7. Adjoint and Inverse of a Matrix

Exercise 7.1

1 A. Question

Find the adjoint of each of the following Matrices.

$$\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}$$

Verify that (adj A) A=|A| I=A (adj A) for the above matrices.

Answer

A = $\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}$

Cofactors of A are

C_{11} = 4

C_{12} = - 2

C_{21} = - 5

C_{22} = - 3

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} 4 & -2 \\ -5 & -3 \end{bmatrix}^T$

= $\begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}$

Now, (adj A)A = $\begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -12-10 & 20-20 \\ 6-6 & -10-12 \end{bmatrix}$

(adj A)A = $\begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

And, |A|.I = $\begin{vmatrix} -3 & 5 \\ 2 & 4 \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (-22)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

Also, A(adj A) = $\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -12-10 & 20-20 \\ 6-6 & -10-12 \end{bmatrix}$

A(adj A) = $\begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

Hence, (adj A)A = |A|.I = A.(adj A)

1 B. Question

Find the adjoint of each of the following Matrices.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Verify that (adj A) A=|A| I=A (adj A) for the above matrices.

Answer

A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Cofactors of A are

C_{11} = d

C_{12} = – c

C_{21} = – b

C_{22} = a

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T$

= $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now, (adj A)A = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ad - bc & bd - bd \\ -ac + ac & -bc + ad \end{bmatrix}$

(adj A)A = $\begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix}$

And, |A|.I = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (ad - bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix}$

Also, A(adj A) = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix}$

Hence, (adj A)A = |A|.I = A.(adj A)

1 C. Question

Find the adjoint of each of the following Matrices.

$$\begin{bmatrix} \cos a & \sin a \\ \sin a & \cos a \end{bmatrix}$$

Verify that (adj A) A=|A| I=A (adj A) for the above matrices.

Answer

A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Cofactors of A are

$C_{11} = \cos\alpha$

$C_{12} = -\sin\alpha$

$C_{21} = -\sin\alpha$

$C_{22} = \cos\alpha$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}^T$

= $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Now, (adj A)A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

= $\begin{bmatrix} -\sin^2\alpha + \cos^2\alpha & \cos\alpha.\sin\alpha - \sin\alpha.\cos\alpha \\ -\cos\alpha\sin\alpha + \sin\alpha\cos\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$

(adj A)A = $\begin{bmatrix} \cos2\alpha & 0 \\ 0 & \cos2\alpha \end{bmatrix}$

And, |A|.I = $\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $(\cos^2\alpha - \sin^2\alpha)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} \cos^2\alpha - \sin^2\alpha & 0 \\ 0 & \cos^2\alpha - \sin^2\alpha \end{bmatrix}$

= $\begin{bmatrix} \cos2\alpha & 0 \\ 0 & \cos2\alpha \end{bmatrix}$

Also, A(adj A) = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & 0 \\ 0 & \cos^2\alpha - \sin^2\alpha \end{bmatrix}$

= $\begin{bmatrix} \cos2\alpha & 0 \\ 0 & \cos2\alpha \end{bmatrix}$

Hence, (adj A)A = |A|.I = A.(adj A)

1 D. Question

Find the adjoint of each of the following Matrices.

$$\begin{bmatrix} 1 & \tan a/2 \\ -\tan a/2 & 1 \end{bmatrix}$$

Verify that (adj A) A=|A| I=A (adj A) for the above matrices.

Answer

A = $\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$

Cofactors of A are

$C_{11} = 1$

$C_{12} = \tan\frac{\alpha}{2}$

$C_{21} = -\tan\frac{\alpha}{2}$

$C_{22} = 1$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}^T$

= $\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

Now, (adj A)A = $\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 + \tan^2\frac{\alpha}{2} & \tan\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} - \tan\frac{\alpha}{2} & 1 + \tan^2\frac{\alpha}{2} \end{bmatrix}$

(adj A)A = $\begin{bmatrix} 1 + \tan^2\frac{\alpha}{2} & 0 \\ 0 & 1 + \tan^2\frac{\alpha}{2} \end{bmatrix}$

And, |A|.I = $\begin{vmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \left(1 + \tan^2\frac{\alpha}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 + \tan^2\frac{\alpha}{2} & 0 \\ 0 & 1 + \tan^2\frac{\alpha}{2} \end{bmatrix}$

Also, A(adj A) = $\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 + \tan^2\frac{\alpha}{2} & \tan\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} - \tan\frac{\alpha}{2} & 1 + \tan^2\frac{\alpha}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 + \tan^2\frac{\alpha}{2} & 0 \\ 0 & 1 + \tan^2\frac{\alpha}{2} \end{bmatrix}$

Hence, (adj A)A = |A|.I = A.(adj A)

2 A. Question

Find the adjoint of each of the following Matrices and Verify that (adj A) A = |A| I = A (adj A)

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Verify that (adj A) A=|A| I=A (adj A) for the above matrices.

Answer

A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

Cofactors of A are:

C_{11} = – 3 C_{21} = 2 C_{31} = 2

C_{12} = 2 C_{22} = – 3 C_{23} = 2

C_{13} = 2 C_{23} = 2 C_{33} = – 3

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

Now, (adj A).A = $\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

= $\begin{bmatrix} -3+4+4 & -6+2+4 & -6+4+2 \\ 2-3+4 & 4-3+4 & 4-6+2 \\ 2+4-6 & 4+2-6 & 4+4-3 \end{bmatrix}$

= $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Also, |A|.I = $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = (- 3 + 4 + 4)$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Then, A.(adj A) = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

= $\begin{bmatrix} -3+4+4 & -6+2+4 & -6+4+2 \\ 2-3+4 & 4-3+4 & 4-6+2 \\ 2+4-6 & 4+2-6 & 4+4-3 \end{bmatrix}$

= $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Since, (adj A).A = |A|.I = A(adj A)

2 B. Question

Find the adjoint of each of the following Matrices and Verify that (adj A) A = |A| I = A (adj A)

$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Verify that (adj A) A=|A| I=A (adj A) for the above matrices.

Answer

A = $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Cofactors of A

C_{11} = 2 C_{21} = 3 C_{31} = - 13

C_{12} = - 3 C_{22} = 6 C_{32} = 9

C_{13} = 5 C_{23} = - 3 C_{33} = - 1

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}^T$

$$\text{adj } A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\text{Now, (adj A).A} = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+6+13 & 4+9-13 & 10+3-13 \\ -3+12-9 & -6+18+9 & -15+6+9 \\ 5-6+1 & 10-9-1 & 25-3-1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$\text{Also, } |A|.I = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1(3-1) - 2(2+1) + 5(2+3)] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (21) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$\text{Then, A.(adj A)} = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6+25 & 3+12-15 & -13+18-5 \\ 4-9+5 & 6+18-3 & -26+27-1 \\ -2-3+5 & -3+6-3 & 13+9-1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

Hence, (adj A).A = |A|.I = A(adj A)

2 C. Question

Find the adjoint of each of the following Matrices and Verify that (adj A) A = |A| I = A (adj A)

$$\begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}$$

Verify that (adj A) A=|A| I=A (adj A) for the above matrices.

