x \sin x + x^2 \cos x) \\ &= 2x \sin x + x^2 \cos x \end{aligned}$$

Q3(v)

We have,

$$f(x) = \sqrt{\sin(3x+1)}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\sqrt{\sin(3(x+h)+1)} - \sqrt{\sin(3x+1)}}{h} \\&= \lim_{h \rightarrow 0} \frac{\sqrt{\sin(3x+3h)+1} - \sqrt{\sin(3x+1)}}{h} \times \frac{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}} \\&= \lim_{h \rightarrow 0} \frac{\sin(3x+3h+1) - \sin(3x+1)}{h \left(\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)} \right)} \\&= \lim_{h \rightarrow 0} 2 \cos \left(3x+1 + \frac{3h}{2} \right) \times \frac{\sin \frac{3h}{2}}{\frac{3h}{2}} \times \frac{3}{2} \times \frac{1}{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}} \\&= \frac{3 \cos(3x+1)}{2 \sqrt{\sin(3x+1)}} \quad \left[\lim_{h \rightarrow 0} \frac{\sin \frac{3h}{2}}{\frac{3h}{2}} = 1 \right]\end{aligned}$$

Q3(vi)

We have,

$$f(x) = \sin x + \cos x$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{\sin(x+h) + \cos(x+h)\} - \sin x + \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{\sin(x+h) + \cos(x+h) - \sin x - \cos x\}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\{\sin(x+h) - \sin x\} + \{\cos(x+h) - \cos x\}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left\{2 \sin \frac{(x+h-x)}{2} \cos \frac{(x+h+x)}{2}\right\} + \left\{-2 \sin \frac{x+h+x}{2} \sin \frac{x-h-x}{2}\right\}}{h}\end{aligned}$$

$$\left[\begin{array}{l} \because \sin A - \sin B = 2 \sin \frac{A-B}{2} \cos \frac{A+B}{2} \\ \text{and } \cos A - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \end{array} \right]$$

$$\begin{aligned}&= \lim_{h \rightarrow 0} \frac{\sin h \cdot \cos \frac{2x+h}{2} - 2 \sin \left(x + \frac{h}{2}\right) \sin h}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin h}{h} \left\{ \cos \frac{x+h}{2} - \sin \left(x + \frac{h}{2}\right) \right\} \\ &= \cos x - \sin x\end{aligned}$$

$$\left[\because \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \right]$$

Q3(vii)

We have,

$$f(x) = x^2 e^x$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 e^x e^h + h^2 e^x e^h + 2xh e^x e^h - x^2 e^x}{h} \\ &= \lim_{h \rightarrow 0} x^2 e^x \frac{(e^h - 1)}{h} + e^x e^h \frac{(h^2 + 2xh)}{h}\end{aligned}$$

$$\left[\because \frac{e^h - 1}{h} \rightarrow 1 \right]$$

$$\begin{aligned}&\therefore x^2 e^x + e^x (0 + 2x) \\ &= x^2 e^x + 2x e^x \\ &= e^x (x^2 + 2x)\end{aligned}$$

Q3(viii)

We have,

$$f(x) = e^{x^2+1}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{(x+h)^2+1} - e^{x^2+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{x^2+h^2+2xh+1} - e^{x^2+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{x^2+1} (e^{2xh+h^2} - 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{x^2+1} (e^{2xh+h^2} - 1)}{2xh+h^2} \times \frac{2xh+h^2}{h}\end{aligned}$$

$$\because h \rightarrow 0$$

$$\Rightarrow 2xh+h^2 = 0$$

$$\text{and } \lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} = 1$$

$$= \lim_{h \rightarrow 0} e^{x^2+1} \cdot 1 \times 2x + h$$

$$= 2xe^{x^2+1}$$

Q3(ix)

We have,

$$f(x) = e^{\sqrt{2x}}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{e^{\sqrt{2(x+h)}} - e^{\sqrt{2x}}}{h} \\&= \lim_{h \rightarrow 0} \frac{e^{\sqrt{2x}} \left(e^{\sqrt{2(x+h)} - \sqrt{2x}} - 1 \right)}{h} \\&= \lim_{h \rightarrow 0} e^{\sqrt{2x}} \frac{\left(e^{\sqrt{2(x+h)} - \sqrt{2x}} - 1 \right)}{\sqrt{2(x+h)} - \sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}\end{aligned}$$

Multiplying Numerator and Denominator by $\sqrt{2(x+h)} - \sqrt{2x}$

$$\therefore h \rightarrow 0 \Rightarrow \sqrt{2(x+h)} - \sqrt{2x} \Rightarrow 0$$

$$\text{and } \lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} = 1$$

$$\therefore \lim_{h \rightarrow 0} e^{\sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

Again Multiplying Numerator and Denominator by $\sqrt{2(x+h)} + \sqrt{2x}$

$$\begin{aligned}\therefore \lim_{h \rightarrow 0} e^{\sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \times \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}} \\= e^{\sqrt{2x}} \times \frac{1}{2\sqrt{2x}}\end{aligned}$$

Q3(x)

We have,

$$f(x) = e^{\sqrt{ax+b}}$$

$$\begin{aligned}\therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{e^{\sqrt{a(x+h)+b}} - e^{\sqrt{ax+b}}}{h} \\&= \lim_{h \rightarrow 0} e^{\sqrt{a(x+h)+b}} \left(\frac{e^{\sqrt{a(x+h)+b} - \sqrt{ax+b}} - 1}{h} \right) \\&= \lim_{h \rightarrow 0} e^{\sqrt{ax+b}} \times \frac{e^{\sqrt{a(x+h)+b} - \sqrt{ax+b}} - 1}{\sqrt{a(x+h)+b} - \sqrt{ax+b}} \times \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{h}\end{aligned}$$

Multiplying Numerator and Denominator by $\sqrt{a(x+h)+b} + \sqrt{ax+b}$

$$\therefore h \rightarrow 0$$

$$\therefore \sqrt{a(x+h)+b} - \sqrt{ax+b} = 0$$

$$\text{and } \lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} = 1$$

$$= \lim_{h \rightarrow 0} e^{\sqrt{ax+b}} \times 1 \times \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{\sqrt{a(x+h)+b} + \sqrt{ax+b}} \times \frac{\sqrt{a(x+h)+b} + \sqrt{ax+b}}{h}$$

Again multiplied Numerator and Denominator by $\sqrt{a(x+h)+b} + \sqrt{ax+b}$

$$\begin{aligned}&= \lim_{h \rightarrow 0} e^{\sqrt{ax+b}} \times \frac{a(x+h) + b - (ax+b)}{h} \times \frac{1}{(\sqrt{a(x+h)+b} + \sqrt{ax+b})} \\&= \frac{e^{\sqrt{ax+b}} \times a}{2\sqrt{ax+b}} \\&= \frac{ae^{\sqrt{ax+b}}}{2\sqrt{ax+b}}\end{aligned}$$

Q3(xi)

$$f(x) = a^{\sqrt{x}} = e^{\sqrt{x} \log a}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{\sqrt{x+h} \log a} - e^{\sqrt{x} \log a}}{h} \\ &= \lim_{h \rightarrow 0} e^{\sqrt{x} \log a} \frac{e^{\sqrt{x+h} \log a - \sqrt{x} \log a} - 1}{h} \\ &= \lim_{h \rightarrow 0} e^{\sqrt{x} \log a} \frac{e^{(\sqrt{x+h} - \sqrt{x}) \log a} - 1}{h} \end{aligned}$$

Multiply numerator and denominator by $(\sqrt{x+h} - \sqrt{x}) \log a$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} e^{\sqrt{x} \log a} \frac{e^{(\sqrt{x+h} - \sqrt{x}) \log a} - 1}{h(\sqrt{x+h} - \sqrt{x}) \log a} (\sqrt{x+h} - \sqrt{x}) \log a \\ &= e^{\sqrt{x} \log a} \lim_{h \rightarrow 0} \frac{e^{(\sqrt{x+h} - \sqrt{x}) \log a} - 1}{(\sqrt{x+h} - \sqrt{x}) \log a} \lim_{h \rightarrow 0} \log a \frac{(\sqrt{x+h} - \sqrt{x})}{h} \\ &= e^{\sqrt{x} \log a} \lim_{h \rightarrow 0} \log a \frac{(\sqrt{x+h} - \sqrt{x})}{h} \end{aligned}$$

Multiply numerator and denominator by $(\sqrt{x+h} + \sqrt{x})$

$$\begin{aligned} f'(x) &= e^{\sqrt{x} \log a} \lim_{h \rightarrow 0} \log a \frac{(\sqrt{x+h} - \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} (\sqrt{x+h} + \sqrt{x}) \\ &= e^{\sqrt{x} \log a} \lim_{h \rightarrow 0} \log a \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\ &= e^{\sqrt{x} \log a} \frac{\log a}{2\sqrt{x}} \\ &= \frac{a^{\sqrt{x}}}{2\sqrt{x}} \log_e a \end{aligned}$$

Q3(xii)

We have,

$$f(x) = 3^{x^2} = e^{x^2 \log 3}$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{(x+h)^2 \log 3} - e^{x^2 \log 3}}{h} \\ &= \lim_{h \rightarrow 0} e^{x^2 \log 3} \left[\frac{e^{((x+h)^2 - x^2) \log 3} - 1}{h} \right] \\ &= \lim_{h \rightarrow 0} e^{x^2 \log 3} \left[\frac{e^{[(x+h)^2 - x^2] \log 3} - 1}{(x+h)^2 - x^2} \times \frac{(x+h)^2 - x^2}{h} \right] \end{aligned}$$

Multiplying Numerator and Denominator by $(x+h)^2 - x^2$

$$\begin{aligned} \therefore \lim_{h \rightarrow 0} e^{x^2 \log 3} \times \frac{(x+h+x)(x+h-x)}{h} \\ &= e^{x^2 \log 3} \times 2x \\ &= 2x e^{x^2 \log 3} \\ &= 2x 3^{x^2 \log 3} \end{aligned}$$

Q4(i)

We have,

$$f(x) = \tan^2 x$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\tan^2(x+h) - \tan^2 x}{h} \\ &= \lim_{h \rightarrow 0} \left\{ \frac{\tan(x+h) + \tan x}{h} \{ \tan(x+h) - \tan x \} \right\} \\ &= \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h+x)}{\cos(x+h)\cos x} \times \frac{\sin(x-h-x)}{\cos(x+h)\cos x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(2x+h)}{h \cos(x+h)\cos x} \times \frac{\sin h}{\cos(x+h)\cos x} \\ &= \lim_{h \rightarrow 0} \frac{\sin h}{h} \times \frac{\sin 2x}{\cos^2 x \cos^2(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{\sin h}{\cos^2 x \cos^2(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin x \cos x}{1 \cos^2 x} \times \frac{1}{\cos^2 x} \\ &= 2 \tan x \sec^2 x \end{aligned}$$

$$[\because \tan^2 A - \tan^2 B = (\tan A + \tan B)(\tan A - \tan B)]$$

$$\begin{aligned} &\left[\because \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \right] \\ &[\sin 2x = 2 \sin x \cos x] \end{aligned}$$

