Ex 8.1

Q1

(i) $2\sin 3\theta \cos\theta$

$= \sin(3\theta+\theta) + \sin(3\theta-\theta)$ $\quad [\because 2\sin A\cos B = \sin(A+B)+\sin(A-B)]$

$= \sin 4\theta + \sin 2\theta$

(ii) $2\cos 3\theta \sin 2\theta$

$\because$ $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$\Rightarrow$ $2\cos 3\theta \sin 2\theta = \sin(3\theta+2\theta) - \sin(3\theta-2\theta)$

$=$ $\sin 5\theta - \sin\theta$

(iii) $2\sin 4\theta \sin 3\theta$

$\because$ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$\Rightarrow$ $2\sin 4\theta \sin 3\theta = \cos(4\theta-3\theta) - \cos(4\theta+3\theta)$

$= \cos\theta - \cos 7\theta$

(iv) $2\cos 7\theta \cos 3\theta$

$\because$ $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$\Rightarrow$ $2\cos 7\theta\cos 3\theta = \cos(7\theta+3\theta) + \cos(7\theta-3\theta)$

$= \cos 10\theta + \cos 4\theta$

Q2

(i) $2\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$\because$ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$\Rightarrow$ $2\sin\frac{5\pi}{12}\sin\frac{\pi}{12} = \cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12}\right) - \cos\left(\frac{6\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{2}\right)$

$= \frac{1}{2} - 0 = \frac{1}{2} = RHS$

(ii) $2\cos\frac{5\pi}{12}\cos\frac{\pi}{12} = \frac{1}{2}$

$\because$ $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$= \cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right) + \cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \cos\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{3}\right)$

$= 0 + \frac{1}{2} = \frac{1}{2} = RHS$

(iii) $2\sin\frac{5\pi}{12}\cos\frac{\pi}{12}$

$\because$ $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$= \sin\left(\frac{5\pi}{12}+\frac{\pi}{12}\right) + \sin\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \sin\frac{\pi}{2} + \sin\frac{\pi}{3}$

$= 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2} = RHS$ (Taking LCM)

Q3(i)

$$\sin 50° \cos 85° = \frac{1-\sqrt{2}\sin 35°}{2\sqrt{2}}$$

$$\text{LHS} = \sin 50° \cos 85° = \frac{2\sin 50° \cos 85°}{2}$$

$\because \quad 2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$$\Rightarrow \quad \frac{2\sin 50° \cos 85°}{2} = \frac{1}{2}\left[\sin(50°+85°) + \sin(50°-85°)\right]$$

$$= \frac{1}{2}\left[\sin 135° + \sin(-35°)\right]$$

$$= \frac{1}{2}\left[\sin(90°+45°) - \sin 35°\right] \qquad \left[\because \sin(-\theta) = -\sin\theta\right]$$

$$= \frac{1}{2}\left[\cos 45° - \sin 35°\right] \qquad \left[\because \sin(90°+\theta) = \cos\theta\right]$$

Now,

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2}} - \sin 35°\right]$$

$$= \frac{1-\sqrt{2}\sin 35°}{2\sqrt{2}}$$

Q3(ii)

LHS $= \sin 25° \cos 115°$

$= \dfrac{2 \sin 25° \cos 115°}{2}$

We Know that

$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

$= \dfrac{1}{2}\left[\sin(25° + 115°) + \sin(25° - 115°)\right]$

$= \dfrac{1}{2}\left[\sin 140° + \sin(-90°)\right]$

$\sin(-\theta) = -\sin\theta$

And, $\sin(90° + \theta) = \cos\theta$

$\Rightarrow \quad \dfrac{1}{2}\left[\sin(90° + 50°) - \sin 90°\right]$

$= \dfrac{1}{2}\left[\cos 50° - 1\right]$

Also,

$\cos\theta = \sin(90° - \theta)$

$\cos 50° = \sin(90° - 50°) = \sin 40°$

$\dfrac{1}{2}\left[\sin 40° - 1\right]$

Q4

We have,

$$\text{LHS} = 4\cos\theta\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)$$

$$= 2\cos\theta\left[2\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\right]$$

$$= 2\cos\theta\left[2\cos\left(\frac{\pi}{3}+\theta+\frac{\pi}{3}-\theta\right)+\cos\left(\frac{\pi}{3}+\theta-\frac{\pi}{3}+\theta\right)\right]$$

$$= 2\cos\theta\left[\cos\frac{2\pi}{3}+\cos 2\theta\right]$$

$$= 2\cos\theta\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)+\cos 2\theta\right]$$

$$= 2\cos\theta\left[-\sin\frac{\pi}{6}+\cos 2\theta\right]$$

$$= 2\cos\theta\left[-\frac{1}{2}+\cos 2\theta\right]$$

$$= -2\cos\theta\times\frac{1}{2}+2\cos\theta\cos 2\theta$$

$$= -\cos\theta+\left[\cos(\theta+2\theta)+\cos(2\theta-\theta)\right]$$

$$= -\cos\theta+\cos 3\theta+\cos\theta$$

$$= \cos 3\theta$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS Hence proved.

Q5(i)

$\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^0 = \frac{3}{16}$

LHS $= \cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ$

$= \cos 30^\circ \cos 10^\circ \cos 50^\circ \cos 70^\circ$

$= \frac{\sqrt{3}}{2}(\cos 10^\circ \cos 50^\circ \cos 70^\circ)$

$= \frac{\sqrt{3}}{2}(\cos 10^\circ \cos 50^\circ)\cos 70^\circ$

$= \frac{\sqrt{3}}{4}(2\cos 10^\circ \cos 50^\circ)\cos 70^\circ$ [Multiiplying and dividing by 2]

Also,

$\Rightarrow$ $2\cos A \cos B = \cos(A+B) + \cos(A-B)$ ---(i)

$= \frac{\sqrt{3}}{4}\cos 70^\circ \{\cos(50^\circ + 10^\circ) + \cos(10^\circ - 50^\circ)\}$

$= \frac{\sqrt{3}}{4}\cos 70^\circ \{\cos 60^\circ + \cos(-40^\circ)\}$

Now,

$\cos(-\theta) = \cos\theta$

$= \frac{\sqrt{3}}{4}\cos 70^\circ \left(\frac{1}{2} + \cos 40^\circ\right)$ $\left[\because \cos 60^\circ = \frac{1}{2}\right]$

$= \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{4}\cos 70^\circ \cos 40^\circ$

$= \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{8}(2\cos 70^\circ \cos 40^\circ)$

$= \frac{\sqrt{3}}{8}[\cos 70^\circ + \cos(70^\circ + 40^\circ) + \cos(70^\circ - 40^\circ)]$ [from (i)]

$= \frac{\sqrt{3}}{8}[\cos 70^\circ + \cos 110^\circ + \cos 30^\circ]$

$= \frac{\sqrt{3}}{8}\left[\cos 70^\circ + \cos(180^\circ - 70^\circ) + \frac{\sqrt{3}}{2}\right]$

$= \frac{\sqrt{3}}{8}\left[\cos 70^\circ - \cos 70^\circ + \frac{\sqrt{3}}{2}\right]$ $[\because \cos(180^\circ - \theta) = -\cos\theta]$

$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

$=$ RHS

Q5(ii)

$\cos 40° \cos 80° \cos 160° = -\frac{1}{8}$

LHS $= \cos 40° \cos 80° \cos 160°$

$= \cos 80° \cos 40° \cos 160°$

Multiplying and dividing by 2

$= \frac{1}{2}\left(\cos 80° \times \left(2\cos 40° \cos 160°\right)\right)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$= \frac{1}{2}\left(\cos 80° \left(\cos(40° + 160°) + \cos(40° - 160°)\right)\right)$

$= \frac{1}{2}\left(\cos 80° \left(\cos 200 + \cos(-120)\right)\right)$

$= \frac{1}{2}\cos 80° \left(\cos(180° + 20°) + \cos(180° - 60°)\right)$

$= \frac{1}{2}\cos 80° \left(\cos 20° + \cos 60°\right)$

$= \frac{1}{2}\cos 80° \cos 20° + \frac{1}{2}\cos 80° + \cos 60°$

$= -\frac{1}{2}\left(2\cos 80° \cos 20°\right) + \frac{1}{2}\cos 80° \cos 60°$

$= -\frac{1}{4}\left[2\cos 80° \cos 20° + \cos 80°\right]$

$= -\frac{1}{4}\left[\cos(80° + 20°) + \cos(80° - 20°) + \cos 80°\right]$

$= -\frac{1}{4}\left[\cos 100° + \cos 60° + \cos 80°\right]$

$= -\frac{1}{4}\left[\cos(180° - 80°) + \cos 60° + \cos 80°\right]$

$= -\frac{1}{4}\left[-\cos 80° + \cos 60° + \cos 80°\right]$

$= -\frac{1}{4}\cos 60°$

$= -\frac{1}{4} \times \frac{1}{2}$

$= -\frac{1}{8}$ RHS

Q5(iii)

$\sin 20° \sin 40° \sin 80°$

$= \frac{1}{2}(2\sin 20° \sin 40°)\sin 80°$

$= \frac{1}{2}\left[\cos(40° - 20°) - \cos(40° + 20°)\right]\sin 80°$ $\quad \left[\because 2\sin A \sin B = \cos(A - B) - \cos(A + B)\right]$

$= \frac{1}{2}\left[\cos 20° - \cos 60°\right]\sin 80°$

$= \frac{1}{2}\left[\cos 20° - \frac{1}{2}\right]\sin 80°$

$= \frac{1}{2}\left[\cos 20° \sin 80°\right] - \frac{1}{4}\sin 80°$

$= \frac{1}{4}\left[2\cos 20° \sin 80° - \sin 80°\right]$

$= \frac{1}{4}\left[\sin(80° + 20°) + \sin(80° - 20°) - \sin 80°\right]$ $\quad \left[\because 2\sin A \cos B = \sin(A + B) + \sin(A - B)\right]$

$= \frac{1}{4}\left[\sin 100° + \sin 60° - \sin 80°\right]$

$= \frac{1}{4}\left[\sin(180° - 80°) + \frac{\sqrt{3}}{2} - \sin 80°\right]$

$= \frac{1}{4}\left[\sin 80° + \frac{\sqrt{3}}{2} - \sin 80°\right]$

$= \frac{\sqrt{3}}{8} \quad = \text{RHS}$

Q5(iv)

$\cos 20° \cos 40° \cos 80°$

$= \frac{1}{2}(2\cos 20° \cos 40°)\cos 80°$

$= \frac{1}{2}\left[\cos(40° + 20°) + \cos(40° - 20°)\right]\cos 80°$ $\quad \left[\because 2\cos A \cos B = \cos(A + B) + \cos(A - B)\right]$