Answer

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}$$

Cofactors of A

C_{11} = – 22 C_{21} = 11 C_{31} = – 11

C_{12} = 4 C_{22} = – 2 C_{32} = 2

C_{13} = 16 C_{23} = – 8 C_{33} = 8

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} -22 & 4 & 16 \\ 11 & -2 & -8 \\ -11 & 2 & 8 \end{bmatrix}^T$

adj A = $\begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix}$

Now, (adj A).A = $\begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix}\begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}$

= $\begin{bmatrix} -44+44+0 & 22+22-44 & -66+55+11 \\ 8-8+0 & -4-4+8 & 12-10-2 \\ 32-32+0 & -16-16+32 & 48-40--8 \end{bmatrix}$

= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Also, |A|.I = $\begin{vmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{vmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= [2(– 2 – 20) + 1(– 4 – 0) + 3(16 – 0)] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= (– 44 – 4 + 48) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Then, A.(adj A) = $\begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}\begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix}$

= $\begin{bmatrix} -44-4+48 & 22+2-24 & -22-2+24 \\ -88+8+80 & 44-4-40 & -44+4+40 \\ 0+16-16 & 0-8+8 & 0+8-8 \end{bmatrix}$

= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Hence, (adj A).A = |A|.I = A(adj A)

2 D. Question

Find the adjoint of each of the following Matrices and Verify that (adj A) A = |A| I = A (adj A)

$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$

Verify that (adj A) A=|A| I=A (adj A) for the above matrices.

Answer

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

Cofactors of A

C_{11} = 3 C_{21} = - 1 C_{31} = - 1

C_{12} = - 15 C_{22} = 7 C_{32} = - 5

C_{13} = 4 C_{23} = - 2 C_{33} = 2

$$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & -15 & 4 \\ -3 & 7 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}$$

$$\text{Now, (adj A).A} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6-5+1 & 0-1+1 & -3+0+3 \\ -30+35-5 & 0+7-5 & 15-0-15 \\ 8-10+2 & 0-2+2 & -4-0+6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\text{Also, } |A|.I = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [2(3 - 0) + 0(15 - 0) - 1(5 - 1)] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (6 - 4) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\text{Then, A.(adj A)} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6+0-4 & -2+0+2 & 2-0-2 \\ 15-15+0 & -5+7+0 & 5-5+0 \\ 3-15+12 & -1+7-6 & 1-5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Hence, (adj A).A = |A|.I = A(adj A)

3. Question

For the matrix A= $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$, show that A(adj A)=O.

Answer

$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$

Cofactors of A

C_{11} = 30 C_{21} = 12 C_{31} = - 3

C_{12} = - 20 C_{22} = - 8 C_{32} = 2

C_{13} = - 50 C_{23} = - 2 0 C_{33} = 5

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 30 & -20 & -50 \\ 12 & -8 & -20 \\ -3 & 2 & 5 \end{bmatrix}^T$

So, adj(A) = $\begin{bmatrix} 30 & 12 & -3 \\ -20 & -8 & 2 \\ -50 & -20 & 5 \end{bmatrix}$

Now, A.(adj A) = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}\begin{bmatrix} 30 & 12 & -3 \\ -20 & -8 & 2 \\ -50 & -20 & 5 \end{bmatrix}$

$= \begin{bmatrix} 30 + 20 - 50 & 12 + 8 - 20 & -3 - 2 + 5 \\ 60 - 60 + 0 & 24 - 24 + 0 & -6 + 6 + 0 \\ 540 - 40 - 500 & 216 - 16 - 200 & -54 + 4 + 50 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Hence, A(adj A) = 0

4. Question

If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, show that adj A=A.

Answer

$A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

Cofactors of A

C_{11} = - 4 C_{21} = - 3 C_{31} = - 3

C_{12} = 1 C_{22} = 0 C_{32} = 1

C_{13} = 4 C_{23} = 4 C_{33} = 3

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

Hence, adj A = A

5. Question

If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, show that adj A=3A^T.

Answer

A = $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Cofactors of A are:

C_{11} = – 3 C_{21} = 6 C_{31} = 6

C_{12} = – 6 C_{22} = 3 C_{32} = – 6

C_{13} = – 6 C_{23} = – 6 C_{33} = 3

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

Now, 3A^T = $3\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

Hence, adj A = 3.A^T

6. Question

Find A (adj A) for the matrix A=$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$.

Answer

A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$

Cofactors of A are:

$C_{11} = 9$ $C_{21} = 19$ $C_{31} = -4$

$C_{12} = 4$ $C_{22} = 14$ $C_{32} = 1$

$C_{13} = 8$ $C_{23} = 3$ $C_{33} = 2$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$

Now, A. adj A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}\begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-8+24 & 19-28+9 & -4-2+6 \\ 0+8-8 & 0+28-3 & 0+2-2 \\ -36+20+16 & -76+70+6 & 16+5+4 \end{bmatrix}$

$= \begin{bmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 25 \end{bmatrix}$

Hence, A. adj A = $25.I_3$

7 A. Question

Find the inverse of each of the following matrices:

$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Answer

Now, $|A| = \cos\theta(\cos\theta) + \sin\theta(\sin\theta)$

$= 1$

Hence, A^{-1} exists.

Cofactors of A are

$C_{11} = \cos\theta$

$C_{12} = \sin\theta$

$C_{21} = -\sin\theta$

$C_{22} = \cos\theta$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^T$

$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj } A$

$A^{-1} = \frac{1}{1}.\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

7 B. Question

Find the inverse of each of the following matrices:

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Answer

Now, $|A| = -1 \neq 0$

Hence, A^{-1} exists.

Cofactors of A are

$C_{11} = 0$

$C_{12} = -1$

$C_{21} = -1$

$C_{22} = 0$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}^T$

$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj } A$

$A^{-1} = -\frac{1}{1}.\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

7 C. Question

Find the inverse of each of the following matrices:

$\begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$

Answer

Now, $|A| = \frac{a+abc}{a} - bc = \frac{a+abc-abc}{a} = 1 \neq 0$

Hence, A^{-1} exists.

Cofactors of A are

$C_{11} = \frac{1+bc}{a}$

$C_{12} = -c$

$C_{21} = -b$

$C_{22} = a$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} \frac{1+bc}{a} & -c \\ -b & a \end{bmatrix}^T$

= $\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$.adj A

$A^{-1} = \frac{1}{1}.\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

7 D. Question

Find the inverse of each of the following matrices:

$$\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

Answer

Now, |A| = 2 + 15 = 17

Hence, A^{-1} exists.