Q4(ii)

We have,

$$f(x) = \tan(2x + 1)$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\tan\{2(x+h)+1\} - \tan(2x+1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(2x+2h+1) \cos(2x+1) - \cos(2x+2h+1) \sin(2x+1)}{h \cos\{2(x+h)+1\} \cos(2x+1)} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin 2h}{2h \cos(2x+2h+1) \cos(2x+1)} \end{aligned}$$

$$\left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B} \right]$$

Multiplying both, Numerator and Denominator by 2.

$$\begin{aligned} \therefore \lim_{h \rightarrow 0} 2 \left(\frac{\sin 2h}{2} \right) \times \frac{1}{\cos(2x+2h+1) \cos(2x+1)} \\ &= \frac{2}{\cos^2(2x+1)} \\ &= 2 \sec^2(2x+1) \\ &= 2 \sec^2(2x+1) \end{aligned}$$

$$\left[\because \lim_{h \rightarrow 0} \frac{\sin 2h}{2} = 1 \right]$$

$$\left[\because \sec^2 x = \frac{1}{\cos^2 x} \right]$$

Q4(iii)

We have,

$$f(x) = \tan 2x$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\tan 2(x+h) - \tan 2x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(2x+2h) \cos 2x - \sin 2x \cos(2x+2h)}{h \cos(2x+2h) \cos 2x} \\ &= \lim_{h \rightarrow 0} \frac{\sin 2h}{h \cos(2x+2h) \cos 2x} \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin 2h}{2h} \right) \times \frac{1 \times 2}{\cos(2h+2x) \cos 2x} \\ &= \frac{2}{\cos 2x \cdot \cos 2x} \\ &= 2 \sec^2 2x \end{aligned}$$

$$\left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A \cdot \cos B} \right]$$

$$\left[\because \lim_{h \rightarrow 0} \frac{\sin 2h}{2h} = 1 \right]$$

$$\left[\because \frac{1}{\cos^2 x} = \sec^2 x \right]$$

Q4(iv)

We have,

$$f(x) = \sqrt{\tan x}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h} \end{aligned}$$

Multiplying Numerator and Denominator by $\sqrt{\tan(x+h)} + \sqrt{\tan x}$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{\tan(x+h) - \tan x}{h \left(\sqrt{\tan(x+h)} + \sqrt{\tan x} \right)} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x+h-x)}{h \cdot \cos(x+h) \cos x \left(\sqrt{\tan(x+h)} + \sqrt{\tan x} \right)} \\ &= \lim_{h \rightarrow 0} \frac{\sinh}{h} \times \frac{1}{\cos(x+h) \cos x \left(\sqrt{\tan(x+h)} + \sqrt{\tan x} \right)} \\ &= \lim_{h \rightarrow 0} \frac{1}{\cos^2 x \cdot 2\sqrt{\tan x}} \\ &= \frac{1}{2} \frac{\sec^2 x}{\sqrt{\tan x}} \end{aligned}$$

$$\left[\lim_{h \rightarrow 0} \frac{\sinh}{h} = 1 \right]$$

$$\left[\therefore \frac{1}{\cos^2 x} = \sec^2 x \right]$$

Q5(i)

let $f(x) = \sin \sqrt{2x}$. Then $f(x+h) = \sin \sqrt{2(x+h)}$

$$\frac{d(f(x))}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin \sqrt{2(x+h)} - \sin \sqrt{2x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin \left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{2} \right) \cos \left(\frac{\sqrt{2(x+h)} + \sqrt{2x}}{2} \right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin \left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{2} \right)}{\left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{2} \right)} \cdot \frac{(\sqrt{2(x+h)} - \sqrt{2x})(\sqrt{2(x+h)} + \sqrt{2x})}{(\sqrt{2(x+h)} + \sqrt{2x})h} \cos \left(\frac{\sqrt{2(x+h)} + \sqrt{2x}}{2} \right)$$

$$= \lim_{h \rightarrow 0} \frac{\sin \left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{2} \right)}{\left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{2} \right)} \cdot \lim_{h \rightarrow 0} \frac{2(x+h) - 2x}{(\sqrt{2(x+h)} + \sqrt{2x})h} \cdot \lim_{h \rightarrow 0} \cos \left(\frac{\sqrt{2(x+h)} + \sqrt{2x}}{2} \right)$$

$$= 1 \times \frac{2}{2\sqrt{2x}} \cos(\sqrt{2x})$$

$$= \frac{\cos(\sqrt{2x})}{\sqrt{2x}}$$

Q5(ii)

We have,

$$f(x) = \cos \sqrt{x}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos \sqrt{x+h} - \cos \sqrt{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \sin \left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right) \left(\sqrt{x+h} - \sqrt{x} \right) \left(\sqrt{x+h} + \sqrt{x} \right)}{\left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right) \left(\sqrt{x+h} + \sqrt{x} \right) h} \times \sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \end{aligned}$$

Multiplied Numerator and Denominator by $(\sqrt{x+h} - \sqrt{x})$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{-\sin \left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right)}{\left(\frac{\sqrt{x+h} - \sqrt{x}}{2} \right)} \times \frac{x+h-x}{\left(\sqrt{x+h} + \sqrt{x} \right) h} \times \sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \\ &= \lim_{h \rightarrow 0} -1 \frac{h}{h \left(\sqrt{x+h} + \sqrt{x} \right)} \times \sin \left(\frac{\sqrt{x+h} + \sqrt{x}}{2} \right) \quad \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\ &= \lim_{h \rightarrow 0} \frac{-1}{\left(\sqrt{x+h} + \sqrt{x} \right)} \sin \frac{\sqrt{x+h} + \sqrt{x}}{2} \\ &= \frac{-\sin \sqrt{x}}{2\sqrt{x}} \end{aligned}$$

Q5(iii)

We have,

$$f(x) = \tan \sqrt{x}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\tan \sqrt{x+h} - \tan \sqrt{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin \sqrt{x+h} - \sqrt{x}}{h \cdot \cos \sqrt{x+h} \cos \sqrt{x}}$$

$$= \lim_{h \rightarrow 0} \frac{\sin \sqrt{x+h} - \sqrt{x}}{(x+h-x) \cos \sqrt{x} \cos \sqrt{x+h}}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(\sqrt{x+h} - \sqrt{x})}{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x}) \cos \sqrt{x} \cos \sqrt{x+h}}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(\sqrt{x+h} - \sqrt{x})}{(\sqrt{x+h} - \sqrt{x})} \times \frac{1}{(\sqrt{x+h} + \sqrt{x}) \cos \sqrt{x} \cos \sqrt{x+h}}$$

$$= 1 \times \frac{1}{2\sqrt{x} \cos \sqrt{x} \cos \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x} \cos^2 x}$$

$$= \frac{\sec^2 x}{2\sqrt{x}}$$

$$\left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B} \right]$$

$$\left[\because \lim_{h \rightarrow 0} \frac{\sin \sqrt{x+h} - \sqrt{x}}{\sqrt{x+h} - \sqrt{x}} = 1 \right]$$

Q5(iv)

We have,

$$f(x) = \tan x^2$$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\tan(x+h)^2 - \tan x^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h)^2}{\cos(x+h)^2} - \frac{\sin x^2}{\cos x^2}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h)^2 \cos x^2 - \cos(x+h)^2 \sin x^2}{\cos(x+h)^2 \cos x^2}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin((x+h)^2 - x^2)}{h \cdot \cos(x+h)^2 \cdot \cos x^2} \\
 &= \lim_{h \rightarrow 0} \frac{\sin(x^2 + h^2 + 2hx - x^2)}{h \cdot \cos(x+h)^2 \cdot \cos x^2} \\
 &= \lim_{h \rightarrow 0} \frac{\sin(h^2 + 2hx)}{h \cdot \cos(x+h)^2 \cdot \cos x^2} \\
 &= \lim_{h \rightarrow 0} \frac{\sinh}{h} \times \frac{(h + 2x)}{\cos(x+h)^2 \cdot \cos x^2} \\
 &= 1 \cdot \frac{2x}{\cos^2(x)^2} \quad \left[\because \lim_{h \rightarrow 0} \frac{\sinh}{h} = 1 \right] \\
 &= 2x \sec^2 x^2
 \end{aligned}$$

Q6(i)

We have,

$$f(x) = (-x)$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{- (x+h) + x}{h} \\ &= \lim_{h \rightarrow 0} \frac{-x - h + x}{h} \\ &= -1 \end{aligned}$$

Q6(ii)

We have,

$$f(x) = (-x)^{-1}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{-1}{x+h} + \frac{1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-x + x + h}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{1}{x^2 + xh} \\ &= \frac{1}{x^2} \end{aligned}$$

Q6(iii)

We have,

$$f(x) = \sin(x+1)$$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin(x+h+1) - \sin(x+1)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2 \cos\left(\frac{x+h+1+x+1}{2}\right) \sin\left(\frac{x+h+1-x-1}{2}\right)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2 \cos\left(\frac{2x+2+h}{2}\right) \sin \frac{h}{2}}{h} \quad \left[\because \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \right] \\
 &= \lim_{h \rightarrow 0} \cos\left(\frac{2x+2+h}{2}\right) \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right) \\
 &= \cos\left(\frac{2(x+1)}{2}\right) \quad \left[\because \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} = 1 \right] \\
 &= \cos(x+1)
 \end{aligned}$$

Q6(iv)

We have,

$$f(x) = \cos\left(x - \frac{\pi}{8}\right)$$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos\left(x+h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2 \sin\left(\frac{2x+h - \frac{2\pi}{8}}{2}\right) \sin\left(\frac{h+x - \frac{\pi}{8} - x + \frac{\pi}{8}}{2}\right)}{h} \end{aligned}$$

$$\left[\because \cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2} \right]$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{-2 \sin\left(\frac{2x+h - \frac{2\pi}{8}}{2}\right) \times \sin\left(\frac{h}{2}\right)}{2 \cdot \frac{h}{2}} \\ &= \lim_{h \rightarrow 0} \frac{-2 \sin\left(\frac{2x+h - \frac{2\pi}{8}}{2}\right)}{2} \times \lim_{h \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \end{aligned}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \sin\left(\frac{2x+h - \frac{2\pi}{8}}{2}\right) \left[\because \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} = 1 \right] \end{aligned}$$

$$= \sin\left(\frac{2x - \frac{2\pi}{8}}{2}\right)$$

$$= \sin\left(x - \frac{\pi}{8}\right)$$

EX 30.3

Q1

We have to differentiate $f(x)$ with respect to x

$$\begin{aligned} & \frac{d}{dx} (x^4 - 2 \sin x + 3 \cos x) \\ &= \frac{d}{dx} (x^4) - 2 \frac{d}{dx} (\sin x) + 3 \frac{d}{dx} (\cos x) \\ &= 4x^3 - 2 \cos x - 3 \sin x \end{aligned}$$