$= \frac{1}{2}\left[\cos 60° + \cos 20°\right]\cos 80°$

$= \frac{1}{2}\left[\frac{1}{2} + \cos 20°\right]\cos 80°$

$= \frac{1}{2}\left[\cos 80° + 2\cos 20° \cos 80°\right]$

$= \frac{1}{4}\left[\cos 80° + \cos(80° + 20°) + \cos(20° - 80°)\right]$

$= \frac{1}{4}\left[\cos 80° + \cos 100° + \cos 60°\right]$

$= \frac{1}{4}\left[\cos 80° + \cos(180° - 80°) + \cos 60°\right]$

$= \frac{1}{4}\left[\cos 80° - \cos 80° + \cos 60°\right]$

$= \frac{1}{4}\left[\frac{1}{2}\right] = \frac{1}{8} = \text{RHS}$

Q5(v)

$\tan 20° \tan 40° \tan 60° \tan 80°$

$$= (\tan 20° \tan 40° \tan 80°)\sqrt{3} \qquad [\because \tan 60° = \sqrt{3}]$$

$$= \left(\frac{\sin 20° \sin 40° \sin 80°}{\cos 20° \cos 40° \cos 80°}\right)\sqrt{3}$$

$$= \frac{(2\sin 20° \sin 40°)\sin 80° \times \sqrt{3}}{(2\cos 20° \cos 40°)\cos 80°}$$

Applying

$\Rightarrow \quad 2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$$= \frac{\{\cos(40° - 20°) - \cos(40° + 20°)\}\sin 80° \times \sqrt{3}}{\{\cos(20° + 40°) + \cos(40° - 20°)\}\cos 80°}$$

$$= \frac{(\cos 20° - \cos 60°)\sin 80° \times \sqrt{3}}{(\cos 60° + \cos 20°)\cos 80°}$$

$$= \frac{\left(\cos 20° - \frac{1}{2}\right)\sin 80° \times \sqrt{3}}{\left(\frac{1}{2} + \cos 20°\right)\cos 80°}$$

$$= \frac{(2\sin 80° \cos 20° - \sin 80°)\sqrt{3}}{\cos 80° + 2\cos 20° \cos 80°}$$

$\Rightarrow \quad 2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$$= \frac{\{\sin(80° + 20°) + \sin(80° - 20°) - \sin 80°\}\sqrt{3}}{\cos 80° + \{\cos(20° + 80°) + \cos(80° - 20°)\}}$$

$$= \frac{(\sin 100° + \sin 60° - \sin 80°)\sqrt{3}}{\cos 80° + \cos 100° + \cos 60°}$$

$$= \frac{\left(\sin(180° - 80°) + \frac{\sqrt{3}}{2} - \sin 80°\right)\sqrt{3}}{\cos 80° + \cos(180° - 80°) + \cos 60°}$$

$$= \frac{\left(\sin 80° + \frac{\sqrt{3}}{2} - \sin 80°\right)\sqrt{3}}{\cos 80° - \cos 80° + \cos 60°}$$

$$= \frac{\frac{3}{2}}{\frac{1}{2}} = 3 = \quad \text{RHS}$$

Q5(vi)

tan 20° tan 30° tan 40° tan 80°

$$\frac{1}{\sqrt{3}}\left(\tan 20^\circ \tan 40^\circ \tan 80^\circ\right) \qquad \left[\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right]$$

$$= \frac{(\sin 20^\circ \sin 40^\circ \sin 80^\circ)}{(\cos 20^\circ \cos 40^\circ \cos 80^\circ)\sqrt{3}}$$

$$= \frac{(2\sin 20^\circ \sin 40^\circ)\sin 80^\circ}{\sqrt{3}\,(2\cos 20^\circ \cos 40^\circ)\cos 80^\circ}$$

Applying

$\Rightarrow \quad 2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$$= \frac{\left(\cos(40^\circ - 20^\circ) - \cos(20^\circ + 40^\circ)\right)\sin 80^\circ}{\cos(20^\circ + 40^\circ) + \cos(40^\circ - 20^\circ)\cos 80^\circ\sqrt{3}}$$

$$= \frac{(\cos 20^\circ - \cos 60^\circ)\sin 80^\circ}{\sqrt{3}\,(\cos 60^\circ + \cos 20^\circ)\cos 80^\circ}$$

$$= \frac{\left(\cos 20^\circ - \frac{1}{2}\right)\sin 80^\circ}{\sqrt{3}\left(\frac{1}{2} + \cos 20^\circ\right)\cos 80^\circ}$$

$$= \frac{2\sin 20^\circ \sin 80^\circ - \sin 80^\circ}{\sqrt{3}\,(\cos 80^\circ + 2\cos 20^\circ \cos 80^\circ)}$$

Now,

$\Rightarrow \quad 2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$$= \frac{\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ) - \sin 80^\circ}{\sqrt{3}\left(\cos 80^\circ + \cos(20^\circ + 80^\circ) + \cos(80^\circ - 20^\circ)\right)}$$

$$= \frac{\sin 100^\circ + \sin 60^\circ - \sin 80^\circ}{\sqrt{3}\,(\cos 80^\circ + \cos 100^\circ + \cos 60^\circ)}$$

$$= \frac{\sin 100^\circ + \sin 60^\circ - \sin(80^\circ - 100^\circ)}{\sqrt{3}\left(\cos 80^\circ + \cos(1800^\circ - 80^\circ) + \sin 60^\circ\right)}$$

$$= \frac{\sin 100^\circ + \frac{\sqrt{3}}{2} - \sin 100^\circ}{\sqrt{3}\,(\cos 80^\circ - \cos 80^\circ + \cos 60^\circ)}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}\left(\frac{1}{2}\right)} = 1 = \qquad \text{RHS}$$

Q5(vii)

$\sin 10° \sin 50° \sin 60° \sin 70° = \frac{\sqrt{3}}{16}$

LHS

$\sin 10° \sin 50° \sin 70° \frac{\sqrt{3}}{2}$ $\left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$

$= \sin(90° - 80°)\sin(90° - 40°)\sin(90° - 20°)\frac{\sqrt{3}}{2}$

$= \cos 80° \cos 40° \cos 20° \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2\times 2}(2\cos 40° \cos 20°)\cos 80°$ $[\because 2\cos A \cos B = \cos(A+B) + \cos(A-B)]$

$= \frac{\sqrt{3}}{2\times 2}[\cos(40° + 20°) + \cos(40° - 20°)]\cos 80°$

$= \frac{\sqrt{3}}{2\times 2}[\cos 60° + \cos 20°]\cos 80°$

$= \frac{\sqrt{3}}{2\times 2}\left[\frac{1}{2} + \cos 20°\right]\cos 80°$

$= \frac{\sqrt{3}}{4}\left[\frac{1}{2}\cos 80° + \cos 20° \cos 80°\right]$

$= \frac{\sqrt{3}}{8}[\cos 80° + 2\cos 20° \cos 80°]$

$= \frac{\sqrt{3}}{8}[\cos 80° + \cos(80° + 20°) + \cos(80° - 20°)]$

$= \frac{\sqrt{3}}{8}[\cos 80° + \cos 100° + \cos 60°]$

$= \frac{\sqrt{3}}{8}[\cos 80° + \cos(180° - 80°) + \cos 60°]$

$= \frac{\sqrt{3}}{8}[\cos 60°] = \frac{\sqrt{3}}{16} = \text{RHS}$

Q5(viii)

LHS $= \sin 20° \sin 40° \sin 60° \sin 80°$

$= \sin 20° \sin 40° \sin 80° \times \frac{\sqrt{3}}{2}$ $\left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$

$= \frac{\sqrt{3}}{2} \times \frac{1}{2}(2 \sin 20° \sin 40°) \sin 80°$

$= \frac{\sqrt{3}}{4}\left[\cos(40° - 20°) - \cos(40° + 20°)\right] \sin 80°$

$= \frac{\sqrt{3}}{4}\left[\cos 20° - \cos 60°\right] \sin 80°$

$= \frac{\sqrt{3}}{4}\left[\cos 20° \sin 80° - \frac{1}{2} \sin 80°\right]$

$= \frac{\sqrt{3}}{8}\left[2 \cos 20° \sin 80° - \sin 80°\right]$

$= \frac{\sqrt{3}}{8}\left[\sin(80° + 20°) + \sin(80° - 20°) - \sin 80°\right]$

$= \frac{\sqrt{3}}{8}\left[\sin 100° + \sin 60° - \sin 80°\right]$

$= \frac{\sqrt{3}}{8}\left[\sin 80° + \sin 60° - \sin 80°\right]$

$= \frac{\sqrt{3}}{8} \times \sin 60° = \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$

3 RHS

Q6(i)

We have,

$$\begin{aligned}
\text{LHS} &= \sin A\sin(B-C)+\sin B\sin(C-A)+\sin C\sin(A-B)\\
&= \frac{1}{2}\left[2\sin A\sin(B-C)+2\sin B\sin(C-A)+2\sin C\sin(A-B)\right]\\
&= \frac{1}{2}\left[\begin{array}{r}\cos(A-B+C)-\cos(A+B-C)+\cos(B-C+A)-\cos(B+C-A)\\ +\cos(C-A+B)-\cos(C+A-B)\end{array}\right]\\
&= \frac{1}{2}\left[\begin{array}{r}\cos(A-B+C)-\cos(A-B+C)-\cos(A+B-C)+\cos(A+B-C)\\ -\cos(B+C-A)+\cos(B+C-A)\end{array}\right]\\
&= \frac{1}{2}\times 0\\
&= 0\\
&= \text{RHS}
\end{aligned}$$

$\therefore\ \sin A\sin(B-C)+\sin B\sin(C-A)+\sin C\sin(A-B)=0$ Hence proved.