Cofactors of A are

$C_{11} = 1$

$C_{12} = 3$

$C_{21} = -5$

$C_{22} = 2$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}^T$

= $\begin{bmatrix} 0 & -5 \\ 3 & 2 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$.adj A

$A^{-1} = \frac{1}{17}.\begin{bmatrix} 0 & -5 \\ 3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{17} \cdot \begin{bmatrix} 0 & -5 \\ 3 & 2 \end{bmatrix}$

8 A. Question

Find the inverse of each of the following matrices.

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Answer

$|A| = 1\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix}$

= 1(6 – 1) – 2(4 – 3) + 3(2 – 9)

= 5 – 2 – 21

= – 18

Hence, A^{-1} exists

Cofactors of A are:

$C_{11} = 5$ $C_{21} = -1$ $C_{31} = -7$

$C_{12} = -1$ $C_{22} = -7$ $C_{32} = 5$

$C_{13} = -7$ $C_{23} = 5$ $C_{33} = -1$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$.adj A

So, $A^{-1} = \frac{1}{(-18)} \cdot \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} \frac{-5}{18} & \frac{1}{18} & \frac{7}{18} \\ \frac{1}{18} & \frac{7}{18} & \frac{-5}{18} \\ \frac{7}{18} & \frac{-5}{18} & \frac{1}{18} \end{bmatrix}$

8 B. Question

Find the inverse of each of the following matrices.

$$\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$$

Answer

$|A| = 1\begin{vmatrix} -1 & -1 \\ 3 & -1 \end{vmatrix} - 2\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} + 5\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}$

= 1(1 + 3) – 2(– 1 + 2) + 5(3 + 2)

= 4 – 2 + 25

= 27

Hence, A^{-1} exists

Cofactors of A are:

$C_{11} = 4$ $C_{21} = 17$ $C_{31} = 3$

$C_{12} = -1$ $C_{22} = -11$ $C_{32} = 6$

$C_{13} = 5$ $C_{23} = 1$ $C_{33} = -3$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj } A$

So, $A^{-1} = \frac{1}{(27)}.\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{3}{27} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{6}{27} \\ \frac{5}{27} & \frac{1}{27} & \frac{-3}{27} \end{bmatrix} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{1}{9} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{2}{9} \\ \frac{5}{27} & \frac{1}{27} & \frac{-1}{9} \end{bmatrix}$

8 C. Question

Find the inverse of each of the following matrices.

$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Answer

$|A| = 2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + 1\begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix}$

= 2(4 – 1) + 1(– 2 + 1) + 1(1 – 2)

= 6 – 2

= – 4

Hence, A^{-1} exists

Cofactors of A are:

C_{11} = 3 C_{21} = 1 C_{31} = - 1

C_{12} = + 1 C_{22} = 3 C_{32} = 1

C_{13} = - 1 C_{23} = 1 C_{33} = 3

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj } A$

So, $A^{-1} = \frac{1}{4}.\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & \frac{-1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ \frac{-1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

8 D. Question

Find the inverse of each of the following matrices.

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Answer

$|A| = 2\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} - 0\begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$

= 2(3 – 0) – 0 – 1(5)

= 6 – 5

= 1

Hence, A^{-1} exists

Cofactors of A are:

C_{11} = 3 C_{21} = - 1 C_{31} = 1

C_{12} = - 15 C_{22} = 6 C_{32} = - 5

C_{13} = - 5 C_{23} = - 2 C_{33} = 2

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj } A$

So, $A^{-1} = \frac{1}{1}.\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

8 E. Question

Find the inverse of each of the following matrices.

$\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

Answer

$|A| = 0\begin{vmatrix} -3 & 0 \\ -3 & 4 \end{vmatrix} - 1\begin{vmatrix} 4 & 4 \\ 3 & 4 \end{vmatrix} - 1\begin{vmatrix} 4 & -3 \\ 3 & -3 \end{vmatrix}$

= 0 – 1(16 – 12) – 1(– 12 + 9)

= – 4 + 3

= – 1

Hence, A^{-1} exists

Cofactors of A are:

$C_{11} = 0$ $C_{21} = -1$ $C_{31} = 1$

$C_{12} = -4$ $C_{22} = 3$ $C_{32} = -4$

$C_{13} = -3$ $C_{23} = 3$ $C_{33} = -4$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 0 & -4 & -3 \\ -1 & 3 & 3 \\ 1 & -4 & -4 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj } A$

So, $A^{-1} = \frac{1}{-1}.\begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

8 F. Question

Find the inverse of each of the following matrices.

$$\begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$$

Answer

$|A| = 0\begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} - 0\begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 1\begin{vmatrix} 3 & 4 \\ -2 & -4 \end{vmatrix}$

= 0 – 0 – 1(– 12 + 8)

= 4

Hence, A^{-1} exists

Cofactors of A are:

$C_{11} = -8$ $C_{21} = 4$ $C_{31} = 4$

$C_{12} = 11$ $C_{22} = -2$ $C_{32} = -3$

$C_{13} = -4$ $C_{23} = 0$ $C_{33} = 0$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 8 & 11 & -4 \\ 4 & -2 & 0 \\ 4 & -3 & 0 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj } A$

So, $A^{-1} = \frac{1}{4}.\begin{bmatrix} 8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 2 & 1 & 1 \\ \frac{11}{4} & \frac{-1}{2} & \frac{-3}{4} \\ -1 & 0 & 0 \end{bmatrix}$

8 G. Question

Find the inverse of each of the following matrices.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & \sin a & -\cos a \end{bmatrix}$$

Answer

$|A| = 1\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} - 0 + 0$

$= (-\cos^2\alpha - \sin^2\alpha)$

= - 1

Hence, A^{-1} exists

Cofactors of A are:

$C_{11} = -1$ $C_{21} = 0$ $C_{31} = 0$

$C_{12} = 0$ $C_{22} = -\cos\alpha$ $C_{32} = -\sin\alpha$

$C_{13} = 0$ $C_{23} = -\sin\alpha$ $C_{33} = \cos\alpha$

$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}^T$

So, $\text{adj } A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj } A$

So, $A^{-1} = \frac{1}{-1}.\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

9 A. Question

Find the inverse of each of the following matrices and verify that $A^{-1}A = I_3$.

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Answer

$|A| = 1\begin{vmatrix} 4 & 3 \\ 3 & 4 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix}$

= 1(16 – 9) – 3(4 – 3) + 3(3 – 4)

= 7 – 3 – 3

= 1

Hence, A^{-1} exists

Cofactors of A are:

$C_{11} = 7$ $C_{21} = -3$ $C_{31} = -3$

$C_{12} = -1$ $C_{22} = -1$ $C_{32} = 0$

$C_{13} = - 1$ $C_{23} = 0$ $C_{33} = 1$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{1}\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Also, $A^{-1}.A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 7-3-3 & 21-12-9 & 21-9-12 \\ -1+1+0 & -3+4+0 & -3+3+0 \\ -1+0+1 & -3+0+3 & -3+0+4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence, $A^{-1}.A = I$

9 B. Question

Find the inverse of each of the following matrices and verify that $A^{-1} A = I_3$.