Q2

We have to differentiate $f(x)$ with respect to x

$$\begin{aligned} & \frac{d}{dx} (3^x + x^3 + 3^3) \\ &= \frac{d}{dx} (3^x) + \frac{d}{dx} (x^3) + \frac{d}{dx} (3^3) \\ &= 3^x \log 3 + 3x^2 + 0 \quad \left[\because \frac{d}{dx} (a^x) = a^x \log a \right] \\ &= 3^x \log 3 + 3x^2 \end{aligned}$$

Q3

We have to differentiate $f(x)$ with respect to x

$$\begin{aligned} & \frac{d}{dx} \left(\frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2} \right) \\ &= \frac{1}{3} \frac{d}{dx} (x^3) - 2 \frac{d}{dx} (\sqrt{x}) + 5 \frac{d}{dx} (x^{-2}) \\ &= \frac{1}{3} \cdot 3x^2 - 2 \cdot \frac{1}{2\sqrt{x}} + 5 \cdot (-2) x^{-3} \\ &= x^2 - x^{-\frac{1}{2}} - 10x^{-3} \\ &= x^2 - \frac{1}{\sqrt{x}} - \frac{10}{x^3} \end{aligned}$$

Q4

We have,

$$\frac{d}{dx} \left(e^{x \log a} + e^{a \log x} + e^{a \log a} \right)$$

$$= \frac{d}{dx} \left(e^{x \log a} \right) + \frac{d}{dx} \left(e^{a \log x} \right) + \frac{d}{dx} \left(e^{a \log a} \right)$$

$$= e^{x \log a} \cdot \log a + e^{a \log x} \cdot \frac{a}{x} + 0 \quad \left[\because e^{a \log a} \text{ is constant} \right]$$

$$= \log a e^{x \log a} + \frac{a}{x} e^{a \log x}$$

$$= \log a a^x + \frac{a}{x} x^a \quad \left[a^x \text{ can be written as } e^{x \log a} \right]$$

$$= a^x \log a + a x^{a-1}$$

Q5

We have,

$$\frac{d}{dx} \left(2x^2 + 1 \right) (3x + 2)$$

$$= (3x + 2) \frac{d}{dx} (2x^2 + 1) + (2x^2 + 1) \frac{d}{dx} (3x + 2) \quad \left[\text{Using product rule} \right]$$

$$= (3x + 2) (4x + 0) + (2x^2 + 1) (3 + 0)$$

$$= (12x^2 + 8x + 6x^2 + 3)$$

$$= 18x^2 + 8x + 3$$

Q6

We have,

$$\frac{d}{dx} f(x) = \frac{d}{dx} (\log_3 x + 3 \log_e x + 2 \tan x)$$

$$= \frac{1}{\log 3} \frac{d}{dx} (\log x) + 3 \frac{d}{dx} (\log_e x) + 2 \frac{d}{dx} (\tan x) \quad \left[\because \log_3 x = \frac{\log x}{\log 3} \right]$$

$$= \frac{1}{\log 3} \times \frac{1}{x} + \frac{3}{x} + 2 \sec^2 x$$

$$= \frac{1}{x \log 3} + \frac{3}{x} + 2 \sec^2 x$$

Q7

We have,

$$\begin{aligned} & \frac{d}{dx} \left(x + \frac{1}{x} \right) \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \\ &= \left(x + \frac{1}{x} \right) \frac{d}{dx} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) + \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \frac{d}{dx} \left(x + \frac{1}{x} \right) \quad [\text{Using product rule}] \\ &= \left(x + \frac{1}{x} \right) \left(\frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}} \right) + \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \left(1 - \frac{1}{x^2} \right) \\ &= \left(\frac{x}{2\sqrt{x}} - \frac{x}{2x^{\frac{3}{2}}} + \frac{1}{2x^{\frac{3}{2}}} - \frac{1}{2x^{\frac{5}{2}}} \right) + \left(\sqrt{x} - \frac{\sqrt{x}}{x^2} + \frac{1}{\sqrt{x}} - \frac{1}{x^{\frac{5}{2}}} \right) \\ &= \left(\frac{1}{2}\sqrt{x} - \frac{1}{2\sqrt{x}} + \frac{1}{2x^{\frac{3}{2}}} - \frac{1}{2x^{\frac{5}{2}}} + \sqrt{x} - \frac{1}{x^{\frac{3}{2}}} + \frac{1}{\sqrt{x}} - \frac{1}{x^{\frac{5}{2}}} \right) \\ &= \left(\frac{3}{2}\sqrt{x} + \frac{1}{2}\sqrt{x} - \frac{1}{2x^{\frac{3}{2}}} - \frac{3}{2x^{\frac{5}{2}}} \right) \\ &= \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} - \frac{3}{2}x^{-\frac{5}{2}} \end{aligned}$$

Q8

We have,

$$\begin{aligned} & \frac{d}{dx} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^3 \\ &= \frac{d}{dx} \left(x^{\frac{1}{2}} + 3x \cdot \frac{1}{\sqrt{x}} + 3\sqrt{x} \cdot \frac{1}{x} + \frac{1}{x^{\frac{3}{2}}} \right) \quad [(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3] \\ &= \frac{d}{dx} \left(x^{\frac{1}{2}} + 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} + x^{-\frac{3}{2}} \right) \\ &= \frac{3}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}} + 3 \cdot \left(-\frac{1}{2} \right) x^{-\frac{3}{2}} - \frac{3}{2}x^{-\frac{5}{2}} \\ &= \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} + \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{5}{2}} \end{aligned}$$

Q9

We have,

$$\begin{aligned}& \frac{d}{dx} \left(\frac{2x^2 + 3x + 4}{x} \right) \\&= \frac{d}{dx} \left(\frac{2x^2}{x} + \frac{3x}{x} + \frac{4}{x} \right) \\&= \frac{d}{dx} (2x + 3 + 4x^{-1}) \\&= 2 - \frac{4}{x^2}\end{aligned}$$

Q10

We have,

$$\begin{aligned}& \frac{d}{dx} \frac{\{(x^3 + 1)(x - 2)\}}{x^2} \\&= \frac{d}{dx} \frac{\{x^4 - 2x^3 + x - 2\}}{x^2} \\&= \frac{d}{dx} (x^2 - 2x + x^{-1} - 2x^{-2}) \\&= \frac{d}{dx} (x^2) - 2 \frac{dx}{dx} + \frac{dx^{-1}}{dx} - 2 \frac{dx^{-2}}{dx} \\&= 2x - 2 - \frac{1}{x^2} + 2 \cdot \frac{2}{x^3} \\&= 2x - 2 - \frac{1}{x^2} + \frac{4}{x^3}\end{aligned}$$

Q11

We have,

$$\begin{aligned}& \frac{d}{dx} \left(\frac{a \cos x + b \sin x + c}{\sin x} \right) \\&= a \frac{d}{dx} \left(\frac{\cos x}{\sin x} \right) + b \frac{d}{dx} (1) + c \frac{d}{dx} \left(\frac{1}{\sin x} \right) \\&= a (-\operatorname{cosec}^2 x) + 0 + c (-\operatorname{cosec} x \cdot \cot x) \\&= -a \operatorname{cosec}^2 x - c \operatorname{cosec} x \cdot \cot x\end{aligned}$$

Q12

We have,

$$\frac{d}{dx} (2 \sec x + 3 \cot x - 4 \tan x)$$

$$= 2 \frac{d}{dx} (2 \sec x) + 3 \frac{d}{dx} (\cot x) - 4 \frac{d}{dx} (\tan x)$$

$$= 2 \sec x \tan x - 3 \operatorname{cosec}^2 x - 4 \sec^2 x$$

Q13

We have,

$$\frac{d}{dx} (a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1} x + a_n)$$

$$= a_0 \frac{d(x)^n}{dx} + a_1 \frac{d(x)^{n-1}}{dx} + a_2 \frac{d(x)^{n-2}}{dx} + \dots + a_{n-1} \frac{d(x)}{dx} + a_n \frac{d(1)}{dx}$$

$$= na_0 x^{n-1} + (n-1)a_1 x^{n-2} + \dots + a_{n-1} + 0$$

$$= na_0 x^{n-1} + (n-1)a_1 x^{n-2} + \dots + a_{n-1}$$

Q14

We have,

$$\frac{d}{dx} \left(\frac{1}{\sin x} + 2^{x+3} + \frac{4}{\log x^3} \right)$$

$$= \frac{d}{dx} \operatorname{cosec} x + 2^3 \frac{d}{dx} (2^x) + \frac{4}{\log 3} \times \frac{d}{dx} (\log x) \left[\because \log_b a = \frac{\log a}{\log b} \right]$$

$$= -\operatorname{cosec} x \cdot \cot x + 8 \cdot 2^x \log 2 + \frac{4}{\log 3} \times \frac{1}{x} \left[\because \frac{d}{dx} (a^x) = a^x \log a \right]$$

$$= -\operatorname{cosec} x \cot x + 2^{x+3} \log 2 + \frac{4}{x \log 3}$$

Q15

We have,

$$\begin{aligned}
 & \frac{d}{dx} \left\{ \frac{(x+5)(2x^2-1)}{x} \right\} \\
 &= \frac{d}{dx} \left(\frac{2x^3 + 10x^2 - x - 5}{x} \right) \\
 &= \frac{d}{dx} (2x^2 + 10x - 1 - 5x^{-1}) \\
 &= 2 \frac{d}{dx} (x^2) + 10 \frac{d}{dx} (x) - \frac{d}{dx} (1) - 5 \frac{d}{dx} (x^{-1}) \\
 &= 2 \times 2x + 10 - 0 + \frac{5}{x^2} \\
 &= 4x + 10 + \frac{5}{x^2}
 \end{aligned}$$

Q16

$$\begin{aligned}
 & \frac{d}{dx} \left\{ \log \left(\frac{1}{\sqrt{x}} \right) + 5x^a - 3a^x + \sqrt[3]{x^2} + 6\sqrt[4]{x^{-3}} \right\} \\
 &= \frac{d}{dx} \log \left(\frac{1}{\sqrt{x}} \right) + 5 \frac{d}{dx} (x^a) - 3(a^x) + \frac{d}{dx} (\sqrt[3]{x^2}) + 6 \frac{d}{dx} (\sqrt[4]{x^{-3}}) \\
 &= \frac{-1}{2} \frac{1}{x} + 5ax^{a-1} - 3a^x \log a + \frac{2x^{-1/3}}{3} + 6x^{-7/4} \left(-\frac{3}{4} \right) \\
 &= \frac{-1}{2x} + 5ax^{a-1} - 3a^x \log a + \frac{2x^{-1/3}}{3} - \frac{9}{2} x^{-7/4}
 \end{aligned}$$

Q17

We have,

$$\begin{aligned}
 & \frac{d}{dx} \{ \cos(x+a) \} \\
 &= \frac{d}{dx} \{ \cos x \cdot \cos a - \sin x \cdot \sin a \} \qquad [\because \cos(x+a) = \cos x \cos a - \sin x \sin a] \\
 &= \cos a \frac{d}{dx} (\cos x) - \sin a \frac{d}{dx} (\sin x) \\
 &= \cos a (-\sin x) - \sin a (\cos x) \\
 &= -\cos x \sin a - \sin x \cos a \\
 &= -(\sin x \cos a + \cos x \sin a) \\
 &= -\sin(x+a)
 \end{aligned}$$