Q6(ii)

We have,

$$\begin{aligned}
\text{LHS} &= \sin(B-C)\cos(A-D)+\sin(C-A)\cos(B-D)+\sin(A-B)\cos(C-D)\\
&= \frac{1}{2}\left[2\sin(B-C)\cos(A-D)+2\sin(C-A)\cos(B-D)+2\sin(A-B)\cos(C-D)\right]\\
&= \frac{1}{2}\left[\begin{array}{r}\sin(B-C+A-D)+\sin(B-C-A+D)+\sin(C-A+B-D)+\sin(C-A-B+D)\\ +\sin(A-B+C-D)+\sin(A-B-C+D)\end{array}\right]\\
&= \frac{1}{2}\left[\begin{array}{r}\sin(A+B-C-D)+\sin(B+D-C-A)+\sin(B+C-A-D)+\sin(C+D-A-B)\\ +\sin(A+C-B-D)+\sin(A+D-B-C)\end{array}\right]\\
&= \frac{1}{2}\left[\begin{array}{r}\sin(A+B-C-D)+\sin(B+D-C-A)+\sin\{-(A+D-B-C)\}+\sin\{-(A+B-C-D)\}\\ +\sin\{-(B+D-A-C)\}+\sin(A+D-B-C)\end{array}\right]\\
&= \frac{1}{2}\left[\begin{array}{r}\sin(A+B-C-D)+\sin(B+D-C-A)-\sin(A+D-B-C)-\sin(A+B-C-D)\\ -\sin(B+D-A-C)+\sin(A+D-B-C)\end{array}\right]\\
&= \frac{1}{2}\times 0 \qquad\qquad [\because \sin(-\theta)=-\sin\theta]\\
&= 0\\
&= \text{RHS}
\end{aligned}$$

$\therefore\ \sin(B-C)\cos(A-D)+\sin(C-A)\cos(B-D)+\sin(A-B)\cos(C-D)=0$ Hence proved.

Q7

We have,

LHS $= \tan\theta \tan(60^\circ - \theta)\tan(60^\circ + \theta)$

$= \dfrac{\sin\theta \sin(60^\circ - \theta)\sin(60^\circ + \theta)}{\cos\theta\cos(60^\circ - \theta)\cos(60^\circ + \theta)}$

$= \dfrac{2\sin\theta \sin(60^\circ - \theta)\sin(60^\circ + \theta)}{2\cos\theta\cos(60^\circ - \theta)\cos(60^\circ + \theta)}$

$= \dfrac{\sin\theta[2\sin(60^\circ - \theta)\sin(60^\circ + \theta)]}{\cos\theta[2\cos(60^\circ - \theta)\cos(60^\circ + \theta)]}$

$= \dfrac{\sin\theta[\cos\{(60^\circ - \theta) - (60^\circ + \theta)\} - \cos\{(60^\circ - \theta) + (60^\circ + \theta)\}]}{\cos\theta[\cos\{(60^\circ - \theta) + (60^\circ + \theta)\} + \cos\{(60^\circ - \theta) - (60^\circ + \theta)\}]}$

$= \dfrac{\sin\theta[\cos(-2\theta) - \cos 120^\circ]}{\cos\theta[\cos 120^\circ + \cos(-2\theta)]}$

$= \dfrac{\sin\theta[\cos 2\theta - \cos 120^\circ]}{\cos\theta[\cos 120^\circ + \cos 2\theta]}$ $\quad [\because \cos(-\theta) = \cos\theta]$

$= \dfrac{\sin\theta[\cos 2\theta - \cos(90^\circ + 30^\circ)]}{\cos\theta[\cos(90^\circ + 30^\circ) + \cos 2\theta]}$

$= \dfrac{\sin\theta[\cos 2\theta + \sin 30^\circ]}{\cos\theta[-\sin 30^\circ + \cos 2\theta]}$ $\quad [\because \cos$ is negative in IInd quadrant$]$

$= \dfrac{\sin\theta\left[\cos 2\theta + \frac{1}{2}\right]}{\cos\theta\left[\frac{-1}{2} + \cos 2\theta\right]}$

$= \dfrac{\sin\theta\cos 2\theta + \frac{1}{2}\sin\theta}{\frac{-1}{2}\cos\theta + \cos\theta\cos 2\theta}$

Q8

Let $y = \cos\alpha\cos\beta$ then,

$$y = \frac{1}{2}(2\cos\alpha\cos\beta)$$

$$= \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$

$$= \frac{1}{2}[\cos 90^\circ + \cos(\alpha-\beta)] \qquad [\because \alpha+\beta = 90^\circ]$$

$$= \frac{1}{2}[0+\cos(\alpha-\beta)]$$

$$= \frac{1}{2}\cos(\alpha-\beta)$$

$$\Rightarrow \quad y = \frac{1}{2}\cos(\alpha-\beta)$$

Now,

$$-1 \le \cos(\alpha-\beta) \le 1$$

$$\Rightarrow \quad \frac{-1}{2} \le \frac{1}{2}\cos(\alpha-\beta) \le \frac{1}{2}$$

$$\Rightarrow \quad \frac{-1}{2} \le y \le \frac{1}{2}$$

$$\Rightarrow \quad \frac{-1}{2} \le \cos\alpha\cos\beta \le \frac{1}{2}$$

Hence, the maximum values of $\cos\alpha\cos\beta$ is $\frac{1}{2}$.

Ex 8.2

Q1

(i) $\sin 12\theta + \sin 4\theta$ $\qquad \left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$$= 2\sin\left(\frac{12\theta + 4\theta}{2}\right)\cos\left(\frac{12\theta - 4\theta}{2}\right)$$
$$= 2\sin 8\theta \cos 4\theta$$

(ii) $\sin 5\theta - \sin\theta$ $\qquad \left[\because \sin C - \sin D = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$

$$= 2\cos\left(\frac{5\theta + \theta}{2}\right)\sin\left(\frac{5\theta - \theta}{2}\right)$$
$$= 2\sin 2\theta \cos 3\theta$$

(iii) $\cos 12\theta + \cos 8\theta$ $\qquad \left[\because \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$$= 2\cos 10\theta \cos 2\theta$$

(iv) $\cos 12\theta - \cos 4\theta$

$$= -2\sin\left(\frac{12\theta + 4\theta}{2}\right)\sin\left(\frac{12\theta - 4\theta}{2}\right) \qquad \left[\because \cos D - \cos C = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$
$$= -2\sin 8\theta \sin 4\theta$$

(v) $\sin 2\theta + \cos 4\theta$

$$= \sin 2\theta + \sin(90 - 4\theta)$$
$$= 2\sin\frac{(2\theta + 90 - 4\theta)}{2}\cos\frac{(2\theta - 90 + 4\theta)}{2}$$
$$= 2\sin\left(\frac{\pi}{4} + \theta\right)\cos\left(\frac{\pi}{4} - 3\theta\right)$$

Q2

$\sin 38° + \sin 22° = \sin 82°$

LHS $= \sin 38° + \sin 22°$

$\because \quad \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\Rightarrow \quad \sin 38° + \sin 22° = 2\sin\frac{60°}{2}\cos\frac{16°}{2}$

$= 2\sin 30° \cos 8°$

$= 2 \times \frac{1}{2}\cos 8°$

$= \cos(90\text{-}82)°$

$= \sin 82° = \text{RHS}$ $\qquad [\because \cos\theta = \sin(90 - \theta)]$

Q2(i)

$\cos 100° + \cos 20° = \cos 40°$

LHS $= \cos 100° + \cos 20°$ $\left[\because \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$\Rightarrow \quad 2\cos\frac{(100° + 20°)}{2}\cos\frac{(100° - 20°)}{2}$

$= 2\cos 60° \cos 40°$

$= 2 \times \frac{1}{2}\cos 40°$ $\left[\because \cos 60° = \frac{1}{2}\right]$

$= \cos 40° =$ RHS

Q2(ii)

$\sin 50° - \sin 10° = \cos 20°$

LHS $= \sin 50° + \sin 10°$ $\left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$\sin 50° + \sin 10° = 2\sin\frac{60°}{2}\cos 20°$

$= 2\sin 30° \cos 20°$

$= 2 \times \frac{1}{2}\cos 20°$

$= \cos 20° =$ RHS $\left[\because \sin 30° = \frac{1}{2}\right]$

Q2(iii)

$\sin 23° + \sin 37° = \cos 7°$

LHS $= \sin 23° + \sin 37°$

$= 2\sin\left(\frac{23° + 37°}{2}\right)\cos\left(\frac{23° - 37°}{2}\right)$ $\left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$= 2\sin(30°)\cos(-7°)$

$= 2 \times \frac{1}{2}\cos 7$ $\left[\because \cos(-\theta) = \cos\theta, \sin 30° = \frac{1}{2}\right]$

$= \cos 7° =$ RHS

Q2(iv)

LHS $= \sin 105° + \cos 105°$

$= \sin 105° + \cos(90° + 15°)$

$= \sin 105° - \sin 15°$

$= 2\sin\left(\frac{105° - 15°}{2}\right)\cos\left(\frac{105° + 15°}{2}\right)$

$= 2\sin 45° \cos 60°$

$= 2\frac{1}{\sqrt{2}}\frac{1}{2}$

$= \frac{1}{\sqrt{2}}$

$= \cos 45°$

Q2(v)

$\sin 40° + \sin 20° = \cos 10°$

LHS $= \sin 40° + \sin 20°$

$= 2\sin\left(\frac{40° + 20°}{2}\right)\cos\left(\frac{40° - 20°}{2}\right)$ $\left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$= 2\sin 30° \cos 10°$

$= 2 \times \frac{1}{2}\cos 10°$

$= \cos 10°$

$= \text{RHS}$ $\left[\because \sin 30° = \frac{1}{2}\right]$

Q3(i)

$\cos 55 + \cos 65 + \cos 175 = 0$

$\cos 175 = -\cos 5$

substitute above value in the equation we get

$\cos 55 + \cos 65 = \cos 5$

applying rule $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\cos 55 + \cos 65 = 2\cos\left(\frac{65+55}{2}\right)\cos\left(\frac{65-55}{2}\right) = 2\cos 60 \cos 5 = 2 \times \frac{1}{2} \times \cos 5 = \cos 5$

Hence Proved

Q3(ii)

$\sin 50° - \sin 70° + \sin 10° = 0$

$(\sin 50° - \sin 70°) + \sin 10°$

$\Rightarrow \left(2\sin\left(\frac{50° - 70°}{2}\right)\cos\left(\frac{50° + 70°}{2}\right)\right) + \sin 10°$ $\left[\because \sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)\right]$

$= 2\sin(-10°)\cos 60° + \sin 10°$

$= -2\sin 10° \times \frac{1}{2} + \sin 10°$ $\left[\because \cos 60° = \frac{1}{2}\right]$

$= 0$

$= \text{RHS}$

Q3(iii)

$\cos 80° + \cos 40° - \cos 20° = 0$

$(\cos 80° + \cos 40°) - \cos 20°$

$= 2\cos\left(\frac{80° + 40°}{2}\right)\cos\left(\frac{80° - 40°}{2}\right) - \cos 20° \qquad \left[\because \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$

$= 2\cos 60° \cos 20° - \cos 20°$

$= 2 \times \frac{1}{2}\cos 20° - \cos 20°$

$= \cos 20° - \cos 20°$

$= 0$

$=$ RHS

Q3(iv)