$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

Answer

$|A| = 2\begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix} - 3\begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 4 \\ 3 & 7 \end{vmatrix}$

= 2(8 – 7) – 3(6 – 3) + 1(21 – 12)

= 2 – 9 + 9

= 2

Hence, A^{-1} exists

Cofactors of A are:

$C_{11} = 1$ $C_{21} = 1$ $C_{31} = - 1$

$C_{12} = - 3$ $C_{22} = 1$ $C_{32} = 1$

$C_{13} = 9$ $C_{23} = - 5$ $C_{33} = - 1$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

Also, $A^{-1}.A = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 2+3-3 & 3+4-7 & 1+1-2 \\ -6+3+3 & -9+4+7 & -3+1+2 \\ 18-15-3 & 27-20-7 & 9-5-2 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence, $A^{-1}.A = I$

10 A. Question

For the following pairs of matrices verify that $(AB)^{-1} = B^{-1}A^{-1}$:

$A=\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix}$

Answer

$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$, $|A| = 1$

Then, adj A = $\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix}$, $|B| = -10$

Then, adj B = $\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}$

$B^{-1} = -\frac{1}{10}\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}$

Also, $A.B = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 12+6 & 18+4 \\ 28+15 & 42+10 \end{bmatrix}$

$AB = \begin{bmatrix} 18 & 22 \\ 43 & 52 \end{bmatrix}$

$|AB| = 936 - 946 = -10$

$Adj(AB) = \begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{-10}\begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} = \begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$

Now $B^{-1}A^{-1} = \frac{1}{-10}\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} 10 + 42 & -4 - 18 \\ -15 - 28 & 6 + 12 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$

Hence, $(AB)^{-1} = B^{-1}A^{-1}$

10 B. Question

For the following pairs of matrices verify that $(AB)^{-1} = B^{-1}A^{-1}$:

$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \text{and } B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

Answer

$|A| = 1$

$\text{Adj } A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

$|B| = -1$

$B^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{-1}\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

Also, $AB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 11 & 14 \\ 29 & 37 \end{bmatrix}$

$|AB| = 407 - 406 = 1$

And, $\text{adj}(AB) = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

$(AB)^{-1} = \frac{\text{adj } AB}{|AB|}$

$= \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

Now, $B^{-1}A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$= \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

Hence, $(AB)^{-1} = B^{-1}A^{-1}$

11. Question

Let $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \text{and } B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$. Find $(AB)^{-1}$.

Answer

$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$

$|A| = 15 - 14 = 1$

$\text{adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$A^{-1} = \frac{\text{adj}}{|A|} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

$|B| = 54 - 56 = -2$ adj $B = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$

$B^{-1} = \frac{\text{adj}B}{|B|} = \frac{1}{-2}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$

Now, $(AB)^{-1} = B^{-1}A^{-1}$

$= \frac{1}{-2}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 45 + 49 & -18 - 21 \\ -40 - 42 & 16 + 18 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$

$(AB)^{-1} = \begin{bmatrix} -47 & \frac{39}{2} \\ 41 & -17 \end{bmatrix}$

12. Question

Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

Answer

$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$|A| = 14 - 12 = 2$ adj $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

To Show: $2A^{-1} = 9I - A$

L.H.S $2A^{-1} = 2.\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

R.H.S $9I - A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

Hence, $2A^{-1} = 9I - A$

13. Question

If $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, then show that $A - 3I = 2(I + 3A^{-1})$.

Answer

A = $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$

|A| = 4 – 10 = – 6 adj A = $\begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{-6}\begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$

To Show: $A - 3I = 2(I + 3A^{-1})$

LHS $A - 3I = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$

R.H.S $2(I + 3A^{-1}) = 2I + 6A^{-1} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 6\frac{1}{6}\begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 5 \\ 2 & -4 \end{bmatrix}$

Hence, $A - 3I = 2(I + 3A^{-1})$

14. Question

Find the inverse of the matrix $A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$ and show that $aA^{-1} = (a^2 + bc + 1)I - aA$.

Answer

$A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$

Now, $|A| = \frac{a+abc}{a} - bc = \frac{a+abc-abc}{a} = 1 \neq 0$

Hence, A^{-1} exists.

Cofactors of A are

$C_{11} = \frac{1+bc}{a}$ $C_{12} = -c$

$C_{21} = -b$ $C_{22} = a$

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} \frac{1+bc}{a} & -c \\ -b & a \end{bmatrix}^T$

$= \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}$.adj A

$A^{-1} = \frac{1}{1}.\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

To show. $aA^{-1} = (a^2 + bc + 1) I - aA$.

LHS $aA^{-1} = a\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

$= \begin{bmatrix} 1 + bc & -ab \\ -ac & a^2 \end{bmatrix}$

RHS $(a^2 + bc + 1) I - aA = \begin{bmatrix} a2 + bc + 1 & 0 \\ 0 & a2 + bc + 1 \end{bmatrix} - \begin{bmatrix} a^2 & ab \\ ac & 1 + bc \end{bmatrix}$

$= \begin{bmatrix} 1 + bc & -ab \\ -ac & a^2 \end{bmatrix}$

Hence, LHS = RHS

15. Question

Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Compute $(AB)^{-1}$.

Answer

$A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

Here , $(AB)^{-1} = B^{-1} A^{-1}$

$|A| = -5 + 4 = -1$

Cofactors of A are:

$C_{11} = -1$ $C_{21} = 8$ $C_{31} = -12$

$C_{12} = 0$ $C_{22} = 1$ $C_{32} = -2$

$C_{13} = 1$ $C_{23} = -10$ $C_{33} = 15$

adj A $= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} -1 & 0 & 1 \\ 8 & 1 & -10 \\ -12 & -2 & 15 \end{bmatrix}^T$

So, adj A $= \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{-1}\begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$

$(AB)^{-1} = B^{-1} A^{-1}$

$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$

$$= \begin{bmatrix} 1+0-3 & -8-3+30 & 12+6-45 \\ 1+0-3 & -8-4+30 & 12+8-45 \\ 1+0-4 & -8-3+40 & 12+6-60 \end{bmatrix}$$

Hence, $= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & 42 \end{bmatrix}$

16 A. Question

Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(\beta) \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$. Show that

$[F(\alpha)]^{-1} = F(-\alpha)$

Answer

$F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$|F(\alpha)| = \cos^2\alpha + \sin^2\alpha = 1$

Cofactors of A are:

$C_{11} = \cos\alpha$ $C_{21} = \sin\alpha$ $C_{31} = 0$

$C_{12} = -\sin\alpha$ $C_{22} = \cos\alpha$ $C_{32} = 0$

$C_{13} = 0$ $C_{23} = -10$ $C_{33} = 1$

adj $F(\alpha) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

So, adj $F(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ...... (i)

Now, $[F(\alpha)]^{-1} = \frac{1}{1} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