Q18

We have,

$$\begin{aligned}\frac{d}{dx} \frac{\{\cos(x-2)\}}{\sin x} \\&= \frac{d}{dx} \frac{(\cos x \cdot \cos 2 + \sin x \cdot \sin 2)}{\sin x} \\&= \cos 2 \frac{d}{dx} (\cot x) + \sin 2 \frac{d}{dx} (1) \\&= -\cos 2 \cdot \operatorname{cosec}^2 x + 0 \\&= -\operatorname{cosec}^2 x \cos 2\end{aligned}$$

Q19

We have,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2 \\&= \frac{d}{dx} \left(\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2} \right) \\&= \frac{d}{dx} (1 + \sin x) \quad \left[\begin{array}{l} \because \sin^2 \theta + \cos^2 \theta = 1 \\ \sin^2 \theta = 2 \sin \theta \cdot \cos \theta \end{array} \right] \\&= 0 + \cos x \\&= \cos x\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} \text{ at } x &= \frac{\pi}{6} \\&= \cos \frac{\pi}{6} \\&= \frac{\sqrt{3}}{2}\end{aligned}$$

Q20

We have,

$$y = \frac{2 - 3 \cos x}{\sin x}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{2 - 3 \cos x}{\sin x} \right) \\ &= \frac{d}{dx} (2 \operatorname{cosec} x - 3 \cot x) \\ &= 2 \frac{d}{dx} (\operatorname{cosec} x) - 3 \frac{d}{dx} (\cot x) \\ &= -2 \operatorname{cosec} x \cdot \cot x + 3 \operatorname{cosec}^2 x\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} \text{ at } x &= \frac{\pi}{4} \\ &= -2 \operatorname{cosec} \frac{\pi}{4} \cdot \cot \frac{\pi}{4} + 3 \operatorname{cosec}^2 \frac{\pi}{4} \\ &= -2\sqrt{2} - 1 + 3 \cdot 2 \\ &= -2\sqrt{2} + 6 \\ &= 6 - 2\sqrt{2}\end{aligned}$$

Q21

Slope of the tangent at a point $x = a$ is the value of the derivative at $x = a$.

We have,

$$\begin{aligned}f(x) &= 2x^6 + x^4 - 1 \\ &= \frac{d}{dx} (2x^6 + x^4 - 1) \\ &= 2 \frac{dx^6}{dx} + \frac{dx^4}{dx} - \frac{d \cdot 1}{dx} \\ &= 12x^5 + 4x^3 - 0 \\ &= 12x^5 + 4x^3\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} \text{ at } x &= 1 \\ &= 12(1)^5 + 4(1)^3 \\ &= 12 + 4 \\ &= 16\end{aligned}$$

∴ The slope of the tangent to the curve $f(x) = 2x^6 + x^4 - 1$ at $x = 1$ is 16.

Q22

We have,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} \right) \\ &= \frac{1}{\sqrt{a}} \frac{d}{dx} \sqrt{x} + \sqrt{a} \frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right) \\ &= \frac{1}{\sqrt{a}} \frac{1}{2\sqrt{x}} + \sqrt{a} \left(\frac{-1}{2} \right) \times \frac{1}{x\sqrt{x}} \\ &= \frac{1}{2x} \left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}} \right)\end{aligned}$$

$$\Rightarrow 2x \frac{dy}{dx} = \sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}$$

Multiplying both side by $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$

$$\begin{aligned}\Rightarrow 2xy \frac{dy}{dx} &= \left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}} \right) \left(\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} \right) \\ &= \left(\frac{x}{a} - \frac{a}{x} \right)\end{aligned}$$

Hence, proved.

Q23

We have,

$$f(x) = x^4 - 2x^3 + 3x^2 + x + 5$$

Differentiate with respect to x

$$\begin{aligned}\frac{df(x)}{dx} &= \frac{d}{dx} (x^4 - 2x^3 + 3x^2 + x + 5) \\ &= 4x^3 - 6x^2 + 6x + 1\end{aligned}$$

Q24

We have,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{2x^9}{3} - \frac{5}{7}x^7 + 6x^3 - x \right) \\&= \frac{2}{3} \frac{dx^9}{dx} - \frac{5}{7} \frac{dx^7}{dx} + 6 \frac{dx^3}{dx} - \frac{dx}{dx} \\&= \frac{2}{3} \cdot 9x^8 - \frac{5}{7} \cdot 7x^6 + 18x^2 - 1 \\&= 6x^8 - 5x^6 + 18x^2 - 1\end{aligned}$$

$$\begin{aligned}\therefore \frac{dy}{dx} \text{ at } x &= 1 \\&= 6(1)^8 - 5(1)^6 + 18(1)^2 - 1 \\&= 6 - 5 + 18 - 1 \\&= 18\end{aligned}$$

Q25

We have,

$$f(x) = \lambda x^2 + \mu x + 12$$

$$\Rightarrow f'(x) = 2\lambda x + \mu \dots\dots\dots(i)$$

$$\text{but, } f'(4) = 15$$

from (i)

$$8\lambda + \mu = 15 \dots\dots\dots(ii)$$

$$\text{also, } f'(12) = 11$$

$$4\lambda + \mu = 11 \dots\dots\dots(iii)$$

(ii) - (iii) gives

$$4\lambda = 4$$

$$\Rightarrow \lambda = 1$$

from (ii)

$$8 \cdot 1 + \mu = 15$$

$$\Rightarrow \mu = 7$$

Hence,

$$\lambda = 1 \text{ and } \mu = 7$$

Q26

We have,

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1$$

Differentiating with respect to x , we get

$$f'(x) = x^{99} + x^{98} + \dots + x + 1 + 0 \dots (i)$$

from (i)

$$\begin{aligned} f'(1) &= 1 + 1 + \dots (100 \text{ times}) \\ &= 100 \end{aligned}$$

Again,

$$\begin{aligned} f'(0) &= 0 + 0 + \dots + 1 \\ &= 1 \end{aligned}$$

Now,

$$f'(1) = 100 = 100 \times 1 = 100 \times f'(0)$$

Hence,

$$f'(1) = 100f'(0)$$

Ex 30.4

Q1

We have,

$$\begin{aligned} & \frac{d}{dx} (x^3 \sin x) \\ &= \sin x \frac{d}{dx} (x^3) + x^3 \frac{d}{dx} (\sin x) \quad [\text{Using product rule}] \\ &= \sin x \cdot 3x^2 + x^3 \cdot \cos x \\ &= x^2 (3 \sin x + x \cos x) \end{aligned}$$

Q2

We have,

$$\begin{aligned} & \frac{d}{dx} (x^3 e^x) \\ &= e^x \frac{d}{dx} (x^3) + x^3 \frac{d}{dx} (e^x) \quad [\text{Using product rule}] \\ &= e^x 3x^2 + x^3 e^x \\ &= x^2 e^x (3 + x) \end{aligned}$$

Q3

We have,

$$\begin{aligned} & \frac{d}{dx} (x^2 e^x \log x) \\ &= e^x \log x \frac{d}{dx} (x^2) + x^2 \log x \frac{d}{dx} (e^x) + x^2 e^x \frac{d}{dx} (\log x) \quad [\text{Using product rule}] \\ &= e^x \log x \cdot 2x + x^2 \log x \cdot e^x + x^2 e^x \cdot \frac{1}{x} \\ &= x e^x (2 \log x + x \log x + 1) \end{aligned}$$

Q4

We have,

$$\begin{aligned} & \frac{d}{dx} (x^n \tan x) \\ &= \tan x \frac{d}{dx} (x^n) + x^n \frac{d}{dx} (\tan x) \quad [\text{Using product rule}] \\ &= \tan x \cdot n x^{n-1} + x^n \sec^2 x \\ &= x^{n-1} (n \tan x + x \sec^2 x) \quad [x^n = x^{n-1} x^1 = x^{n-1+1}] \end{aligned}$$

Q5

We have,

$$\begin{aligned}& \frac{d}{dx} (x^n \log_a x) \\&= \log_a x \frac{d}{dx} (x^n) + x^n \frac{d}{dx} (\log_a x) \quad [\text{Using product rule}] \\&= nx^{n-1} \log_a x + \frac{x^n}{\log a} \cdot \frac{1}{x} \quad \left[\because \log_a x = \frac{\log x}{\log a} \right] \\&= x^{n-1} \left[n \log_a x + \frac{1}{\log a} \right]\end{aligned}$$

Q6

We have,

$$\begin{aligned}& \frac{d}{dx} (x^3 + x^2 + 1) \sin x \\&= \sin x \frac{d}{dx} (x^3 + x^2 + 1) + (x^3 + x^2 + 1) \frac{d}{dx} (\sin x) \quad [\text{Using product rule}] \\&= \sin x (3x^2 + 2x) + (x^3 + x^2 + 1) \cos x \\&\therefore (x^3 + x^2 + 1) \cos x + (3x^2 + 2x) \sin x\end{aligned}$$

Q7

We have,

$$\begin{aligned}& \frac{d}{dx} (\sin x \times \cos x) \\&= \cos x \frac{d}{dx} (\sin x) + \sin x \frac{d}{dx} (\cos x) \quad [\text{using product rule}] \\&= \cos x (\cos x) + \sin x (-\sin x) \\&= \cos^2 x - \sin^2 x \quad [\because \cos 2x = \cos^2 x - \sin^2 x] \\&= \cos 2x\end{aligned}$$

Q8

We have,

$$\begin{aligned}
& \frac{d}{dx} \left(2^x \times \cot x \times x^{-\frac{1}{2}} \right) \\
&= \cot x \times \frac{1}{\sqrt{x}} \times \frac{d}{dx} (2^x) + 2^x \times \frac{1}{\sqrt{x}} \times \frac{d}{dx} (\cot x) + 2^x \times \cot x \times \frac{d}{dx} (x^{-\frac{1}{2}}) \quad [\text{Using product rule}] \\
&= \frac{\cot x}{\sqrt{x}} \times 2^x \times \log 2 + \frac{2^x}{\sqrt{x}} (-\operatorname{cosec}^2 x) + 2^x \times \cot x \left(-\frac{1}{2} \right) \frac{1}{2^{\frac{1}{2}}} \\
&= \frac{2^x}{\sqrt{x}} \left(\cot x \times \log 2 - \operatorname{cosec}^2 x - \frac{\cot x}{2} \right)
\end{aligned}$$

Q9

$$\begin{aligned}
& \frac{d}{dx} (x^2 \sin x \log x) \\
&= \sin x \log x \frac{d}{dx} (x^2) + x^2 \log x \frac{d}{dx} (\sin x) + x^2 \sin x \frac{d}{dx} (\log x) \quad [\text{Using product rule}] \\
&= \sin x \log x \times 2x + x^2 \log x \times \cos x + x^2 \sin x \times \frac{1}{x} \\
&= 2x \times \sin x \times \log x + x^2 \times \cos x \times \log x + x \sin x
\end{aligned}$$