$\cos 20° + \cos 100° + \cos 140° = 0$

$\Rightarrow \quad (\cos 20° + \cos 100°) + \cos 140°$

$= 2\cos\left(\frac{20° + 100°}{2}\right)\cos\left(\frac{20° - 100°}{2}\right) + \cos 140° \qquad \left[\because \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$

$= 2\cos 60° \cos(-40°) + \cos 140°$

$= 2 \times \frac{1}{2}\cos 40° + \cos 140° \qquad \left[\because \cos 60° = \frac{1}{2}\right]$

$= \cos 40° + \cos(180° - 40°)$

$= \cos 40° - \cos 40°$

$= 0$

$=$ RHS

Q3(v)

$\sin\frac{5\pi}{18} - \cos\frac{4\pi}{9} = \sqrt{3}\sin\frac{\pi}{9}$

LHS $= \sin\frac{5\pi}{18} - \cos\frac{4\pi}{9}$

$= \sin 50° - \cos 80°$

$= \sin 50° - \sin 10°$

$= 2\sin\left(\frac{50° - 10°}{2}\right)\cos\left(\frac{50° + 10°}{2}\right)$

$= 2\sin 20° \cos 30°$

$= 2\sin 20° \times \frac{\sqrt{3}}{2}$

$= \sqrt{3}\sin\frac{\pi}{9}$

Q3(vi)

$\cos\frac{\pi}{12} - \sin\frac{\pi}{12} = \frac{1}{\sqrt{2}}$

Multiplying and dividing by $\sqrt{2}$ on LHS

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\frac{\pi}{12} - \frac{1}{\sqrt{2}}\sin\frac{\pi}{12}\right)$

$= \sqrt{2}\left(\sin\frac{\pi}{4}\cos\frac{\pi}{12} - \cos\frac{\pi}{4}\sin\frac{\pi}{12}\right)$ $\left[\because \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} = \sin\frac{\pi}{4}\right]$

$= \sqrt{2}\left(\sin\left(\frac{\pi}{4} - \frac{\pi}{12}\right)\right)$ $\left[\because \sin(A-B) = \sin A\cos B - \cos A\sin B\right]$

$= \sqrt{2}\left(\sin\frac{\pi}{6}\right)$

$= \sqrt{2} \times \frac{1}{2}$

$= \frac{1}{\sqrt{2}}$

= RHS

Q3(vii)

$\sin 80° - \cos 70° = \cos 50°$

LHS $= \sin 80° = \cos 50° + \cos 70°$

Now,

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

RHS $= \cos 50° + \cos 70°$

$= 2\cos\left(\frac{50° + 70°}{2}\right)\cos\left(\frac{50° - 70°}{2}\right)$

$= 2\cos 60°\cos(-10°)$

$= 2 \times \frac{1}{2}\cos 10°$ $\left[\cos(-\theta) = \cos\theta\right]$

$= \cos 10°$

$= \sin 80°$

= LHS $\left[\because \cos\theta = \sin(90 - \theta)\right]$

Q3(viii)

$\sin 51° + \cos 81° = \cos 21°$

$\sin 51° = \cos 21° - \cos 81°$

RHS $= \cos 21° - \cos 81°$

$= -2\sin(51°)\sin(-30°)$ $\quad \left[\because \cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$

$= +2\sin 51° \sin 30°$

$= 2\sin 51° \times \frac{1}{2}$

$= \sin 51°$

$=$ LHS

Q4

We have,

LHS $= \cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right)$

$= -\left[\cos\left(\frac{3\pi}{4}-x\right) - \cos\left(\frac{3\pi}{4}+x\right)\right]$

$= -\left[2\sin\frac{3\pi}{4}\sin x\right]$ $\quad \left[\because \cos(A-B)-\cos(A+B) = 2\sin A\sin B\right]$

$= -2\sin\frac{3\pi}{4}\sin x$

$= -2\sin\left(\frac{\pi}{2}+\frac{\pi}{4}\right)\sin x$

$= -2\cos\frac{\pi}{4}\sin x$

$= -2 \times \frac{1}{\sqrt{2}} \times \sin x$

$= -\frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}}\sin x$

$= -\sqrt{2}\sin x$

$=$ RHS

$\therefore \cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}\sin x$ Hence proved.

Q4(i)

We have,

LHS $= \cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)$

$= 2\cos\frac{\pi}{4}\cos x$ $\quad\left[\because \cos(A+B)+\cos(A-B) = 2\cos A\cos B\right]$

$= 2\times\frac{1}{\sqrt{2}}\times\cos x$

$= \frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}}\cos x$

$= \sqrt{2}\cos x$

= RHS

$\therefore\quad \cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right) = \sqrt{2}\cos x.$

Q5(i)

We have,

LHS $= \sin 65° + \cos 65°$

$= \sin(45°+20°)+\cos(90°-25°)$

$= \sin(45°+20°)+\sin 25°$

$= \sin(45°+20°)+\sin(45°-20°)$

$= 2\sin 45°\cos 20°$

$= 2\times\frac{1}{\sqrt{2}}\cos 20°$

$= \frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}}\times\cos 20°$

$= \sqrt{2}\cos 20°$

= RHS

$\therefore\quad \sin 65° + \cos 65° = \sqrt{2}\cos 20°$ Hence proved.

Q5(ii)

We have,

LHS $= \sin 47° + \cos 77°$

$= \sin(90° - 43°) + \cos 77°$

$= \cos 43° + \cos 77°$

$= \cos(60° - 17°) + \cos(60° + 17°)$

$= 2\cos 60° \cos 17°$

$= 2 \times \frac{1}{2} \times \cos 17°$

$= \cos 17°$

$=$ RHS

$\therefore \quad \sin 47° + \cos 77° = \cos 17°$ Hence proved.

Q6(i)

We have,

LHS $= \cos 3A + \cos 5A + \cos 7A + \cos 15A$

$= [\cos 5A + \cos 3A] + [\cos 15A + \cos 7A]$

$= \left[2\cos\frac{(5A + 3A)}{2}\cos\frac{(5A - 3A)}{2}\right] + \left[2\cos\frac{(15A + 7A)}{2}\cos\frac{(15A - 7A)}{2}\right]$

$= 2\cos 4A \cos A + 2\cos 11A \cos 4A$

$= 2\cos 4A[\cos A + \cos 11A]$

$= 2\cos 4A[\cos 11A + \cos A]$

$= 2\cos 4A\left[2\cos\frac{(11A + A)}{2}\cos\frac{(11A - A)}{2}\right]$

$= 4\cos A[\cos 6A \cos 5A]$

$= 4\cos 4A \cos 5A \cos 6A$

$=$ RHS

$\therefore \quad \cos 3A + \cos 5A + \cos 7A + \cos 15A = 4\cos 4A \cos 5A \cos 6A$ Hence proved.

Q6(ii)

We have,

LHS $= \cos A + \cos 3A + \cos 5A + \cos 7A$

$= (\cos 3A + \cos A) + (\cos 7A + \cos 5A)$

$= \left[2\cos\left(\frac{3A + A}{2}\right)\cos\left(\frac{3A - A}{2}\right)\right] + \left[2\cos\left(\frac{7A + 5A}{2}\right)\cos\left(\frac{7A - 5A}{2}\right)\right]$

$= 2\cos 2A \cos A + 2\cos 6A \cos A$

$= 2\cos A[\cos 2A + \cos 6A]$

$= 2\cos A[\cos 6A + \cos 2A]$

$= 2\cos A\left[2\cos\left(\frac{6A + 2A}{2}\right)\cos\left(\frac{6A - 2A}{2}\right)\right]$

$= 4\cos A[\cos 4A \cos 2A]$

= RHS

$\therefore \quad \cos A + \cos 3A + \cos 5A + \cos 7A = 4\cos A \cos 2A \cos 4A$. Hence proved.

Q6(iii)

We have,

LHS $= \sin A + \sin 2A + \sin 4A + \sin 5A$

$= (\sin 2A + \sin A) + (\sin 5A + \sin 4a)$

$= \left[2\sin\left(\frac{2A + A}{2}\right)\cos\left(\frac{2A - A}{2}\right)\right] + \left[2\sin\left(\frac{5A + 4A}{2}\right)\cos\left(\frac{5A - 4A}{2}\right)\right]$

$= 2\sin\frac{3A}{2}\cos\frac{A}{2} + 2\sin\frac{9A}{2}\cos\frac{A}{2}$

$= 2\cos\frac{A}{2}\left[\sin\frac{3A}{2} + \sin\frac{9A}{2}\right]$

$= 2\cos\frac{A}{2}\left[\sin\frac{9A}{2} + \sin\frac{3A}{2}\right]$

$= 2\cos\frac{A}{2}\left[2\sin\left\{\frac{1}{2}\left(\frac{9A}{2} + \frac{3A}{2}\right)\right\}\cos\left\{\frac{1}{2}\left(\frac{9A}{2} - \frac{3A}{2}\right)\right\}\right]$

$= 4\cos\frac{A}{2}\left[\sin\frac{12A}{4}\cos\frac{6A}{4}\right]$

$= 4\cos\frac{A}{2}\sin 3A \cos\frac{3A}{2}$

$= 4\cos\frac{A}{2}\cos\frac{3A}{2}\sin 3A$

= RHS

$\therefore \quad \sin A + \sin 2A + \sin 4A + \sin 5A = 4\cos\frac{A}{2}\cos\frac{3A}{2}\sin 3A$. Hence proved.

Q6(iv)

We have,

$$\begin{aligned}
LHS &= \sin 3A + \sin 2A - \sin A \\
&= \sin 3A - \sin A + \sin 2A \\
&= 2\sin\left(\frac{3A - A}{2}\right)\cos\left(\frac{3A + A}{2}\right) + \sin 2A \\
&= 2\sin A\cos 2A + \sin 2A \\
&= 2\sin A\cos 2A + 2\sin A\cos A \\
&= 2\sin A\left[\cos 2A + \cos A\right] \\
&= 2\sin A\left[2\cos\left(\frac{2A + A}{2}\right)\cos\left(\frac{2A - A}{2}\right)\right] \\
&= 4\sin A\cos\frac{3A}{2}\cos\frac{A}{2} \\
&= 4\sin A\cos\frac{A}{2}\cos\frac{3A}{2} \\
&= RHS
\end{aligned}$$

$\therefore\ \sin 3A + \sin 2A - \sin A = 4\sin A\cos\frac{A}{2}\cos\frac{3A}{2}$. Hence proved.