And, $F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & \sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ...... (ii)

$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, $[F(\alpha)]^{-1} = F(-\alpha)$

16 B. Question

Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(\beta) \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$. Show that

$[G(\beta)]^{-1} = G(-\beta)$

Answer

$$G(\beta)\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$|G(\beta)| = \cos^2\beta + \sin^2\beta = 1$

Cofactors of A are:

$C_{11} = \cos\beta$ $C_{21} = \sin\alpha$ $C_{31} = \sin\beta$

$C_{12} = 0$ $C_{22} = 1$ $C_{32} = 0$

$C_{13} = \sin\beta$ $C_{23} = 0$ $C_{33} = \cos\beta$

$$\text{Adj } G(\beta) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}^T$$

$$\text{So, adj } G(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \text{...... (i)}$$

$$\text{Now, } [G(\beta)]^{-1} = \frac{1}{1}\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$\text{And, } G(-\beta) = \begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) \\ 0 & 1 & 0 \\ \sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

Hence, $[G(\beta)]^{-1} = G(-\beta)$

16 C. Question

Let $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(\beta)\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$. Show that

$[F(\alpha)G(\beta)]^{-1} = G - (-\beta) F(-\alpha)$.

Answer

We have to show that

$[F(\alpha)G(\beta)]^{-1} = G(-\beta) F(-\alpha)$

We have already shown that

$[G(\beta)]^{-1} = G(-\beta)$

$[F(\alpha)]^{-1} = F(-\alpha)$

And LHS = $[F(\alpha)G(\beta)]^{-1}$

$= [G(\beta)]^{-1}[F(\alpha)]^{-1}$

$= G(-\beta)\,F(-\alpha)$

Hence = RHS

17. Question

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, verify that $A^2 - 4A + I = O$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Hence, find A^{-1}.

Answer

$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$

$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

$4A = 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now, $A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7-8+1 & 12-2+0 \\ 4-4+0 & 7-8+1 \end{bmatrix}$

Hence, $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Now, $A^2 - 4A + I = 0$

$A.A - 4A = -I$

Multiply by A^{-1} both sides

$A.A(A^{-1}) - 4A\,A^{-1} = -IA^{-1}$

$AI - 4I = -A^{-1}$

$A^{-1} = 4I - A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

18. Question

Show that $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation $A^2 + 4A - 42I = O$. Hence, find A^{-1}.

Answer

$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 64+10 & -40+20 \\ -16+8 & 10+16 \end{bmatrix}$

$= \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix}$

$4A = 4\begin{bmatrix}-8 & 5\\ 2 & 4\end{bmatrix} = \begin{bmatrix}-32 & 20\\ 8 & 16\end{bmatrix}$

$42I = 42\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}42 & 0\\ 0 & 42\end{bmatrix}$

Now,

$A^2 + 4A - 42I = \begin{bmatrix}74 & -20\\ -8 & 26\end{bmatrix} + \begin{bmatrix}-32 & 20\\ 8 & 16\end{bmatrix} - \begin{bmatrix}42 & 0\\ 0 & 42\end{bmatrix}$

$= \begin{bmatrix}74 - 74 & -20 + 20\\ -8 + 8 & 42 - 42\end{bmatrix}$

Hence, $= \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

Now, $A^2 + 4A - 42I = 0$

$= A^{-1}. A . A + 4 A^{-1}.A - 42 A^{-1}.I = 0$

$= IA + 4I - 42A^{-1} = 0$

$= 42A^{-1} = A + 4I$

$= A^{-1} = \frac{1}{42}[A + 4I]$

$= \frac{1}{42}\left[\begin{bmatrix}-8 & 5\\ 2 & 4\end{bmatrix} + \begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}\right]$

$A^{-1} = \frac{1}{42}\left[\begin{bmatrix}-4 & 5\\ 2 & 8\end{bmatrix}\right]$

19. Question

If $A = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence, find A^{-1}.

Answer

$A = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$A^2 = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix} = \begin{bmatrix}9 - 1 & 3 + 2\\ -3 - 2 & -1 + 4\end{bmatrix}$

$= \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$

Now, $A^2 - 5A + 7I = \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix} - 5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix} + 7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$= \begin{bmatrix}8 - 15 + 7 & 5 - 5 + 0\\ -5 + 5 + 0 & 3 - 10 + 7\end{bmatrix}$

$= \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

So, $A^2 - 5A + 7I = 0$

Multiply by A^{-1} both sides

$= A.A. A^{-1} - 5A. A^{-1} + 7I. A^{-1} = 0$

$= A - 5I + 7 A^{-1} = 0$

$= A^{-1} = \frac{1}{7}[5I - A]$

$= A^{-1} = \frac{1}{7}.\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix} - \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$

$= A^{-1} = \frac{1}{7}.\begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}$

20. Question

If $A = \begin{bmatrix}4 & 3\\2 & 5\end{bmatrix}$ find x and y such $A^2 - xA + yI = O$. Hence, evaluate A^{-1}.

Answer

$A = \begin{bmatrix}4 & 3\\2 & 5\end{bmatrix}$

$A^2 = \begin{bmatrix}4 & 3\\2 & 5\end{bmatrix}\begin{bmatrix}4 & 3\\2 & 5\end{bmatrix} = \begin{bmatrix}16 + 6 & 12 + 15\\8 + 10 & 6 + 25\end{bmatrix}$

$= \begin{bmatrix}22 & 27\\18 & 31\end{bmatrix}$

Now, $A^2 - xA + yI = \begin{bmatrix}22 & 27\\18 & 31\end{bmatrix} - X\begin{bmatrix}4 & 3\\2 & 5\end{bmatrix} + Y\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$= 22 - 4x + y = 0$ or $4x - y = 22$

$= 18 - 2x = 0$ or $X = 9$

$= Y = 14$

So, $A^2 - 5A + 7I = 0$

Multiply by A^{-1} both sides

$= A.A.\ A^{-1} - 9A.\ A^{-1} + 14I.\ A^{-1} = 0$

$= A - 9I + 14\ A^{-1} = 0$

$= A^{-1} = \frac{1}{14}[9I - A]$

$= A^{-1} = \frac{1}{14}.\begin{bmatrix}9 & 0\\0 & 9\end{bmatrix} - \begin{bmatrix}4 & 3\\2 & 5\end{bmatrix}$

$= A^{-1} = \frac{1}{7}.\begin{bmatrix}5 & -3\\-2 & 4\end{bmatrix}$

21. Question

If $A = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$, find the value of λ so that $A^2 = \lambda A - 2I$. Hence, find A^{-1}.