Q10

We have,

$$\begin{aligned}
& \frac{d}{dx} (x^5 e^x + x^6 \log x) \\
&= \frac{d}{dx} (x^5 e^x) + \frac{d}{dx} (x^6 \log x) \\
&= e^x \frac{dx^5}{dx} + x^5 \frac{de^x}{dx} + \log x \frac{d}{dx} (x^6) + x^6 \frac{d}{dx} (\log x) \quad [\text{Using product rule}] \\
&= e^x \times 5x^4 + x^5 \times e^x + \log x \times 6x^5 + x^6 \times \frac{1}{x} \\
&= 5x^4 \times e^x + x^5 \times e^x + 6x^5 \times \log x + x^5 \\
&= x^4 (5e^x + ex^x + 6x \log x + x)
\end{aligned}$$

Q11

We have,

$$\frac{d}{dx} \left\{ (x \sin x + \cos x) (x \cos x - \sin x) \right\}$$

We will apply product rule,

$$\begin{aligned} &= (x \cos x - \sin x) \frac{d}{dx} (x \sin x + \cos x) + (x \sin x + \cos x) \frac{d}{dx} (x \cos x - \sin x) \\ &= (x \cos x - \sin x) \left\{ \frac{d}{dx} (x \sin x) + \frac{d}{dx} (\cos x) \right\} + (x \sin x + \cos x) \left\{ \frac{d}{dx} (x \cos x) - \frac{d}{dx} (\sin x) \right\} \end{aligned}$$

Again apply product rule,

$$\begin{aligned} &= (x \cos x - \sin x) \left\{ \left(\sin x \frac{dx}{dx} + x \frac{d \sin x}{dx} \right) \right\} + (-\sin x) + (x \cos x + \sin x) \left\{ \left(\sin x \frac{dx}{dx} + x \frac{d \cos x}{dx} - \cos x \right) \right\} \\ &= (x \cos x - \sin x) \{ (\sin x + x \cos x) - \sin x \} - \sin x + (x \cos x + \sin x) \{ (\cos x - x \sin x - \cos x) \} \\ &= (x \cos x - \sin x) \times x \cos x + (x \sin x + \cos x) (-x \sin x) \\ &= (x^2 \cos^2 x - x \sin x \times \cos x) + (-x^2 \sin^2 x - x \sin x \times \cos x) \\ &= x^2 (\cos^2 x - \sin^2 x) - x (\sin x \cos x + \sin x \cos x) \\ &= x^2 - \cos 2x - x \times 2 \sin x \cos x \\ &= x^2 \cos 2x - x \sin 2x \\ &= x (x \cos 2x - \sin 2x) \end{aligned}$$

Q12

We have,

$$\frac{d}{dx} \left\{ (x \sin x + \cos x) (e^x + x^2 \log x) \right\}$$

We will apply product rule,

$$\begin{aligned} &= (e^x + x^2 \log x) \frac{d}{dx} (x \sin x + \cos x) + (x \sin x + \cos x) \frac{d}{dx} (e^x + x^2 \log x) \\ &= (e^x + x^2 \log x) \left\{ \frac{d}{dx} (x \sin x) + \frac{d}{dx} (\cos x) \right\} + (x \sin x + \cos x) \times \left\{ \frac{d}{dx} (e^x) + \frac{d}{dx} (x^2 \log x) \right\} \end{aligned}$$

Again apply product rule,

$$\begin{aligned} &= (e^x + x^2 \log x) \left\{ \left(\sin x \frac{d}{dx} (x) + x \frac{d}{dx} (\sin x) \right) - \sin x + (x \sin x + \cos x) \left\{ e^x + \left(\log x \frac{d}{dx} (x^2) + x^2 \frac{d}{dx} (\log x) \right) \right\} \right\} \\ &= (e^x + x^2 \log x) (\sin x + x \cos x - \sin x) + (x \sin x + \cos x) \left(e^x + \log x \times 2x + x^2 \frac{1}{x} \right) \\ &= (e^x + x^2 \log x) x \cos x + (x \sin x + \cos x) (e^x + 2x \log x + x) \\ &= x \cos x e^x + e^x \cos x \log x + x e^x \sin x + e^x \cos x + 2x^2 \sin x \log x + 2x \cos x \log x + x^2 \sin x + x \cos x \\ &= x \cos x (e^x + x^2 \log x) + (x \sin x + \cos x) (e^x + x + 2x \log x) \end{aligned}$$

Q13

We have,

$$\begin{aligned}& \frac{d}{dx} \{(1 - 2 \tan x)(5 + 4 \sin x)\} \\&= (5 + 4 \sin x) \frac{d}{dx} (1 - 2 \tan x) + (1 - 2 \tan x) \frac{d}{dx} (5 + 4 \sin x) \quad [\text{Using product rule,}] \\&= (5 + 4 \sin x)(0 - 2 \sec^2 x) + (1 - 2 \tan x)(0 + 4 \cos x) \\&= -10 \sec^2 x - 8 \sin x \times \sec^2 x + 4 \cos x - 8 \cos x \times \tan x \\&= 4 \left(\frac{-5}{2} \sec^2 x - 2 \sin x \times \frac{1}{\cos^2 x} + \cos x - 2 \cos x \times \frac{\sin x}{\cos x} \right) \\&= 4 \left(\frac{-5}{2} \sec^2 x - 2 \tan x \sec x + \cos x - 2 \sin x \right) \\&= 4 \left(\cos x - 2 \sin x - 2 \tan x \sec x - \frac{5}{2} \sec^2 x \right)\end{aligned}$$

Q14

We have,

$$\begin{aligned}& \frac{d}{dx} \{(1 + x^2) \cos x\} \\&= \cos x \frac{d}{dx} (1 + x^2) + (1 + x^2) \frac{d}{dx} (\cos x) \quad (\text{using product rule}) \\&= \cos x \times 2x + (1 + x^2)(-\sin x) \\&= 2x \cos x - (1 + x^2) \sin x\end{aligned}$$

Q15

We have,

$$\begin{aligned}& \frac{d}{dx} (\sin^2 x) \\&= \frac{d}{dx} (\sin x)(\sin x) \\&= \sin x \frac{d}{dx} (\sin x) + \sin x \frac{d}{dx} (\sin x) \quad [\text{Using product rule}] \\&= \sin x \times \cos x + \sin x \times \cos x \\&= 2 \sin x \cos x \\&= \sin 2x \quad [\because \sin 2A = 2 \sin A \cos A]\end{aligned}$$

Q16

We have,

$$\frac{d}{dx}(\log_{x^2} x)$$

$$\begin{aligned}\log_{x^2} x &= \frac{\log x}{\log x^2} \\ &= \frac{\log x}{2 \log x} \\ &= \frac{1}{2}\end{aligned}$$

$$\frac{d}{dx}\left(\frac{1}{2}\right) = 0$$

$$\therefore \frac{d}{dx}(\log_{x^2} x) = 0$$

Q17

$$\frac{d}{dx}(e^x \log \sqrt{x} \tan x)$$

Apply product rule,

$$\begin{aligned}&= \log \sqrt{x} \times \tan x \frac{d}{dx}(e^x) + e^x \times \tan x \frac{d}{dx}(\log \sqrt{x}) + e^x \log \sqrt{x} \frac{d}{dx}(\tan x) \\ &= \log \sqrt{x} \times \tan x e^x + e^x \tan x \frac{1}{2x} + e^x \log \sqrt{x} \times \sec^2 x \\ &= \frac{1}{2} \log x \times \tan x \times e^x + \frac{1 \tan x}{2x} e^x + e^x \frac{1}{2} \log x \sec^2 x \quad \left[\because \log \sqrt{x} = \frac{1}{2} \log x \right] \\ &= \frac{1}{2} e^x \left(\log x \times \tan x + \frac{\tan x}{x} + \log x \sec^2 x \right)\end{aligned}$$

Q18

We have,

$$\begin{aligned}&\frac{d}{dx}(x^3 e^x \cos x) \\ &= e^x \cos x \frac{d}{dx}(x^3) + x^3 \cos x \frac{d}{dx}(e^x) + x^3 e^x \frac{d}{dx}(\cos x) \quad [\text{Using product rule}] \\ &= e^x \cos x \times 3x^2 + x^3 \cos x \times e^x + x^3 e^x (-\sin x) \\ &= x^2 e^x (3 \cos x + x \cos x + x (-\sin x)) \\ &= x^2 e^x (3 \cos x + x \cos x - x \sin x)\end{aligned}$$

Q19

We have,

$$\begin{aligned}& \frac{d}{dx} \left(x^2 \cos \frac{\pi}{4} \times \operatorname{cosec} x \right) \\&= \cos \frac{\pi}{4} \operatorname{cosec} x \frac{d}{dx} (x^2) + x^2 \operatorname{cosec} x \frac{d}{dx} \left(\cos \frac{\pi}{4} \right) + x^2 \cos \frac{\pi}{4} \frac{d}{dx} (\operatorname{cosec} x) \quad [\text{Using product rule,}] \\&= \cos \frac{\pi}{4} \operatorname{cosec} x \times 2x + x^2 \operatorname{cosec} x \times 0 + x^2 \cos \frac{\pi}{4} (-\operatorname{cosec} x \cot x) \quad \left[\because \frac{d}{dx} \left(\cos \frac{\pi}{4} \right) = 0 \right] \\&= \left(\frac{2x}{\sin x} - \frac{x^2 \operatorname{cosec} x}{\sin^2 x} \right) \cos \frac{\pi}{4} \\&\therefore \cos \frac{\pi}{4} \left(\frac{2x}{\sin x} - \frac{x^2 \cos x}{\sin x} \right)\end{aligned}$$

Q20

We have,

$$\begin{aligned}&= \frac{d}{dx} \left\{ x^4 (5 \sin x - 3 \cos x) \right\} \\&= \frac{d}{dx} x^4 5 \sin x - 3x^4 \cos x \\&= 5 \frac{d}{dx} (x^4 \sin x) - 3 \frac{d}{dx} (x^4 \cos x) \\&= 5 \left(\sin x \frac{d}{dx} (x^4) + x^4 \frac{d}{dx} (\sin x) \right) - 3 \left(\cos x \frac{d}{dx} (x^4) + x^4 \frac{d}{dx} (\cos x) \right) \quad [\text{Apply product rule,}] \\&= 5 (\sin x \times 4x^3 + x^4 \times \cos x) - 3 (\cos x \times 4x^3 + x^4 (-\sin x)) \\&= 20x^3 \times \sin x + 5x^4 \cos x - 12x^3 \cos x + 3x^4 \sin x\end{aligned}$$

Q21

We have,

$$\begin{aligned}& \frac{d}{dx} (2x^2 - 3) \sin x \\&= \sin x \frac{d}{dx} (2x^2 - 3) + (2x^2 - 3) \frac{d}{dx} (\sin x) \quad [\text{Using product rule}] \\&= \sin x \times 4x + (2x^2 - 3) \cos x \\&= 4x \sin x + (2x^2 - 3) \cos x\end{aligned}$$