Q6(v)

We have,

$$\begin{aligned}
LHS &= \cos 20^\circ\cos 100^\circ + \cos 100^\circ\cos 140^\circ - \cos 140^\circ\cos 200^\circ \\
&= \frac{1}{2}\left[2\cos 100^\circ\cos 20^\circ + 2\cos 140^\circ\cos 100^\circ - 2\cos 200^\circ\cos 140^\circ\right] \\
&= \frac{1}{2}\left[\cos(100^\circ + 20^\circ) + \cos(100^\circ - 20^\circ) + \cos(140^\circ + 100^\circ) + \cos(140^\circ - 100^\circ) - \left\{\cos(200^\circ + 140^\circ) + \cos(200^\circ - 140^\circ)\right\}\right] \\
&= \frac{1}{2}\left[\cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \cos 340^\circ - \cos 60^\circ\right] \\
&= \frac{1}{2}\left[\cos(90^\circ + 30^\circ) + \cos 80^\circ + \cos 40^\circ - \cos(180^\circ + 60^\circ) - \cos(360^\circ - 20^\circ) - \frac{1}{2}\right] \\
&= \frac{1}{2}\left[-\sin 30^\circ + 2\cos\left(\frac{80^\circ + 40^\circ}{2}\right)\cos\left(\frac{80^\circ - 40^\circ}{2}\right) - \cos 60^\circ - \cos 20^\circ - \frac{1}{2}\right] \\
&= \frac{1}{2}\left[-\frac{1}{2} + 2\cos 60^\circ\cos 20^\circ - \frac{1}{2} - \cos 20^\circ - \frac{1}{2}\right] \\
&= \frac{1}{2}\left[-\frac{3}{2} + 2\times\frac{1}{2}\times\cos 20^\circ - \cos 20^\circ\right] \\
&= \frac{1}{2}\left[-\frac{3}{2} + \cos 20^\circ - \cos 20^\circ\right] \\
&= \frac{1}{2}\left[-\frac{3}{2} + 0\right] \\
&= -\frac{3}{4} \\
&= RHS
\end{aligned}$$

$\therefore\ \cos 20^\circ\cos 100^\circ + \cos 100^\circ\cos 140^\circ - \cos 140^\circ\cos 200^\circ = -\frac{3}{4}$ Hence proved.

Q6(vi)

We have,

$$\text{LHS} = \sin\frac{\theta}{2}\sin\frac{7\theta}{2} + \sin\frac{3\theta}{2}\sin\frac{11\theta}{2}$$

$$= \frac{1}{2}\left[2\sin\frac{7\theta}{2}\sin\frac{\theta}{2} + 2\sin\frac{11\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{7\theta}{2}-\frac{\theta}{2}\right) - \cos\left(\frac{7\theta}{2}+\frac{\theta}{2}\right) + \cos\left(\frac{11\theta}{2}-\frac{3\theta}{2}\right) - \cos\left(\frac{11\theta}{2}+\frac{3\theta}{2}\right)\right]$$

$$= \frac{1}{2}\left[\cos\frac{6\theta}{2} - \cos\frac{8\theta}{2} + \cos\frac{8\theta}{2} - \cos\frac{14\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos 3\theta - \cos 4\theta + \cos 4\theta - \cos 7\theta\right]$$

$$= \frac{1}{2}\left[\cos 3\theta - \cos 7\theta\right]$$

$$= \frac{-1}{2}\left[\cos 7\theta - \cos 3\theta\right]$$

$$= -\frac{1}{2}\left[-2\sin\left(\frac{7\theta+3\theta}{2}\right)\sin\left(\frac{7\theta-3\theta}{2}\right)\right]$$

$$= \sin\frac{10\theta}{2}\sin\frac{4\theta}{2}$$

$$= \sin 5\theta \sin 2\theta$$

$$= \sin 2\theta \sin 5\theta$$

$$= \text{RHS}$$

$\therefore \sin\frac{\theta}{2}\sin\frac{7\theta}{2} + \sin\frac{3\theta}{2}\sin\frac{11\theta}{2} = \sin 2\theta \sin 5\theta.$ Hence proved.

Q7(i)

We have,

$$\text{LHS} = \frac{\sin A + \sin 3A}{\cos A - \cos 3A}$$

$$= \frac{2\sin\left(\frac{A+3A}{2}\right)\cos\left(\frac{A-3A}{2}\right)}{-2\sin\left(\frac{A+3A}{2}\right)\sin\left(\frac{A-3A}{2}\right)}$$

$$= \frac{-\sin 2A \times \cos(-A)}{\sin 2A \sin(-A)}$$

$$= \frac{-\cos(-A)}{\sin(-A)}$$

$$= \frac{-\cos A}{-\sin A} \qquad [\because \cos(-\theta) = \cos\theta \text{ and } \sin(-\theta) = -\sin\theta]$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A$$

$$= \text{RHS}$$

$\therefore \frac{\sin A + \sin 3A}{\cos A - \cos 3A} = \cot A.$ Hence proved.

Q7(ii)

We have,

$$\text{LHS} = \frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A}$$

$$= \frac{2\sin\left(\frac{9A-7A}{2}\right)\cos\left(\frac{9A+7A}{2}\right)}{-2\sin\left(\frac{7A+9A}{2}\right)\sin\left(\frac{7A-9A}{2}\right)}$$

$$= \frac{-\sin A\cos 8A}{\sin 8A\sin(-A)}$$

$$= \frac{-\sin A\cos 8A}{-\sin A \times \sin 8A} \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$= \frac{\cos 8A}{\sin 8A}$$

$$= \cot 8A$$

= RHS

$\therefore \quad \frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A} = \cot 8A.$ Hence proved.

Q7(iii)

We have,

$$\text{LHS} = \frac{\sin A - \sin B}{\cos A + \cos B}$$

$$= \frac{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A-B}{2}\right)}$$

$$= \tan\left(\frac{A-B}{2}\right)$$

= RHS

$\therefore \quad \frac{\sin A - \sin B}{\cos A + \cos B} = \tan\left(\frac{A-B}{2}\right).$ Hence proved.

Q7(iv)

We have,

LHS $= \dfrac{\sin A + \sin B}{\sin A - \sin B}$

$= \dfrac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}$

$= \dfrac{\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$

$= \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$

= RHS

$\therefore \dfrac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$. Hence proved.

Q7(v)

We have,

LHS $= \dfrac{\cos A + \cos B}{\cos B - \cos A}$

$= \dfrac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{B+A}{2}\right)\sin\left(\frac{B-A}{2}\right)}$

$= \dfrac{\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)}$ $\left[\because \sin(-\theta) = -\sin\theta\right]$

$= \dfrac{-\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$

$= \cot\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$

= RHS

$\therefore \dfrac{\cos A + \cos B}{\cos B - \cos A} = \cot\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$. Hence proved.

Q8(i)

We have,

$$\text{LHS} = \frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A}$$

$$= \frac{(\sin 5A + \sin A) + \sin 3A}{(\cos 5A + \cos A) + \cos 3A}$$

$$= \frac{2\sin\left(\frac{5A + A}{2}\right)\cos\left(\frac{5A - A}{2}\right) + \sin 3A}{2\cos\left(\frac{5A + A}{2}\right)\cos\left(\frac{5A - A}{2}\right) + \cos 3A}$$

$$= \frac{2\sin 3A\cos 2A + \sin 3A}{2\cos 3A\cos 2A + \cos 3A}$$

$$= \frac{\sin 3A\,(2\cos 2A + 1)}{\cos 3A\,(2\cos 2A + 1)}$$

$$= \frac{\sin 3A}{\cos 3A}$$

$$= \tan 3A$$

$$= \text{RHS}$$

$\therefore \quad \dfrac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$ Hence proved.

Q8(ii)

We have,

$$\text{LHS} = \frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A}$$

$$= \frac{(\cos 7A + \cos 3A) + 2\cos 5A}{(\cos 5A + \cos A) + 2\cos 3A}$$

$$= \frac{2\cos\left(\frac{7A + 3A}{2}\right)\cos\left(\frac{7A - 3A}{2}\right) + 2\cos 5A}{2\cos\left(\frac{5A + A}{2}\right)\cos\left(\frac{5A - A}{2}\right) + \cos 3A}$$

$$= \frac{2\cos 5A\cos 2A + 2\cos 5A}{2\cos 3A\cos 2A + 2\cos 3A}$$

$$= \frac{2\cos 5A\,(\cos 2A + 1)}{2\cos 3A\,(\cos 2A + 1)}$$

$$= \frac{\cos 5A}{\cos 3A}$$

$$= \text{RHS}$$

$\therefore \quad \dfrac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A} = \dfrac{\cos 5A}{\cos 3A}$ Hence proved.

Q8(iii)

We have,

$$\text{LHS} = \frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A}$$

$$= \frac{(\cos 4A + \cos 2A) + \cos 3A}{(\sin 4A + \sin 2A) + \sin 3A}$$

$$= \frac{2\cos\left(\frac{4A+2A}{2}\right)\cos\left(\frac{4A-2A}{2}\right) + \cos 3A}{2\sin\left(\frac{4A+2A}{2}\right)\cos\left(\frac{4A-2A}{2}\right) + \sin 3A}$$

$$= \frac{2\cos 3A \cos A + \cos 3A}{2\sin 3A \cos A + \sin 3A}$$

$$= \frac{\cos 3A(2\cos A + 1)}{\sin 3A(2\cos A + 1)}$$

$$= \frac{\cos 3A}{\sin 3A}$$

$$= \cot 3A$$

$$= \text{RHS}$$

$\therefore \frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A - \sin 2A} = \cot 3A$ Hence proved.

Q8(iv)

We have,

$$\text{LHS} = \frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A}$$

$$= \frac{(\sin 9A + \sin 3A) + (\sin 7A + \sin 5A)}{(\cos 9A + \cos 3A) + (\cos 7A + \cos 5A)}$$

$$= \frac{2\sin\left(\frac{9A+3A}{2}\right)\cos\left(\frac{9A-3A}{2}\right) + 2\sin\left(\frac{7A+5A}{2}\right)\cos\left(\frac{7A-5A}{2}\right)}{2\cos\left(\frac{9A+3A}{2}\right)\cos\left(\frac{9A-3A}{2}\right) + 2\cos\left(\frac{7A+5A}{2}\right)\cos\left(\frac{7A-5A}{2}\right)}$$

$$= \frac{2\sin 6A \cos 3A + 2\sin 6A \cos A}{2\cos 6A \cos 3A + 2\cos 6A \cos A}$$

$$= \frac{2\sin 6A(\cos 3A + \cos A)}{2\cos 6A(\cos 3A + \cos A)}$$

$$= \frac{\sin 6A}{\cos 6A}$$

$$= \tan 6A$$

$$= \text{RHS}$$

$\therefore \frac{\sin 3A + \sin 5A + \sin 7A - \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A$ Hence proved.