Answer

$A = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$

$A^2 = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} = \begin{bmatrix}9 - 8 & -6 + 4\\12 - 8 & -8 + 4\end{bmatrix}$

$= \begin{bmatrix}1 & -2\\4 & -4\end{bmatrix}$

Now, $A^2 = \lambda A - 2I$

$= \lambda A = A^2 + 2I$

$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \lambda \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 3\lambda & -2\lambda \\ 4\lambda & -2\lambda \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= 3\lambda = 3$ or $\lambda = 1$

So, $A^2 = A - 2I$

Multiply by A^{-1} both sides

$= A.A.\ A^{-1} = A.\ A^{-1} - 2I.\ A^{-1} = 0$

$= 2A^{-1} = I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{2}.\begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$

22. Question

Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $x^2 - 3A - 7 = 0$. Thus, find A^{-1}.

Answer

$A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 25 - 3 & 15 - 6 \\ -5 + 2 & -3 + 4 \end{bmatrix}$

$= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$

Now, $A^2 - 3A - 7 = 0$

$= \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 22 - 15 - 7 & 9 - 9 - 0 \\ -3 + 3 - 0 & 1 + 6 - 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

So, $A^2 - 3A - 7I = 0$

Multiply by A^{-1} both sides

$= A.A.\ A^{-1} - 3A.\ A^{-1} - 7I.\ A^{-1} = 0$

$= A - 3I - 7A^{-1} = 0$

$= 7A^{-1} = A - 3I$

$= A^{-1} = \frac{1}{7}.\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{7}.\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

23. Question

Show that $A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ satisfies the equation $x^2 - 12x + 1 = 0$. Thus, find A^{-1}

Answer

$A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$

We have $A^2 - 12A + I = 0$

$A^2 = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 36 + 35 & 30 + 30 \\ 42 + 42 & 35 + 36 \end{bmatrix}$

$= \begin{bmatrix} 71 & 60 \\ 84 & 71 \end{bmatrix}$

Now, $A^2 - 12A + 1 = 0$

$= \begin{bmatrix} 71 & 60 \\ 84 & 71 \end{bmatrix} - 12\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 71 - 72 + 1 & 60 - 60 + 0 \\ 84 - 82 + 0 & 71 - 72 + 1 \end{bmatrix}$

Hence, $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Also, $A^2 - 12A + 1 = 0$

$= A - 12I + A^{-1} = 0$

$= A^{-1} = 12I - A$

$= 12\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$

$= \begin{bmatrix} 12 - 6 & 0 - 5 \\ 0 - 7 & 12 - 6 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}$

24. Question

For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Show that $A^3 - 6A^2 + 5A + 11I_3 = O$.Hence, find A^{-1}.

Answer

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$A^3 = A^2.A$

$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 + 1 + 2 & 1 + 2 - 1 & 1 - 3 + 3 \\ 1 + 2 - 6 & 1 + 4 + 3 & 1 - 6 - 9 \\ 2 - 1 + 6 & 2 - 2 - 3 & 2 + 3 + 9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$

$A^2.A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$

Now, $A^3 - 6A^2 + 5A + 11I$

$\begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8-24 & 7-12 & 1-6 \\ -23+18 & 27-48 & -69+84 \\ 32-42 & -13+18 & 58-84 \end{bmatrix} + \begin{bmatrix} 5+11 & 5+0 & 5+0 \\ 5+0 & 10+11 & -15+0 \\ 10+0 & -5+0 & 15+11 \end{bmatrix}$

$= \begin{bmatrix} -16 & -5 & -5 \\ -5 & -21 & 15 \\ -10 & 5 & 26 \end{bmatrix} + \begin{bmatrix} 16 & 5 & 5 \\ 5 & 21 & -15 \\ 10 & -5 & 26 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Thus, $A^3 - 6A^2 + 5A + 11I$

Now, $(AAA)\,A^{-1}. - 6(A.A)\,A^{-1} + 5.A\,A^{-1} + 11I.A^{-1} = 0$

$AA(A^{-1}A) - 6A(A^{-1}A) + 5(A^{-1}A) = -1(A^{-1}I)$

$A^2 - 6A + 5I = 11\,A^{-1}$

$= A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I)$

Now,

$A^2 - 6A + 5I$

$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 9 & 2 & 1 \\ -3 & 13 & -14 \\ 7 & -3 & 19 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix}$

$= \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$

Hence, $A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$

25. Question

Show that the matrix, $A = \begin{bmatrix} 1 & 0 & -1 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ satisfies the equation, $A^3 - A^2 - 3A - I_3 = O$. Hence, find A^{-1}.

Answer

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$A^3 = A^2.A$

$$A^2 = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0-6 & 0+0-8 & -2+0-2 \\ -2+2+6 & 0+1+8 & 4-2+2 \\ 3-8+3 & 0-4+4 & -6+8+1 \end{bmatrix} = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}$$

$$A^2.A = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5+16-12 & 0-8+16 & 10-16-4 \\ 6-18+12 & 0-9+16 & -12+18+4 \\ -2-0+9 & 0-0-12 & 4+0+3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix}$$

Now, $A^3 - A^2 - 3A - I$

$$\begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+5 & -8+8 & -10+4 \\ 0-6 & 7-9 & 10-4 \\ 7+2 & 12-0 & 7-3 \end{bmatrix} + \begin{bmatrix} -3-1 & -0-0 & 6-0 \\ 6-0 & +3-1 & -6-0 \\ -9-0 & -12+0 & -3-1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & -6 \\ -6 & -2 & 6 \\ 9 & 12 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 6 \\ 6 & 2 & -6 \\ -9 & -12 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus, $A^3 - A^2 - 3A - I$

Now, $(AAA)\,A^{-1}. - (A.A)\,A^{-1} - 3.A\,A^{-1} - I.A^{-1} = 0$

$AA(A^{-1}A) - A(A^{-1}A) - 3(A^{-1}A) = -1(A^{-1}I)$

$A^2 - A - 3A - I = 0$

$= A^{-1} = (A^2 - A - 3I)$

Now,

$$(A^2 - A - 3I) = \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5-1-3 & -8-0-0 & -4+2-0 \\ 6+2-0 & 7+1-3 & 4-2-0 \\ -2-3-0 & 0-4-0 & 3-1-3 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

26. Question

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence fid A^{-1}.

Answer

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$A^3 = A^2.A$

$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4+1+1 & -2-2-2 & 2+1+2 \\ -2-2-2 & 1+2+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$A^2.A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now, $A^3 - 6A^2 + 9A - 4I$

$$\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36+18-4 & -21+30-9-0 & 21-30+9-0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9-0 \\ 21-30+9-0 & -21+30-9-0 & 22-36+18-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus, $A^3 - 6A^2 + 9A - 4I$

Now, (AAA) A^{-1}. – 6(A.A) A^{-1} + 9.A A^{-1} – 4I.A^{-1} = 0

$A^2 - 6A + 9I = 4A^{-1}$

$= A^{-1} = \frac{1}{4}(A^2 - 6A + 9I)$

Now,

$$(A^2 - 6A + 9I) = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 6 - 12 + 9 & -5 + 6 + 0 & 5 - 6 + 0 \\ -5 + 6 + 0 & 6 - 12 + 9 & -5 + 6 + 0 \\ 5 - 6 + 0 & -5 - 6 + 0 & 6 - 12 + 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

27. Question

If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.