Q22

We have,

$$\begin{aligned} & \frac{d}{dx} x^5 (3 - 6x^{-9}) \\ &= (3 - 6x^{-9}) \frac{d}{dx} (x^5) + x^5 \frac{d}{dx} (3 - 6x^{-9}) \quad [\text{Using product rule}] \\ &= (3 - 6x^{-9}) 5x^4 + x^5 (54x^{-10}) \\ &= 15x^4 - 30x^5 + 54x^{-5} \\ &= 15x^4 + 24x^{-5} \end{aligned}$$

Q23

We have,

$$\begin{aligned} & \frac{d}{dx} \{x^{-4} (3 - 4x^{-5})\} \\ &= (3 - 4x^{-5}) \frac{d}{dx} (x^{-4}) + x^{-4} \frac{d}{dx} (3 - 4x^{-5}) \quad [\text{Using product rule}] \\ &= (3 - 4x^{-5}) (-4x^{-5}) + (x^{-4}) 20x^{-6} \\ &= -12x^{-5} + 16x^{-10} + 20x^{-10} \\ &= -12x^{-5} + 36x^{-10} \end{aligned}$$

Q24

We have,

$$\frac{d}{dx} \{x^{-3} (5 + 3x)\}$$

Apply product rule,

$$\begin{aligned} &= 5 + 3x \frac{d}{dx} (x^{-3}) + x^{-3} \frac{d}{dx} (5 + 3x) \\ &= (5 + 3x) (-3x^{-4}) + x^{-3} (3) \\ &= -15x^{-4} - 9x^{-3} + 3x^{-3} \\ &= -15x^{-4} - 6x^{-3} \end{aligned}$$

Q25

$$\begin{aligned}
\frac{d}{dx} \left(\frac{(ax+b)}{(cx+d)} \right) &= \left(\frac{1}{cx+d} \right) \frac{d}{dx} (ax+b) + (ax+b) \frac{d}{dx} \left(\frac{1}{cx+d} \right) \\
&= \left(\frac{1}{cx+d} \right) (a) + (ax+b) \frac{d}{dx} (cx+d)^{-1} \\
&= \left(\frac{1}{cx+d} \right) (a) + (ax+b) \left(-1 \times (cx+d)^{-2} \times c \right) \\
&= \left(\frac{a}{cx+d} \right) - \left(\frac{c(ax+b)}{(cx+d)^2} \right) \\
&= \frac{a(cx+d) - c(ax+b)}{(cx+d)^2} \\
&= \frac{ad - bc}{(cx+d)^2}
\end{aligned}$$

Q26

$$\begin{aligned}
\frac{d}{dx} (ax+b)^n (cx+d)^m &= (cx+d)^m \frac{d}{dx} (ax+b)^n + (ax+b)^n \frac{d}{dx} (cx+d)^m \\
&= (cx+d)^m \left[n \times (ax+b)^{n-1} \times a \right] + (ax+b)^n \left[m \times (cx+d)^{m-1} \times c \right] \\
&= na(cx+d)^m (ax+b)^{n-1} + mc(ax+b)^n (cx+d)^{m-1} \\
&= na(cx+d)^{m-1} (ax+b)^{n-1} [na(cx+d) + mc(ax+b)]
\end{aligned}$$

Q27

Using product rule

$$\begin{aligned}
 \frac{d}{dx}(1+2\tan x)(5+4\cos x) &= (1+2\tan x)\frac{d}{dx}(5+4\cos x) + (5+4\cos x)\frac{d}{dx}(1+2\tan x) \\
 &= (1+2\tan x)[0+4(-\sin x)] + (5+4\cos x)[0+2(\sec^2 x)] \\
 &= -4(1+2\tan x)\sin x + 2(5+4\cos x)\sec^2 x \\
 &= -4\sin x - 8\tan x \sin x + 10\sec^2 x + 8\cos x \sec^2 x \\
 &= -4\sin x - \frac{8\sin^2 x}{\cos x} + 10\sec^2 x + \frac{8}{\cos x} \\
 &= -4\sin x + 10\sec^2 x + \frac{8[1-\sin^2 x]}{\cos x} \\
 &= -4\sin x + 10\sec^2 x + \frac{8\cos^2 x}{\cos x} \\
 &= -4\sin x + 10\sec^2 x + 8\cos x
 \end{aligned}$$

Using alternate method

$$\begin{aligned}
 \frac{d}{dx}(1+2\tan x)(5+4\cos x) &= \frac{d}{dx}(5+4\cos x+10\tan x+8\tan x \cos x) \\
 &= \frac{d}{dx}(5+4\cos x+10\tan x+8\sin x) \\
 &= -0+4(-\sin x)+10(\sec^2 x)+8\cos x \\
 &= -4\sin x+10\sec^2 x+8\cos x
 \end{aligned}$$

Q28(i)

Using product rule

$$\begin{aligned}
 \frac{d}{dx}(3x^2+2)^2 &= (3x^2+2)\frac{d}{dx}(3x^2+2) + (3x^2+2)\frac{d}{dx}(3x^2+2) \\
 &= (3x^2+2)(6x+0) + (3x^2+2)(6x+0) \\
 &= 18x^3+12x+18x^3+12x \\
 &= 36x^3+24x
 \end{aligned}$$

Using alternate method

$$\begin{aligned}
 \frac{d}{dx}(3x^2+2)^2 &= \frac{d}{dx}(9x^4+12x^2+4) \\
 &= (36x^3+24x+0) \\
 &= 36x^3+24x
 \end{aligned}$$

Q28(ii)

Using product rule

$$\begin{aligned}\frac{d}{dx}(x+2)(x+3) &= (x+2)\frac{d}{dx}(x+3) + (x+3)\frac{d}{dx}(x+2) \\ &= (x+2)(1+0) + (x+3)(1+0) \\ &= x+2+x+3 \\ &= 2x+5\end{aligned}$$

Using alternate method

$$\begin{aligned}\frac{d}{dx}(x+2)(x+3) &= \frac{d}{dx}(x^2+5x+6) \\ &= (2x+5+0) \\ &= 2x+5\end{aligned}$$

Q28(iii)

Using product rule

$$\begin{aligned}&\frac{d}{dx}(3\sec x - 4\operatorname{cosec} x)(-2\sin x - 5\cos x) \\ &= (3\sec x - 4\operatorname{cosec} x)\frac{d}{dx}(-2\sin x + 5\cos x) + (-2\sin x + 5\cos x)\frac{d}{dx}(3\sec x - 4\operatorname{cosec} x) \\ &= (3\sec x - 4\operatorname{cosec} x)(-2\cos x + 5(-\sin x)) + (-2\sin x + 5\cos x)(3\sec x \tan x - 4(-\operatorname{cosec} x \cot x)) \\ &= -6\sec x \cos x - 15\sec x \sin x + 8\operatorname{cosec} x \cos x - 20\operatorname{cosec} x \sin x \\ &\quad - 6\sin x \sec x \tan x - 8\sin x \operatorname{cosec} x \cot x + 15\cos x \sec x \tan x + 20\cos x \operatorname{cosec} x \cot x \\ &= -6 - 15\tan x + 8\cot x + 20 - 6\tan^2 x - 8\cot x + 15\tan x + 20\cot^2 x \\ &= -6 - 5\tan^2 x + 20 + 20\cot^2 x \\ &= -6(1 + \tan^2 x) + 20(1 + \cot^2 x) \\ &= -6\sec^2 x + 20\operatorname{cosec}^2 x\end{aligned}$$

Using alternate method

$$\begin{aligned}&\frac{d}{dx}(3\sec x - 4\operatorname{cosec} x)(-2\sin x - 5\cos x) \\ &= \frac{d}{dx}(-6\sec x \sin x + 15\sec x \cos x + 8\operatorname{cosec} x \sin x - 20\operatorname{cosec} x \cos x) \\ &= \frac{d}{dx}(-6\tan x + 15 + 8 - 20\cot x) \\ &= -6\sec^2 x + 0 + 0 - 20(-\operatorname{cosec}^2 x) \\ &= -6\sec^2 x + 20\operatorname{cosec}^2 x\end{aligned}$$

EX – 30.5

Q1

Using quotient rule, we have

$$\begin{aligned}\frac{d}{dx} \left(\frac{x^2+1}{x+1} \right) &= \frac{(x+1) \frac{d}{dx} (x^2+1) - (x^2+1) \frac{d}{dx} (x+1)}{(x+1)^2} \\&= \frac{(x+1) \times 2x - (x^2+1) \times 1}{(x+1)^2} \\&= \frac{2x^2+2x-x^2-1}{(x+1)^2} \\&= \frac{x^2+2x-1}{(x+1)^2}\end{aligned}$$

Q2

Using quotient rule, we have get,

$$\begin{aligned}\frac{d}{dx} \left(\frac{2x-1}{x^2+1} \right) &= \frac{(x^2+1) \frac{d}{dx} (2x-1) - (2x-1) \frac{d}{dx} (x^2+1)}{(x^2+1)^2} \\&= \frac{(x^2+1) \times 2 - (2x-1) \times 2x}{(x^2+1)^2} \\&= \frac{2x^2+2-4x^2+2x}{(x^2+1)^2} \\&= \frac{-2x^2+2x+2}{(x^2+1)^2} \\&= \frac{2(-x^2+x+1)}{(x^2+1)^2} \\&= \frac{2(1+x-x^2)}{(1+x^2)^2}\end{aligned}$$

Q3

By using quotient rule, we have,

$$\begin{aligned}& \frac{d}{dx} \left(\frac{x + e^x}{1 + \log x} \right) \\&= \frac{(1 + \log x) \frac{d}{dx} (x + e^x) - (x + e^x) \frac{d}{dx} (1 + \log x)}{(1 + \log x)^2} \\&= \frac{(1 + \log x) (1 + e^x) - (x + e^x) \times \frac{d}{dx}}{(1 + \log x)^2} \\&= \frac{x (1 + \log x + e^x + e^x \log x) - x - e^x}{x (1 + \log x)^2} \\&= \frac{x + x \log x + x e^x + x e^x \log x - x - e^x}{x (1 + \log x)^2} \\&= \frac{x \log x (1 + e^x) - e^x (1 - x)}{x (1 + \log x)^2}\end{aligned}$$

Q4

Using quotient rule, we have,

$$\begin{aligned}& \frac{d}{dx} \left(\frac{e^x - \tan x}{\cot x - x^n} \right) \\&= \frac{(\cot x - x^n) \frac{d}{dx} (e^x - \tan x) - (e^x - \tan x) \frac{d}{dx} (\cot x - x^n)}{(\cot x - x^n)^2} \\&= \frac{(\cot x - x^n) (e^x - \sec^2 x) - (e^x - \tan x) (-\operatorname{cosec}^2 x - nx^{n-1})}{(\cot x - x^n)^2} \\&= \frac{(\cot x - x^n) (e^x - \sec^2 x) + (e^x - \tan x) (\operatorname{cosec}^2 x + nx^{n-1})}{(\cot x - x^n)^2}\end{aligned}$$