Q8(v)

We have,

$$\text{LHS} = \frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A}$$

$$= \frac{-(\sin 7A - \sin 5A) + (\sin 8A - \sin 4A)}{-(\cos 7A - \cos 5A) - (\cos 8A - \cos 4A)}$$

$$= \frac{-\left[2\sin\left(\frac{7A-5A}{2}\right)\cos\left(\frac{7A+5A}{2}\right)\right] + \left[2\sin\left(\frac{8A-4A}{2}\right)\cos\left(\frac{8A+4A}{2}\right)\right]}{-2\sin\left(\frac{7A+5A}{2}\right)\sin\left(\frac{7A-5A}{2}\right) - \left[-2\sin\left(\frac{8A+4A}{2}\right)\sin\left(\frac{8A-4A}{2}\right)\right]}$$

$$= \frac{-2\sin A\cos 6A + 2\sin 2A\cos 6A}{-2\sin 6A\sin A + 2\sin 6A\sin 2A}$$

$$= \frac{2\cos 6A\left[-\sin A + \sin 2A\right]}{2\sin 6A\left[-\sin A + \sin 2A\right]}$$

$$= \frac{\cos 6A}{\sin 6A}$$

$$= \cot 6A$$

$$= \text{RHS}$$

$$\therefore \frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} = \cot 6A$$ Hence proved.

Q8(vi)

We have,

$$\text{LHS} = \frac{\sin 5A\cos 2A - \sin 6A\cos A}{\sin A\sin 2A - \cos 2A\cos 3A}$$

$$= \frac{2(\sin 5A\cos 2A - \sin 6A\cos A)}{2(\sin A\sin 2A - \cos 2A\cos 3A)}$$

$$= \frac{2\sin 5A\cos 2A - 2\sin 6A\cos A}{2\sin A\sin 2A - 2\cos 2A\cos 3A}$$

$$= \frac{\sin(5A+2A) + \sin(5A-2A) - [\sin(6A+A) + \sin(6A-A)]}{\cos(2A-A) - \cos(2A+A) - [\cos(3A+2A) + \cos(3A-2A)]}$$

$$= \frac{\sin 7A + \sin 3A - \sin 7A - \sin 5A}{\cos A - \cos 3A - \cos 5A - \cos A}$$

$$= \frac{\sin 3A - \sin 5A}{-\cos 3A - \cos 5A}$$

$$= \frac{-(\sin 5A - \sin 3A)}{-(\cos 5A + \cos 3A)}$$

$$= \frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$$

$$= \frac{2\sin\left(\frac{5A-3A}{2}\right)\cos\left(\frac{5A+3A}{2}\right)}{2\cos\left(\frac{5A+3A}{2}\right)\cos\left(\frac{5A-3A}{2}\right)}$$

$$= \frac{\sin A\cos 4A}{\cos 4A\cos A}$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

$$= \text{RHS}$$

$$\therefore \frac{\sin 5A\cos 2A - \sin 6A\cos A}{\sin A\sin 2A - \cos 2A\cos 3A} = \tan A$$ Hence proved.

Q8(vii)

We have,

$$\text{LHS} = \frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A}$$

$$= \frac{2(\sin 11A \sin A + \sin 7A \sin 3A)}{2(\cos 11A \sin A + \cos 7A \sin 3A)}$$

$$= \frac{2\sin 11A \sin A + 2\sin 7A \sin 3A}{2\cos 11A \sin A + 2\cos 7A \sin 3A}$$

$$= \frac{\cos(11A - A) - \cos(11A + A) + \cos(7A - 3A) - \cos(7A + 3A)}{\sin(11A + A) - \sin(11A - A) + \sin(7A + 3A) - \sin(7A - 3A)}$$

$$= \frac{\cos 10A - \cos 12A + \cos 4A - \cos 10A}{\sin 12A - \sin 10A + \sin 10A - \sin 4A}$$

$$= \frac{-(\cos 12A - \cos 4A)}{\sin 12A - \sin 4A}$$

$$= \frac{-\left[-2\sin\left(\frac{12A + 4A}{2}\right)\sin\left(\frac{12A - 4A}{2}\right)\right]}{2\sin\left(\frac{12A - 4A}{2}\right)\cos\left(\frac{12A + 4A}{2}\right)}$$

$$= \frac{2\sin 8A \sin 4A}{2\sin 4A \cos 8A}$$

$$= \frac{\sin 8A}{\cos 8A}$$

$$= \tan 8A$$

$$= \text{RHS}$$

$$\therefore \quad \frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$$ Hence proved.

Q8(viii)

$$LHS = \frac{\sin 3A\cos 4A - \sin A\cos 2A}{\sin 4A\sin A + \cos 6A\cos A}$$

$$= \frac{2(\sin 3A\cos 4A - \sin A\cos 2A)}{2(\sin 4A\sin A + \cos 6A\cos A)}$$

$$= \frac{2\sin 3A\cos 4A - 2\sin A\cos 2A}{2\sin 4A\sin A + 2\cos 6A\cos A}$$

$$= \frac{\sin(4A+3A) - \sin(4A-3A) - [\sin(2A+A) - \sin(2A-A)]}{\cos(4A-A) - \cos(4A+A) + \cos(6A+A) + \cos(6A-A)}$$

$$= \frac{\sin(7A) - \sin(A) - \sin(3A) + \sin(A)}{\cos(3A) - \cos(5A) + \cos(7A) + \cos(5A)}$$

$$= \frac{\sin(7A) - \sin(3A)}{\cos(3A) - \cos(7A)}$$

$$= \frac{2\sin\left(\frac{7A-3A}{2}\right)\cos\left(\frac{7A+3A}{2}\right)}{2\cos\left(\frac{7A-3A}{2}\right)\cos\left(\frac{7A-3A}{2}\right)}$$

$$= \frac{\sin 2A}{\cos 2A}$$

$$= \tan 2A$$

$$= RHS$$

Q8(ix)

We have

$$LHS = \frac{\sin A\sin 2A + \sin 3A\sin 6A}{\sin A\cos 2A + \sin 3A\cos 6A}$$

$$= \frac{2[\sin A\sin 2A + \sin 3A\sin 6A]}{2[\sin A\cos 2A + \sin 3A\cos 6A]}$$

$$= \frac{2\sin 2A\sin A + 2\sin 6A\sin 3A}{2\cos 2A\sin A + 2\cos 6A\sin 3A}$$

$$= \frac{\cos(2A-A) - \cos(2A+A) + \cos(6A-3A) - \cos(6A+3A)}{\sin(2A+A) - \sin(2A-A) + \sin(6A+3A) - \sin(6A-3A)}$$

$$= \frac{\cos A - \cos 3A + \cos 3A - \cos 9A}{\sin 3A - \sin A + \sin 9A - \sin 3A}$$

$$= \frac{\cos A - \cos 9A}{\sin 9A - \sin A}$$

$$= \frac{-(\cos 9A - \cos A)}{\sin 9A - \sin A}$$

$$= \frac{-\left(-2\sin\left(\frac{9A+A}{2}\right)\times\sin\left(\frac{9A-A}{2}\right)\right)}{2\sin\left(\frac{9A-A}{2}\right)\times\cos\left(\frac{9A+A}{2}\right)}$$

$$= \frac{\sin 5A\sin 4A}{\sin 4A\cos 5A}$$

$$= \tan 5A$$

$$= RHS$$

$$\frac{\sin A\sin 2A + \sin 3A\sin 6A}{\sin A\cos 2A + \sin 3A\cos 6A} = \tan 5A$$ Hence proved.

Q8(x)

We have,

$$\text{LHS} = \frac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A - \sin 7A}$$

$$= \frac{\sin 5A + \sin A + 2\sin 3A}{\sin 7A + \sin 3A + 2\sin 5A}$$

$$= \frac{2\sin\left(\frac{5A - A}{2}\right)\cos\left(\frac{5A - A}{2}\right) - 2\sin 3A}{2\sin\left(\frac{7A + 3A}{2}\right)\cos\left(\frac{7A - 3A}{2}\right) + 2\sin 5A}$$

$$= \frac{2\sin 3A\cos 2A + 2\sin 3A}{2\sin 5A\cos 2A + 2\sin 5A}$$

$$= \frac{2\sin 3A(\cos 2A + 1)}{2\sin 5A(\cos 2A + 1)}$$

$$= \frac{\sin 3A}{\sin 5A}$$

$= \text{RHS}$

$$\therefore \frac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A} = \frac{\sin 3A}{\sin 5A}$$ Hence proved.

Q8(xi)

We have,

$$\text{LHS} = \frac{\sin(\theta + \phi) - 2\sin\theta + \sin(\theta - \phi)}{\cos(\theta + \phi) - 2\cos\theta + \cos(\theta - \phi)}$$

$$= \frac{\sin(\theta + \phi) + \sin(\theta - \phi) - 2\sin\theta}{\cos(\theta + \phi) + \cos(\theta - \phi) - 2\cos\theta}$$

$$= \frac{2\sin\left[\frac{(\theta + \phi) + (\theta - \phi)}{2}\right]\cos\left[\frac{(\theta + \phi) - (\theta - \phi)}{2}\right] - 2\sin\theta}{2\cos\left[\frac{(\theta + \phi) + (\theta - \phi)}{2}\right]\cos\left[\frac{(\theta + \phi) - (\theta - \phi)}{2}\right] - 2\cos\theta}$$

$$= \frac{2\sin(\theta)\cos(\phi) - 2\sin\theta}{2\cos(\theta)\cos(\phi) - 2\cos\theta}$$

$$= \frac{2\sin\theta(\cos\phi - 1)}{2\cos\theta(\cos\phi - 1)}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$= \text{RHS}$

$$\therefore \frac{\sin(\theta + \phi) - 2\sin\theta + \sin(\theta - \phi)}{\cos(\theta + \phi) - 2\cos\theta + \cos(\theta - \phi)} = \tan\theta$$ Hence proved.

Q9(i)

We have,

LHS $= \sin\alpha + \sin\beta - \sin\gamma - \sin(\alpha+\beta-\gamma)$

$= (\sin\alpha + \sin\beta) + \{\sin\gamma - \sin(\alpha+\beta+\gamma)\}$

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + 2\sin\left(\frac{\gamma-(\alpha+\beta-\gamma)}{2}\right)\cos\left(\frac{\gamma+\alpha+\beta+\gamma}{2}\right)$

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + 2\sin\left(\frac{-\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)$

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) - 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)$

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right) - \cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[2\sin\left[\frac{\frac{\alpha}{2}-\frac{\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\right]\sin\left[\frac{\frac{\alpha}{2}-\frac{\beta}{2}-\frac{\alpha+\beta+2\gamma}{2}}{2}\right]\right]$

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[2\sin\left[\frac{2\alpha+2\gamma}{2\times2}\right]\sin\left[\frac{-2\beta-2\gamma}{2\times2}\right]\right]$

$= 4\sin\left(\frac{\alpha-\beta}{2}\right)\left[\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left(\frac{-(\beta+\gamma)}{2}\right)\right]$

$= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)$

$= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)$

$=$ RHS

$\therefore \sin\alpha + \sin\beta - \sin\gamma - \sin(\alpha+\beta+\gamma) = 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\gamma}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)$ Hence proved.