Answer

$A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ $A^T = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$

$|A| = \frac{1}{9}[-8(16 + 56) - 1(16 - 7) + 4(-32 - 4)]$

= – 81

Cofactors of A are:

$C_{11} = 72$ $C_{21} = -36$ $C_{31} = -9$

$C_{12} = -9$ $C_{22} = -36$ $C_{32} = 72$

$C_{13} = -36$ $C_{23} = -63$ $C_{33} = -36$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$$= \begin{bmatrix} 72 & -9 & -36 \\ -36 & -36 & -63 \\ -9 & 72 & -36 \end{bmatrix}^T$$

So, adj A = $\begin{bmatrix} 72 & -36 & -9 \\ -9 & -36 & 72 \\ -36 & -63 & -36 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{-81}\begin{bmatrix} 72 & -36 & -9 \\ -9 & -36 & 72 \\ -36 & -63 & -36 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix} = A^T$

28. Question

If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, show that $A^{-1} = A^3$.

Answer

$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$|A| = 3 + 6 - 8 = 1$

Cofactors of A are:

$C_{11} = 1$ $C_{21} = -1$ $C_{31} = 0$

$C_{12} = -2$ $C_{22} = 3$ $C_{32} = -4$

$C_{13} = -2$ $C_{23} = 3$ $C_{33} = -3$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

Also, $A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9-6+0 & -9+9-4 & 12-12+4 \\ 6-6+0 & -6+9-4 & 8-12+4 \\ 0-2+0 & 0+3-1 & 0-4+1 \end{bmatrix}$

$= \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$

$A^3 = A^2.A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

Hence, $A^{-1} = A^3$

29. Question

If $A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, Show that $A^2 = A^{-1}$.

Answer

$|A| = -1\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} + 0$

$|A| = -1(0 - 1) - 2(0) + 0$

$= 1 - 0 + 0$

$|A| = 1$

$A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$A^2 = A.A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1-2+0 & -2+2+0 & 0+2+0 \\ 1-1+1 & -2+1+1 & -1+1-0 \\ 0-1+0 & 0+1-0 & 0+1-0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Cofactors of A are:

$C_{11} = -1$ $C_{21} = 0$ $C_{31} = 2$

$C_{12} = 0$ $C_{22} = 0$ $C_{32} = 1$

$C_{13} = -1$ $C_{23} = 1$ $C_{33} = 1$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{1}\begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} = A^2$

30. Question

Solve the matrix equation $\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$, where X is a 2x2 matrix.

Answer

Let A = $\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

So, AX = B

Or, X = $A^{-1}B$

|A| = 1

Cofactors of A are

C_{11} = 1 C_{12} = – 1

C_{21} = – 4 C_{22} = 5

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

(adj A) = $\begin{bmatrix} 1 & -1 \\ -4 & 5 \end{bmatrix}^T$

= $\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$

So, X = $\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

Hence, X = $\begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$

31. Question

Find the matrix X satisfying the matrix equation: $X\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

Answer

Let A = $\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ B = $\begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

So, AX = B

Or, X = $A^{-1}B$

|A| = – 7

Cofactors of A are

C_{11} = – 2 C_{12} = 1

C_{21} = – 3 C_{22} = 5

Since, adj A = $\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$(\text{adj } A) = \begin{bmatrix} -2 & 1 \\ -3 & 5 \end{bmatrix}^T$

$= \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

So, $X = \frac{1}{7} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix} \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

Hence, $X = \frac{1}{7} \begin{bmatrix} 28 + 21 & 14 + 21 \\ -14 - 35 & -7 - 35 \end{bmatrix}$

$X = \begin{bmatrix} 7 & 5 \\ -7 & -6 \end{bmatrix}$

32. Question

Find the matrix X for which: $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$.

Answer

Let $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

Then The given equations becomes as

AXB = C

$= X = A^{-1}CB^{-1}$

$|A| = 35 - 14 = 21$

$|B| = -1 + 2 = 1$

$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{21} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$B^{-1} = \frac{\text{adj}(B)}{|B|} = \frac{1}{1} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$= X = A^{-1}CB^{-1} = \frac{1}{21} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$= \frac{1}{21} \begin{bmatrix} 10 + 0 & -5 - 8 \\ -14 + 0 & 7 + 12 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$= \frac{1}{21} \begin{bmatrix} 10 & -13 \\ -14 & 19 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$= \frac{1}{21} \begin{bmatrix} 10 - 26 & -10 + 13 \\ -14 + 38 & 14 - 19 \end{bmatrix}$

Hence, $X = \frac{1}{21} \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$

33. Question

Find the matrix X satisfying the equation: $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} X \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Answer

Let A = $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ B = $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ C = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then The given equations becomes as

AXB = I

= X = $A^{-1}B^{-1}$

|A| = 6 – 5 = 1

|B| = 10 – 9 = 1

$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$B^{-1} = \frac{\text{adj}(B)}{|B|} = \frac{1}{1}\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$

= X = $A^{-1}B^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 6+3 & -9-5 \\ -10-6 & 15+10 \end{bmatrix}$

$= \begin{bmatrix} 9 & -14 \\ -16 & 25 \end{bmatrix}$

Hence, X = $\begin{bmatrix} 9 & -14 \\ -16 & 25 \end{bmatrix}$

34. Question

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, find A^{-1} and prove that A^2 – 4A–5I = O.

Answer

A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

A^2 – 4A + 5I = 0

$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Also, $A^2 - 4A - 5I = 0$

Now, $6(A.A)\,A^{-1} - 4.A\,A^{-1} - 5I.A^{-1} = 0$

$= A - 4I - 5A^{-1} = 0$

$= A^{-1} = \frac{1}{5}(A - 4I)$

$= \frac{1}{5}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 1-4 & 2-0 & 2-0 \\ 2-0 & 1-4 & 2-0 \\ 2-0 & 2-0 & 1-4 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

35. Question

If A is a square matrix of order n, prove that $|A \text{ adj } A| = |A|^n$.

Answer

$|A \text{ adj } A| = |A|^n$

LHS $|A \text{ adj } A|$

$|A|.|\text{adj } A|$

$|A|.|A|^{n-1}$

$|A|^{n-1+1}$

$|A|^n$ = RHS

Hence, LHS = RHS

36. Question

If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Answer

$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$|B| = 1(3 - 0) - 2(-1 - 0) - 2(2 - 0)$

$= 3 + 2 - 4$

$|B| = 1$

Now, $B^{-1} = \frac{1}{|B|}\text{adj } B$

Cofactors of B are:

$C_{11} = -3$ $C_{21} = 2$ $C_{31} = 6$

$C_{12} = 1$ $C_{22} = 1$ $C_{32} = 2$

$C_{13} = 2$ $C_{23} = 2$ $C_{33} = 5$

adj B = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T$

So, adj B = $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Now, $B^{-1} = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

$(AB)^{-1} = B^{-1} A^{-1}$

$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$

Hence, = $\begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

37. Question

If $A = \begin{bmatrix} 1-2 & 3 \\ 0-1 & 4 \\ -2 \;\; 2 & 1 \end{bmatrix}$, find $(A^T)^{-1}$.