Q5

Using quotient rule, we have,

$$\begin{aligned}& \frac{d}{dx} \left(\frac{ax^2 + bx + c}{px^2 + qx + r} \right) \\&= \frac{(px^2 + qx + r) \frac{d}{dx} (ax^2 + bx + c) - (ax^2 + bx + c) \frac{d}{dx} (px^2 + qx + r)}{(px^2 + qx + r)^2} \\&= \frac{(px^2 + qx + r)(2ax + b) - (ax^2 + bx + c)(2px + q)}{(px^2 + qx + r)^2} \\&= \frac{2apx^3 + 2aqx^2 + 2axr + bpx^2 + bq x + br - (2apx^3 + 2pbx^2 + 2pcx + qax^2 + bq x + cq)}{(px^2 + qx + r)^2} \\&= \frac{2apx^3 - 2apx^3 + 2aqx^2 + bpx^2 - 2pbx^2 - qax^2 + 2arx + bq x - 2pcx - bq x + br - cq}{(px^2 + qx + r)^2} \\&= \frac{aqx^2 - bpx^2 + 2arx - 2pcx + br - cq}{(px^2 + qx + r)^2} \\&= \frac{x^2(aq - bp) + 2(ar - pc)x + br - cq}{(px^2 + qx + r)^2} \\&= \frac{(aq - bp)x^2 + 2(ar - pc)x + br - cq}{(px^2 + qx + r)^2}\end{aligned}$$

Q6

Using quotient rule, we have,

$$\begin{aligned}& \frac{d}{dx} \left(\frac{x}{1 + \tan x} \right) \\&= \frac{(1 + \tan x) \frac{d}{dx} (x) - x \frac{d}{dx} (1 + \tan x)}{(1 + \tan x)^2} \\&= \frac{(1 + \tan x) - x(\sec^2 x)}{(1 + \tan x)^2} \\&= \frac{1 + \tan x - x \sec^2 x}{(1 + \tan x)^2}\end{aligned}$$

Q7

Using quotient rule, we have

$$\begin{aligned}& \frac{d}{dx} \left(\frac{1}{ax^2 + bx + c} \right) \\&= \frac{(ax^2 + bx + c) \frac{d}{dx} (1) - 1 \times \frac{d}{dx} (ax^2 + bx + c)}{(ax^2 + bx + c)^2} \\&= \frac{-(2ax + b)}{(ax^2 + bx + c)^2} \\&\therefore \frac{d}{dx} \frac{1}{ax^2 + bx + c} = \frac{-(2ax + b)}{(ax^2 + bx + c)^2}\end{aligned}$$

Q8

We have,

$$\frac{d}{dx} \left(\frac{e^x}{1+x^2} \right)$$

Using quotient rule,

$$\begin{aligned}&= \frac{(1+x^2) \frac{d}{dx} (e^x) - (e^x) \frac{d}{dx} (1+x^2)}{(1+x^2)^2} \\&= \frac{(1+x^2) e^x - e^x \times 2x}{(1+x^2)^2} \\&= \frac{e^x (1+x^2 - 2x)}{(1+x^2)^2} \\&= \frac{e^x (1-x)^2}{(1+x^2)^2}\end{aligned}$$

Q9

We have,

$$\frac{d}{dx} \left(\frac{e^x + \sin x}{1 + \log x} \right)$$

Using quotient rule, we get

$$\begin{aligned} &= \frac{(1 + \log x) \frac{d}{dx} (e^x + \sin x) - (e^x + \sin x) \frac{d}{dx} (1 + \log x)}{(1 + \log x)^2} \\ &= \frac{(1 + \log x) (e^x + \cos x) - (e^x + \sin x) \frac{1}{x}}{(1 + \log x)^2} \\ &= \frac{x(1 + \log x) (e^x + \cos x) - (e^x + \sin x)}{x(1 + \log x)^2} \end{aligned}$$

Q10

We have,

$$\frac{d}{dx} \left(\frac{x \tan x}{\sec x + \tan x} \right)$$

Using quotient rule, we get

$$\begin{aligned} &= \frac{(\sec x + \tan x) \frac{d}{dx} (x \tan x) - (x \tan x) \frac{d}{dx} (\sec x + \tan x)}{(\sec x + \tan x)^2} \\ &= \frac{(\sec x + \tan x) \{x \sec^2 x + \tan x\} - (x \tan x) \{\sec x \tan x + \sec^2 x\}}{(\sec x + \tan x)^2} \quad [\text{Used product rule}] \\ &= \frac{(\sec x + \tan x) \{x \sec^2 x + \tan x\} - x \sec x + \tan^2 x - x \tan x \sec^2 x}{(\sec x + \tan x)^2} \\ &= \frac{(\sec x + \tan x) \{x \sec^2 x + \tan x\} - x \tan x \{\sec x \tan x + \sec^2 x\}}{(\sec x + \tan x)^2} \\ &= \frac{(\sec x + \tan x) \{x \sec^2 x + \tan x\} - x \tan x \sec x (\sec x + \tan x)}{(\sec x + \tan x)^2} \\ &= \frac{(x \sec^2 x + \tan x - x \tan x \sec x) (\sec x + \tan x)}{(\sec x + \tan x)^2} \\ &= \frac{(x \sec^2 x + \tan x - x \tan x \sec x)}{(\sec x + \tan x)} \\ &= \frac{x \sec x (\sec x - \tan x) + \tan x}{(\sec x + \tan x)} \end{aligned}$$

Q11

We have,

$$\frac{d}{dx} \left(\frac{x \sin x}{1 + \cos x} \right)$$

Using quotient rule, we get

$$\begin{aligned} &= \frac{(1 + \cos x) \frac{d}{dx} (x \sin x) - (x \sin x) \frac{d}{dx} (1 + \cos x)}{(1 + \cos x)^2} \\ &= \frac{(1 + \cos x) \left(x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x \right) - x \sin x (-\sin x)}{(1 + \cos x)^2} \quad [\text{Used product rule}] \\ &= \frac{(1 + \cos x) (x \cos x + \sin x) + x \sin^2 x}{(1 + \cos x)^2} \\ &= \frac{x \cos x + x \cos^2 x + \sin x + \sin x \cos x + x \sin^2 x}{(1 + \cos x)^2} \\ &= \frac{(x \cos x + \sin x + \sin x \cos x) + x (\sin^2 x + \cos^2 x)}{(1 + \cos x)^2} \\ &= \frac{\sin x + \sin x \cos x + x (\cos x + \sin^2 x + \cos^2 x)}{(1 + \cos x)^2} \\ &= \frac{\sin x (1 + \cos x) + x (\cos x + 1)}{(1 + \cos x)^2} \\ &= \frac{(x + \sin x) (\cos x + 1)}{(1 + \cos x)^2} \end{aligned}$$

Q12

We have,

$$\frac{d}{dx} \left(\frac{2^x \cot x}{\sqrt{x}} \right)$$

Using quotient rule, we get

$$\begin{aligned} & \frac{\sqrt{x} \frac{d}{dx} (2^x \cot x) - (2^x \cot x) \frac{d}{dx} (\sqrt{x})}{(\sqrt{x})^2} \\ &= \frac{\sqrt{x} \left(2^x \frac{d}{dx} \cot x + \cot x \frac{d}{dx} 2^x \right) - 2^x \cot x \times \frac{1}{2} x^{-1/2}}{(\sqrt{x})^2} \\ &= \frac{\sqrt{x} (2^x - \operatorname{cosec}^2 x + \cot x \times \log 2 \times 2^x) - 2^x \cot x \times \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} \\ &= \frac{2^x \left\{ -x \operatorname{cosec}^2 x + x \cot x \times \log 2 - \left(\frac{1}{2} \right) \cot x \right\}}{(\sqrt{x})^2 \times \sqrt{x}} \\ &= \frac{2^x \left(-x \operatorname{cosec}^2 x + x \cot x \times \log 2 - \left(\frac{1}{2} \right) \cot x \right)}{x^{3/2}} \end{aligned}$$

Q13

We have,

$$\frac{d}{dx} \left(\frac{\sin x - x \cos x}{x \sin x + \cos x} \right)$$

Apply quotient rule, we get

$$\begin{aligned} & \frac{(x \sin x + \cos x) \frac{d}{dx} (\sin x - x \cos x) - (\sin x - x \cos x) \frac{d}{dx} (x \sin x + \cos x)}{(x \sin x + \cos x)^2} \\ &= \frac{(x \sin x + \cos x) \left\{ \cos x - \left(\frac{dx}{dx} \cos x + \cos x \frac{dx}{dx} \right) \right\} - (\sin x - x \cos x) \left(\frac{dx}{dx} \sin x + \sin x \frac{dx}{dx} \right) + \frac{d}{dx} \cos x}{(x \sin x + \cos x)^2} \\ &= \frac{(x \sin x + \cos x) (\cos x + x \sin x - \cos x) - (\sin x - x \cos x) (x \cos x + \sin x - \sin x)}{(x \sin x + \cos x)^2} \\ &= \frac{(x \sin x + \cos x) x \sin x - (\sin x - x \cos x) x \cos x}{(x \sin x + \cos x)^2} \\ &= \frac{x^2 \sin^2 x + x \sin x \cos x - x \sin x \cos x + x^2 \cos^2 x}{(x \sin x + \cos x)^2} \\ &= \frac{x^2 (\sin^2 x + \cos^2 x)}{(x \sin x + \cos x)^2} \quad \left(\because \sin^2 x + \cos^2 x = 1 \right) \\ &= \frac{x^2}{x \sin x + \cos x} \end{aligned}$$

Q14

We have,

$$\frac{d}{dx} \left(\frac{x^2 - x + 1}{x^2 + x + 1} \right)$$

Using quotient rule,

$$\begin{aligned} & \frac{(x^2 + x + 1) \frac{d}{dx} (x^2 - x + 1) - (x^2 - x + 1) \frac{d}{dx} (x^2 + x + 1)}{(x^2 + x + 1)^2} \\ &= \frac{(x^2 + x + 1)(2x - 1) - (x^2 - x + 1)(2x + 1)}{(x^2 + x + 1)^2} \\ &= \frac{(x^2 + 1 - x)(2x - 1) - (x^2 - x + 1)(2x + 1)}{(x^2 + x + 1)^2} \\ &= \frac{2x^3 + 2x + 2x^2 - x^2 - 1 - x - 2x^3 + 2x^2 - 2x - x^2 + x - 1}{(x^2 + x + 1)^2} \\ &= \frac{2x^2 - 2}{(x^2 + x + 1)^2} \\ &= \frac{2(x^2 - 1)}{(x^2 + x + 1)^2} \end{aligned}$$

Q15

We have,

$$\frac{d}{dx} \left(\frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}} \right)$$