Q9(ii)

We have,

LHS $= \cos(A+B+C) + \cos(A-B+C) + \cos(A+B-C) + \cos(-A+B+C)$

$= [\cos(A+B-C) + \cos(A-B+C)] + [\cos(A+B-C) + \cos(-A+B+C)]$

$= 2\cos\left\{\frac{A+B+C+A-B+C}{2}\right\}\cos\left\{\frac{A+B+C-A+B-C}{2}\right\} + 2\left\{\cos\left\{\frac{A+B-C-A+B+C}{2}\right\}\cos\left\{\frac{A+B-C+A-B-C}{2}\right\}\right\}$

$= 2\cos\left\{\frac{2A+2C}{2}\right\}\cos\left\{\frac{2B}{2}\right\} + 2\cos\left\{\frac{2B}{2}\right\}\cos\left\{\frac{2A-2C}{2}\right\}$

$= 2\cos(A+C)\cos(B) + 2\cos(B)\cos(A-C)$

$= 2\cos(B)[\cos(A+C) + \cos(A-C)]$

$= 2\cos(B)\left[2\cos\left(\frac{A+C+A-C}{2}\right)\cos\left(\frac{A+C-A+C}{2}\right)\right]$

$= 2\cos(B)[2\cos A\cos C]$

$= 4\cos A\cos B\cos C.$

Q10

We have,

$$\cos A + \cos B = \frac{1}{2}$$

and, $\sin A + \sin B = \frac{1}{4}$

Now,

$$\frac{\sin A + \sin B}{\cos A + \cos B} = \frac{\frac{1}{4}}{\frac{1}{2}} \qquad \text{[On dividing]}$$

$$\Rightarrow \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)} = \frac{1}{2}$$

$$\Rightarrow \frac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} = \frac{1}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \frac{1}{2}$$ Hence proved.

Q11

We have,

$$\operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$$

$$\Rightarrow \sec A - \sec B = \operatorname{cosec} B - \operatorname{cosec} A$$

$$\Rightarrow \frac{1}{\cos A} - \frac{1}{\cos B} = \frac{1}{\sin B} - \frac{1}{\sin A}$$

$$\Rightarrow \frac{\cos B - \cos A}{\cos A \cos B} = \frac{\sin A - \sin B}{\sin A \sin B}$$

$$\Rightarrow \frac{\sin A \sin B}{\cos A \cos B} = \frac{\sin A - \sin B}{\cos B - \cos A}$$

$$\Rightarrow \tan A \tan B = \frac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{-2\sin\left(\frac{B-A}{2}\right)\sin\left(\frac{B+A}{2}\right)}$$

$$\Rightarrow \tan A \tan B = \frac{-\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)} \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$\Rightarrow \tan A \tan B = \cot\left(\frac{A+B}{2}\right)$$ Hence proved.

Q12

We have,

$$\sin 2A = \lambda \sin 2B$$

$$\Rightarrow \quad \lambda = \frac{\sin 2A}{\sin 2B}$$

Now,

$$\frac{\lambda + 1}{\lambda - 1} = \frac{\frac{\sin 2A}{\sin 2B} + 1}{\frac{\sin 2A}{\sin 2B} - 1}$$

$$= \frac{\frac{\sin 2A + \sin 2B}{\sin 2B}}{\frac{\sin 2A - \sin 2B}{\sin 2B}}$$

$$= \frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B}$$

$$= \frac{2 \sin\left(\frac{2A + 2B}{2}\right) \cos\left(\frac{2A - 2B}{2}\right)}{2 \sin\left(\frac{2A - 2B}{2}\right) \cos\left(\frac{2A + 2B}{2}\right)}$$

$$= \frac{\sin(A + B)\cos(A - B)}{\sin(A - B)\cos(A + B)}$$

$$= \frac{\sin(A + B)\cos(A - B)}{\cos(A + B)\sin(A - B)}$$

$$= \frac{\tan(A + B)}{\tan(A - B)}$$

$$\therefore \quad \frac{\lambda + 1}{\lambda - 1} = \frac{\tan(A + B)}{\tan(A - B)}$$

$$\Rightarrow \quad \frac{\tan(A + B)}{\tan(A - B)} = \frac{\lambda + 1}{\lambda - 1}$$

Hence proved.

Q13(i)

We have,

$$\text{LHS} = \frac{\cos(A+B+C)+\cos(-A+B+C)+\cos(A-B+C)+\cos(A+B-C)}{\sin(A+B+C)+\sin(-A+B+C)+\sin(A-B+C)-\sin(A+B-C)}$$

$$= \frac{2\cos\left\{\frac{A+B+C-A+B+C}{2}\right\}\cos\left\{\frac{A+B+C+A-B-C}{2}\right\}+2\cos\left\{\frac{A-B+C+A+B-C}{2}\right\}\cos\left\{\frac{A-B+C-A-B+C}{2}\right\}}{2\sin\left\{\frac{A+B+C-A+B+C}{2}\right\}\cos\left\{\frac{A+B+C+A-B-C}{2}\right\}+2\sin\left\{\frac{A-B+C-A-B+C}{2}\right\}\cos\left\{\frac{A-B+C+A+B-C}{2}\right\}}$$

$$= \frac{2\cos(B+C)\cos A+2\cos A\cos(C-B)}{2\sin(B+C)\cos A+2\sin(C-B)\cos A}$$

$$= \frac{2\cos A\left[\cos(B+C)+\cos(C-B)\right]}{2\cos A\left[\sin(B+C)+\sin(C-B)\right]}$$

$$= \frac{\cos(B+C)+\cos(C-B)}{\sin(B+C)+\sin(C-B)}$$

$$= \frac{2\cos\left\{\frac{B+C+C-B}{2}\right\}\cos\left\{\frac{B+C-C+B}{2}\right\}}{2\sin\left\{\frac{B+C+C-B}{2}\right\}\cos\left\{\frac{B+C-C+B}{2}\right\}}$$

$$= \frac{2\cos C\cos B}{2\sin C\cos B}$$

$$= \frac{\cos C}{\sin C}$$

$$= \cot C$$

$$= \text{RHS}$$

$$\therefore \frac{\cos(A+B+C)+\cos(-A+B+C)+\cos(A-B+C)+\cos(A+B-C)}{\sin(A+B+C)+\sin(-A+B+C)+\sin(A-B+C)-\sin(A+B-C)} = \cot C.$$

Hence proved.

Q13(ii)

We have,

$$\begin{aligned}
LHS &= \sin(B-C)\cos(A-D)+\sin(C-A)\cos(B-D)+\sin(A-B)\cos(C-D)\\
&= \frac{1}{2}\left[2\sin(B-C)\cos(A-D)+2\sin(C-A)\cos(B-D)+2\sin(A-B)\cos(C-D)\right]\\
&= \frac{1}{2}\left[\begin{array}{r}\sin(B-C+A-D)+\sin(B-C-A+D)+\sin(C-A+B-D)+\sin(C-A-B+D)\\ +\sin(A-B+C-D)+\sin(A-B-C+D)\end{array}\right]\\
&= \frac{1}{2}\left[\begin{array}{r}\sin(A+B-C-D)+\sin(B+D-C-A)+\sin(B+C-A-D)+\sin(C+D-A-B)\\ +\sin(A+C-B-D)+\sin(A+D-B-C)\end{array}\right]\\
&= \frac{1}{2}\left[\begin{array}{r}\sin(A+B-C-D)-\sin(A+C-B-D)-\sin(A+D-B-C)-\sin(A+B-C-D)\\ +\sin(A+C-B-D)+\sin(A+D-B-C)\end{array}\right]\\
&= \frac{1}{2}[0]\\
&= 0\\
&= RHS
\end{aligned}$$

$\sin(B-C)\cos(A-D)+\sin(C-A)\cos(B-D)+\sin(A-B)\cos(C-D)=0$ Hence proved.

Q14

We have,

$$\frac{\cos(A-B)}{\cos(A+B)}+\frac{\cos(C+D)}{\cos(C-D)}=0$$

$$\Rightarrow \frac{\cos(A-B)}{\cos(A+B)}=-\frac{\cos(C+D)}{\cos(C-D)} \qquad \text{(i)}$$

Now,

$$\frac{\cos(A-B)}{\cos(A+B)}=-\frac{\cos(C+D)}{\cos(C-D)}$$

$$\Rightarrow \frac{\cos(A-B)}{\cos(A+B)}+1=\frac{-\cos(C+D)}{\cos(C-D)}+1$$

$$\Rightarrow \frac{\cos(A-B)+\cos(A+B)}{\cos(A+B)}=\frac{-\cos(C+D)+\cos(C-D)}{\cos(C-D)}$$

$$\Rightarrow \frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)}=\frac{-[\cos(C+D)-\cos(C-D)]}{\cos(C-D)} \qquad \text{—(I)}$$

Again,

$$\frac{\cos(A-B)}{\cos(A+B)}=\frac{-\cos(C+D)}{\cos(C-D)} \qquad \text{[By equation (i)]}$$

$$\Rightarrow \frac{\cos(A-B)}{\cos(A+B)}-1=\frac{-\cos(C+D)}{\cos(C-D)}-1$$

$$\Rightarrow \frac{\cos(A-B)-\cos(A+B)}{\cos(A+B)}=\frac{-\cos(C+D)-\cos(C-D)}{\cos(C-D)}$$

$$\Rightarrow \frac{-[\cos(A+B)-\cos(A-B)]}{\cos(A+B)}=\frac{-[\cos(C+D)+\cos(C-D)]}{\cos(C-D)}$$

$$\Rightarrow \frac{\cos(A+B)-\cos(A-B)}{\cos(A+B)}=\frac{\cos(C+D)+\cos(C-D)}{\cos(C-D)} \qquad \text{—(II)}$$

Dividing equation (I) by equation (II), we get

$$\frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)-\cos(A-B)}=\frac{-[\cos(C+D)-\cos(C-D)]}{\cos(C+D)+\cos(C-D)}$$

$$\Rightarrow \frac{2\cos\left\{\frac{A+B+A-B}{2}\right\}\cos\left\{\frac{A+B-A+B}{2}\right\}}{-2\sin\left\{\frac{A+B+A-B}{2}\right\}\sin\left\{\frac{A+B-A+B}{2}\right\}}=\frac{-\left[-2\sin\left\{\frac{C+D+C-D}{2}\right\}\sin\left\{\frac{C+D-C+D}{2}\right\}\right]}{2\cos\left\{\frac{C+D+C-D}{2}\right\}\cos\left\{\frac{C+D-C+D}{2}\right\}}$$

$$\Rightarrow \frac{\cos A\cos B}{-\sin A\sin B}=\frac{\sin C\sin D}{\cos C\cos D}$$

$$\Rightarrow \frac{1}{-\tan A\tan B}=\tan C\tan D$$

$$\Rightarrow -1=\tan A\tan B\tan C\tan D$$

$\therefore\ \tan A\tan B\tan C\tan D=-1$ Hence proved.