Using quotient rule,

$$\begin{aligned} & \frac{(\sqrt{a} - \sqrt{x}) \frac{d}{dx} (\sqrt{a} + \sqrt{x}) - (\sqrt{a} + \sqrt{x}) \frac{d}{dx} (\sqrt{a} - \sqrt{x})}{(\sqrt{a} - \sqrt{x})^2} \\ &= \frac{(\sqrt{a} - \sqrt{x}) \times \frac{1}{2\sqrt{x}} - (\sqrt{a} + \sqrt{x}) \times \frac{-1}{2\sqrt{x}}}{(\sqrt{a} - \sqrt{x})^2} \\ &= \frac{\sqrt{a} - \sqrt{x} + \sqrt{a} + \sqrt{x}}{2\sqrt{x} (\sqrt{a} - \sqrt{x})^2} \\ &= \frac{\sqrt{a}}{\sqrt{x} (\sqrt{a} - \sqrt{x})^2} \end{aligned}$$

Q16

We have,

$$\frac{d}{dx} \left(\frac{a + \sin x}{1 + a \sin x} \right)$$

Using quotient rule, we get

$$\begin{aligned} & \frac{(1 + a \sin x) \frac{d}{dx} (a + \sin x) - (a + \sin x) \frac{d}{dx} (1 + a \sin x)}{(1 + a \sin x)^2} \\ &= \frac{(1 + a \sin x) \cos x - (a + \sin x) a \cos x}{(1 + a \sin x)^2} \\ &= \frac{\cos x + a \sin x \cos x - a^2 \cos x - a \sin x \cos x}{(1 + a \sin x)^2} \\ &= \frac{(1 - a^2) \cos x}{(1 + a \sin x)^2} \end{aligned}$$

Q17

We have,

$$\frac{d}{dx} \left(\frac{10^x}{\sin x} \right)$$

Using quotient rule,

$$\begin{aligned} & \frac{(\sin x) \frac{d}{dx} (10^x) - (10^x) \frac{d}{dx} (\sin x)}{(\sin x)^2} \\ &= \frac{\sin x \times 10^x \log 10 - 10^x \cos x}{(\sin x)^2} \\ &= 10^x \operatorname{cosec} x \log 10 - 10^x \operatorname{cosec} x \cot x \\ &= 10^x \operatorname{cosec} x (\log 10 - \cot x) \end{aligned}$$

Q18

We have,

$$\frac{d}{dx} \left(\frac{1+3^x}{1-3^x} \right)$$

Using quotient rule,

$$\begin{aligned} & \frac{(1-3^x) \frac{d}{dx} (1+3^x) - (1+3^x) \frac{d}{dx} (1-3^x)}{(1-3^x)^2} \\ &= \frac{(1-3^x) 3^x \log 3 + (1+3^x) 3^x \log 3}{(1-3^x)^2} \\ &= \frac{3^x \log 3 - 3^x \times 3^x \log 3 + 3^x \log 3 + 3^x \times 3^x \log 3}{(1-3^x)^2} \\ &= \frac{2 \times 3^x \log 3}{(1-3^x)^2} \end{aligned}$$

Q19

We have,

$$\frac{d}{dx} \left(\frac{3^x}{x + \tan x} \right)$$

Applying quotient rule,

$$\begin{aligned} & \frac{(x + \tan x) \frac{d}{dx} (3^x) - 3^x \frac{d}{dx} (x + \tan x)}{(x + \tan x)^2} \\ &= \frac{(x + \tan x) \times 3^x \log 3 - 3^x (1 + \sec^2 x)}{(x + \tan x)^2} \\ &= \frac{3^x \{(x + \tan x) \log 3 - (1 + \sec^2 x)\}}{(x + \tan x)^2} \end{aligned}$$

Q20

We have,

$$\frac{d}{dx} \left(\frac{1 + \log x}{1 - \log x} \right)$$

Using quotient rule,

$$\begin{aligned} & \frac{(1 - \log x) \frac{d}{dx} (1 + \log x) - (1 + \log x) \frac{d}{dx} (1 - \log x)}{(1 - \log x)^2} \\ &= \frac{(1 - \log x) \times \frac{1}{x} - (1 + \log x) \left(-\frac{1}{x} \right)}{(1 - \log x)^2} \\ &= \frac{1 - \log x + 1 + \log x}{x (1 - \log x)^2} \\ &= \frac{2}{x (1 - \log x)^2} \end{aligned}$$

Q21

We have,

$$\frac{d}{dx} \left(\frac{4x + 5 \sin x}{3x + 7 \cos x} \right)$$

Using quotient rule, we get

$$\begin{aligned} & \frac{(3x + 7 \cos x) \frac{d}{dx} (4x + 5 \sin x) - (4x + 5 \sin x) \frac{d}{dx} (3x + 7 \cos x)}{(3x + 7 \cos x)^2} \\ &= \frac{(3x + 7 \cos x) (4x + 5 \sin x) - (4x + 5 \sin x) (3x + 7(-\sin x))}{(3x + 7 \cos x)^2} \\ &= \frac{12x + 28 \cos x + 15x \cos x + 13 \cos^2 x - 12x - 15 \sin x + 28x \sin x + 25 \sin^2 x}{(3x + 7 \cos x)^2} \\ &= \frac{15x \cos x + 28x \sin x + 28 \cos x - 15 \sin x + 35 (\sin^2 x + \cos^2 x)}{(3x + 7 \cos x)^2} \quad (\because \sin^2 x + \cos^2 x = 1) \\ &\therefore \frac{15x \cos x + 28x \sin x + 28 \cos x - 15 \sin x + 35}{(3x + 7 \cos x)^2} \end{aligned}$$

Q22

$$\begin{aligned} & \frac{d}{dx} \left(\frac{1}{ax^2 + bx + c} \right) \\ &= \frac{d}{dx} (ax^2 + bx + c)^{-1} \\ &= \frac{-(2ax + b)}{(ax^2 + bx + c)^2} \end{aligned}$$

Q23

We have,

$$\frac{d}{dx} \left(\frac{a + b \sin x}{c + d \cos x} \right)$$

Using quotient rule, we get

$$\begin{aligned} & \frac{(c + d \cos x) \frac{d}{dx} (a + b \sin x) - (a + b \sin x) \frac{d}{dx} (c + d \cos x)}{(c + d \cos x)^2} \\ &= \frac{(c + d \cos x) (b \cos x) - (a + b \sin x) (-d \sin x)}{(c + d \cos x)^2} \\ &= \frac{bc \cos x + bd \cos^2 x + ad \sin x + bd \sin^2 x}{(c + d \cos x)^2} \\ &= \frac{bc \cos x + ad \sin x + bd (\sin^2 x + \cos^2 x)}{(c + d \cos x)^2} \\ &= \frac{bc \cos x + ad \sin x + bd}{(c + d \cos x)^2} \end{aligned}$$

Q24

We have,

$$\frac{d}{dx} \left(\frac{px^2 + qx + r}{ax + b} \right)$$

Using quotient rule, we get

$$\begin{aligned} & \frac{(ax + b) \frac{d}{dx} (px^2 + qx + r) - (px^2 + qx + r) \frac{d}{dx} (ax + b)}{(ax + b)^2} \\ &= \frac{(ax + b) (2px + q) - (px^2 + qx + r) a}{(ax + b)^2} \\ &= \frac{2apx^2 + 2pbx + aqx + bq - apx^2 - aqx - ar}{(ax + b)^2} \\ &= \frac{apx^2 + 2pbx + bq - ar}{(ax + b)^2} \end{aligned}$$

Q25

We have,

$$\frac{d}{dx} \left(\frac{\sec x - 1}{\sec x + 1} \right)$$

$$\begin{aligned} & \frac{(\sec x + 1) \frac{d}{dx} (\sec x - 1) - (\sec x - 1) \frac{d}{dx} (\sec x + 1)}{(\sec x + 1)^2} \\ &= \frac{(\sec x + 1) (\sec x \tan x) - (\sec x - 1) (\sec x \tan x)}{(\sec x + 1)^2} \\ &= \frac{\sec x \tan x (\sec x + 1 - \sec x + 1)}{(\sec x + 1)^2} \\ &= \frac{2 \sec x \tan x}{(\sec x + 1)^2} \end{aligned}$$

Q26

We have,

$$\frac{d}{dx} \left(\frac{x^5 - \cos x}{\sin x} \right)$$

Using quotient rule, we get

$$\begin{aligned} & \frac{(\sin x) \frac{d}{dx} (x^5 - \cos x) - (x^5 - \cos x) \frac{d}{dx} (\sin x)}{(\sin^2 x)} \\ &= \frac{\sin x (5x^4 \sin x) - (x^5 - \cos x) \cos x}{(\sin^2 x)} \\ &= \frac{5x^4 \sin x + \sin^2 x - x^5 \cos x + \cos^2 x}{(\sin^2 x)} \quad (\because \sin^2 x + \cos^2 x = 1) \\ &= \frac{-x^5 \cos x + 5x^4 \sin x + 1}{(\sin^2 x)} \end{aligned}$$

Q27

We have,

$$\frac{d}{dx} \left(\frac{x + \cos x}{\tan x} \right)$$

Using quotient rule, we get

$$\begin{aligned} & \frac{(\tan x) \frac{d}{dx} (x + \cos x) - (x + \cos x) \frac{d}{dx} (\tan x)}{(\tan^2 x)} \\ &= \frac{\tan x \{1 + (-\sin x)\} - (x + \cos x) \sec^2 x}{(\tan^2 x)} \\ &= \frac{(1 - \sin x) \tan x - (x + \cos x) \sec^2 x}{(\tan^2 x)} \end{aligned}$$

Q28

$$\begin{aligned} & \frac{d}{dx} \left(\frac{x^n}{\sin x} \right) \\ &= x^n \frac{d}{dx} (\sin x)^{-1} + \frac{1}{\sin x} \frac{d}{dx} (x^n) \\ &= x^n \frac{-1}{\sin^2 x} + \frac{1}{\sin x} n x^{n-1} \\ &= \frac{\sin x (n x^{n-1}) - x^n (\cos x)}{\sin^2 x} \end{aligned}$$

Q29

$$\begin{aligned} \frac{d}{dx} \left(\frac{ax+b}{px^2+qx+r} \right) &= \frac{(px^2+qx+r) \frac{d}{dx} (ax+b) - (ax+b) \frac{d}{dx} (px^2+qx+r)}{(px^2+qx+r)^2} \\ &= \frac{(px^2+qx+r)(a) - (ax+b)(2px+q)}{(px^2+qx+r)^2} \\ &= \frac{(apx^2+aqx+ar) - (2apx^2+aqx+2bpx+bq)}{(px^2+qx+r)^2} \\ &= \frac{-(apx^2+2bpx+bq-ar)}{(px^2+qx+r)^2} \end{aligned}$$

Q30

$$\begin{aligned}& \frac{d}{dx} \left(\frac{1}{ax^2 + bx + c} \right) \\&= \frac{(ax^2 + bx + c) \frac{d}{dx}(1) - (1) \frac{d}{dx}(ax^2 + bx + c)}{(ax^2 + bx + c)^2} \\&= \frac{(ax^2 + bx + c)(0) - (1)(2ax + b)}{(ax^2 + bx + c)^2} \\&= \frac{-(2ax + b)}{(ax^2 + bx + c)^2}\end{aligned}$$