Q15

We have,

$$\cos(\alpha+\beta)\sin(\gamma+\delta)=\cos(\alpha-\beta)\sin(\gamma-\delta)$$

$$\Rightarrow \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)} \quad \text{---(i)}$$

Now,

$$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}$$

$$\Rightarrow \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}+1=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}+1$$

$$\Rightarrow \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)+\sin(\gamma+\delta)}{\sin(\gamma+\delta)} \quad \text{---(ii)}$$

Again,

$$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)} \quad [\text{By equation (i)}]$$

$$\Rightarrow \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}-1=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}-1$$

$$\Rightarrow \frac{\cos(\alpha+\beta)-\cos(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)-\sin(\gamma+\delta)}{\sin(\gamma+\delta)} \quad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)+\sin(\gamma+\delta)}{\sin(\gamma-\delta)-\sin(\gamma+\delta)}$$

$$\Rightarrow \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}=-\left[\frac{\sin(\gamma+\delta)+\sin(\gamma-\delta)}{\sin(\gamma+\delta)-\sin(\gamma-\delta)}\right]$$

$$\Rightarrow \frac{2\cos\left\{\frac{\alpha+\beta+\alpha-\beta}{2}\right\}\cos\left\{\frac{\alpha+\beta-\alpha+\beta}{2}\right\}}{-2\sin\left\{\frac{\alpha+\beta+\alpha-\beta}{2}\right\}\sin\left\{\frac{\alpha+\beta-\alpha+\beta}{2}\right\}}=-\left[\frac{2\sin\left\{\frac{\gamma+\delta+\gamma-\delta}{2}\right\}\cos\left\{\frac{\gamma+\delta-\gamma+\delta}{2}\right\}}{2\sin\left\{\frac{\gamma+\delta-\gamma+\delta}{2}\right\}\cos\left\{\frac{\gamma+\delta+\gamma-\delta}{2}\right\}}\right]$$

$$\Rightarrow \frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}=\frac{\sin\gamma\cos\delta}{\sin\delta\cos\gamma}$$

$$\Rightarrow \cot\alpha\cot\beta=\frac{\sin\gamma\cos\delta}{\cos\gamma\sin\delta}$$

$$\Rightarrow \cot\alpha\cot\beta=\frac{\cot\delta}{\cot\gamma}$$

$$\Rightarrow \cot\alpha\cot\beta\cot\gamma=\cot\delta$$

$\therefore \cot\alpha\cot\beta\cot\gamma=\cot\delta$ Hence proved.

Q16

We have,

$$y \sin\phi = x \sin(2\theta + \phi)$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta + \phi)} = \frac{x}{y} \qquad \text{---(i)}$$

Now,

$$\frac{\sin\phi}{\sin(2\theta + \phi)} = \frac{x}{y}$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta + \phi)} + 1 = \frac{x}{y} + 1$$

$$\Rightarrow \quad \frac{\sin\phi + \sin(2\theta + \phi)}{\sin(2\theta + \phi)} = \frac{x + y}{y} \qquad \text{---(ii)}$$

Again,

$$\frac{\sin\phi}{\sin(2\theta + \phi)} = \frac{x}{y} \qquad \text{[By equation (i)]}$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta + \phi)} - 1 = \frac{x}{y} - 1$$

$$\Rightarrow \quad \frac{\sin\phi - \sin(2\theta + \phi)}{\sin(2\theta + \phi)} = \frac{x - y}{y} \qquad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\sin\phi + \sin(2\theta + \phi)}{\sin\phi - \sin(2\theta + \phi)} = \frac{x + y}{x - y}$$

$$\Rightarrow \quad \frac{2\sin\left(\frac{\phi + 2\theta + \phi}{2}\right)\cos\left(\frac{\phi - 2\theta - \phi}{2}\right)}{2\sin\left(\frac{\phi - 2\theta - \phi}{2}\right)\cos\left(\frac{\phi + 2\theta + \phi}{2}\right)} = \frac{x + y}{x - y}$$

$$\Rightarrow \quad \frac{\sin(\theta + \phi)\cos(\theta - \phi)}{\sin(-\theta)\cos(\theta + \phi)} = \frac{x + y}{x - y}$$

$$\Rightarrow \quad \frac{\sin(\theta + \phi)\cos(\theta)}{\cos(\theta + \phi)[-\sin(\theta)]} = \frac{x + y}{x - y}$$

$$\Rightarrow \quad \frac{-\cot(\theta)}{\cot(\theta + \phi)} = \frac{x + y}{x - y}$$

$$\Rightarrow \quad -(x - y)\cot\theta = (x + y)\cot(\theta + \phi)$$

$$\Rightarrow \quad (y - x)\cot\theta = (x + y)\cot(\theta + \phi)$$

$$\Rightarrow \quad (x + y)\cot(\theta + \phi) = (y - x)\cot\theta \qquad \text{Hence proved.}$$

Q17

We have,

$$\cos(A+B)\sin(C-D) = \cos(A-B)\sin(C+D)$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)} \qquad \text{---(i)}$$

Now,

$$\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)}$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)} + 1 = \frac{\sin(C+D)}{\sin(C-D)} + 1$$

$$\Rightarrow \quad \frac{\cos(A+B) + \cos(A-B)}{\cos(A-B)} = \frac{\sin(C+D) + \sin(C-D)}{\sin(C-D)} \qquad \text{---(ii)}$$

Again,

$$\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)} \qquad [\text{By equation (i)}]$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)} - 1 = \frac{\sin(C+D)}{\sin(C-D)} - 1$$

$$\Rightarrow \quad \frac{\cos(A+B) - \cos(A-B)}{\cos(A-B)} = \frac{\sin(C+D) - \sin(C-D)}{\sin(C-D)} \qquad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\cos(A+B) + \cos(A-B)}{\cos(A+B) - \cos(A-B)} = \frac{\sin(C+D) + \sin(C-D)}{\sin(C+D) - \sin(C-D)}$$

$$\Rightarrow \quad \frac{2\cos\left\{\frac{A+B+A-B}{2}\right\}\cos\left\{\frac{A+B-A+B}{2}\right\}}{-2\sin\left\{\frac{A+B+A-B}{2}\right\}\sin\left\{\frac{A+B-A+B}{2}\right\}} = \frac{2\sin\left\{\frac{C+D+C-D}{2}\right\}\cos\left\{\frac{C+D-C+D}{2}\right\}}{2\sin\left\{\frac{C+D-C+D}{2}\right\}\cos\left\{\frac{C+D+C-D}{2}\right\}}$$

$$\Rightarrow \quad \frac{\cos A \cos B}{-\sin A \sin B} = \frac{\sin C \cos D}{\sin D \cos C}$$

$$\Rightarrow \quad \frac{1}{-\tan A \tan B} = \frac{\sin C \cos D}{\cos C \sin D}$$

$$\Rightarrow \quad \frac{-1}{\tan A \tan B} = \frac{\tan C}{\tan D}$$

$$\Rightarrow \quad -\tan D = \tan A \tan B \tan C$$

$$\Rightarrow \quad \tan A \tan B \tan C = -\tan D$$

$$\Rightarrow \quad \tan A \tan B \tan C + \tan D = 0$$

Hence proved.

Q18

Given $x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right) = k(say)$

$$x = \frac{k}{\cos\theta}$$

$$y = \frac{k}{\cos\left(\theta + \frac{2\pi}{3}\right)}$$

$$z = \frac{k}{\cos\left(\theta + \frac{4\pi}{3}\right)}$$

$$xy + yz + zx = k^2\left[\frac{1}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)} + \frac{1}{\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)} + \frac{1}{\cos\left(\theta + \frac{4\pi}{3}\right)\cos\theta}\right]$$

$$= k^2\left[\frac{\cos\left(\theta + \frac{4\pi}{3}\right) + \cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= k^2\left[\frac{\cos\theta\cos\frac{4\pi}{3} - \sin\theta\sin\frac{4\pi}{3} + \cos\theta + \cos\theta\cos\frac{2\pi}{3} - \sin\theta\sin\frac{2\pi}{3}}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= k^2\left[\frac{\cos\theta\left(\frac{-1}{2}\right) - \sin\theta\left(\frac{-\sqrt{3}}{2}\right) + \cos\theta + \cos\theta\left(\frac{-1}{2}\right) - \sin\theta\left(\frac{\sqrt{3}}{2}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= k^2\left[\frac{-\cos\theta + \sin\theta\left(\frac{\sqrt{3}}{2}\right) + \cos\theta + -\sin\theta\left(\frac{\sqrt{3}}{2}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= 0$$

Hence Proved

Q19

Given that $m\sin\theta = n\sin(\theta + 2a)$,

We need to prove that $\tan(\theta + a) = \frac{m+n}{m-n}\tan a$

$m\sin\theta = n\sin(\theta + 2a)$

$\Rightarrow \frac{\sin(\theta + 2a)}{\sin\theta} = \frac{m}{n}$

Using Componendo − Dividendo, we have,

$\Rightarrow \frac{\sin(\theta + 2a) + \sin\theta}{\sin(\theta + 2a) - \sin\theta} = \frac{m+n}{m-n}$ ....(1)

We know that,

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

and

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

Applying the above formulae in equation (1), we have,

$$\frac{2\sin\frac{\theta+2a+\theta}{2}\cos\frac{\theta+2a-\theta}{2}}{2\cos\frac{\theta+2a+\theta}{2}\sin\frac{\theta+2a-\theta}{2}} = \frac{m+n}{m-n}$$

$\Rightarrow \frac{2\sin(\theta + a)\cos a}{2\cos(\theta + a)\sin a} = \frac{m+n}{m-n}$

$\Rightarrow \frac{\tan(\theta + a)}{\tan a} = \frac{m+n}{m-n}$

$\Rightarrow \tan(\theta + a) = \frac{m+n}{m-n} \times \tan a$

Hence proved.