Answer

$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$

Let $B = A^T = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{vmatrix}$

$= (-1-8) - 0 - 2(-8+3) = -9 + 10 = 1$

Cofactors of B are:

$C_{11} = -9$ $C_{21} = 8$ $C_{31} = -5$

$C_{12} = -8$ $C_{22} = 7$ $C_{32} = -4$

$C_{13} = -2$ $C_{23} = 2$ $C_{33} = -1$

adj B = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & 4 \\ -2 & 2 & -1 \end{bmatrix}^T$

So, adj B = $\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & 4 & -1 \end{bmatrix}$

Now, $B^{-1} = \frac{1}{1}\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & 4 & -1 \end{bmatrix}$

Hence, $(A^T)^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & 4 & -1 \end{bmatrix}$

38. Question

Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that A(adj A) = |A| I_3.

Answer

$A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$

= – 1(1 – 4) + 2(2 + 4) – 2(– 4 – 2)

= 3 + 12 + 12

|A| = 27

Cofactors of A

C_{11} = – 3 C_{21} = – 6 C_{31} = 6

C_{12} = – 6 C_{22} = 3 C_{32} = – 6

C_{13} = – 6 C_{23} = – 6 C_{33} = 3

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$= \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & 6 & 3 \end{bmatrix}$

A(adj A) = $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & 6 & 3 \end{bmatrix}$

$= \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$

A(adj A) = $27\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

Hence, A(adj A) = |A|I

39. Question

If $A = \begin{bmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{bmatrix}$, find A^{-1} and show that $A^{-1} = 1/2(A^2 - 3I)$.

Answer

$A = \begin{bmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{bmatrix}$ |A| = 0 – 1(0 – 1) + 1(1 – 0) = 0 + 1 + 1 = 2

Cofactors of A are:

$C_{11} = -1$ $C_{21} = 1$ $C_{31} = 1$

$C_{12} = 1$ $C_{22} = -1$ $C_{32} = 1$

$C_{13} = 1$ $C_{23} = 1$ $C_{33} = -1$

adj A = $\begin{bmatrix}C_{11} & C_{12} & C_{13}\\C_{21} & C_{22} & C_{23}\\C_{31} & C_{32} & C_{33}\end{bmatrix}^T$

$= \begin{bmatrix}-1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}^T$

So, adj A = $\begin{bmatrix}-1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

Now, $A^{-1} = \frac{1}{2}\begin{bmatrix}-1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

$A^2 - 3I = \begin{bmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{bmatrix} - 3\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

$= \begin{bmatrix}0+1+1 & 0+0+1 & 0+1+0\\0++0+1 & 1+0+1 & 1+0+0\\0+1+0 & 1+0+0 & 1+1+0\end{bmatrix} - 3\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

$= \begin{bmatrix}2 & 1 & 1\\1 & 2 & 1\\1 & 1 & 2\end{bmatrix} - \begin{bmatrix}3 & 0 & 0\\0 & 3 & 0\\0 & 0 & 3\end{bmatrix}$

$= \begin{bmatrix}-1 & 1 & 1\\1 & -1 & 1\\1 & 1 & -1\end{bmatrix}$

Hence, $A^{-1} = \frac{1}{2}(A^2 - 3I)$

Exercise 7.2

1. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 7 & 1 \\ 4 & -3 \end{bmatrix}$$

Answer

Given:- 2 x 2 square matrix

Tip:- Algorithm to find Inverse of a square matrix of 'n' order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_2A

Where I_2 is 2 x 2 elementary matrix

$$\Rightarrow \begin{bmatrix} 7 & 1 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathrm{A}$$

Applying $r_1 \to \frac{1}{7} r_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{7} \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{bmatrix} \mathrm{A}$$

Applying $r_2 \to r_2 - 4r_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{7} \\ 0 & \frac{-25}{7} \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ -\frac{4}{7} & 1 \end{bmatrix} \mathrm{A}$$

Applying $r_2 \to -\frac{7}{25} r_2$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{7} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix} \mathrm{A}$$

Applying $r_1 \to r_1 - \frac{1}{7} r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{21}{175} & \frac{1}{25} \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix} \mathrm{A}$$

Hence, it is of the form

I = BA

So, as we know that

I = A^{-1}A

Therefore

$A^{-1} = B$

$\Rightarrow A^{-1} = \begin{bmatrix} \frac{21}{175} & \frac{1}{25} \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix}$ inverse of A

2. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

Answer

Given:- 2 x 2 square matrix

Tip:- Algorithm to find Inverse of a square matrix of 'n' order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write $A = I_nA$

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

$I_n = BA$

(iv) Write $A^{-1} = B$

Now,

We have,

$A = I_2A$

Where I_2 is 2 x 2 elementary matrix

$\Rightarrow \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_1 \to \frac{1}{5} r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{2}{5} \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_2 \to r_2 - 2r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{2}{5} \\ 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ -\frac{2}{5} & 1 \end{bmatrix} A$

Applying $r_2 \to 5r_2$

$\Rightarrow \begin{bmatrix} 1 & \frac{2}{5} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ -2 & 5 \end{bmatrix} A$

$\Rightarrow$

Applying $r_1 \to r_1 - \frac{2}{5} r_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} A$

Hence, it is of the form

I = BA

So, as we know that

I = A^{-1}A

Therefore

A^{-1} = B

⇒ $A^{-1} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ inverse of A

3. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Answer

Given:- 2 x 2 square matrix

Tip:- Algorithm to find Inverse of a square matrix of 'n' order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_2A

Where I_2 is 2 x 2 elementary matrix

⇒ $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathrm{A}$

Applying $r_2 \to r_2 - 2r_1$

⇒ $\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \mathrm{A}$

Applying $r_2 \to -\frac{1}{5}r_2$

⇒ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} \mathrm{A}$

Applying $r_1 \to r_1 - 2r_2$

⇒ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} \mathrm{A}$

Hence, it is of the form

I = BA

So, as we know that

I = $A^{-1}A$

Therefore

A^{-1} = B

$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$ inverse of A

4. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Answer

Given:- 2 x 2 square matrix

Tip:- Algorithm to find Inverse of a square matrix of 'n' order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_2A

Where I_2 is 2 x 2 elementary matrix

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathrm{A}$

Applying $r_1 \to \frac{1}{2} r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} \mathrm{A}$

Applying $r_2 \to r_2 - r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} \mathrm{A}$

Applying $r_2 \to 2r_2$

$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & 2 \end{bmatrix} \mathrm{A}$

Applying $r_1 \to r_1 - \frac{5}{2} r_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$

Hence, it is of the form

I = BA

So, as we know that

$I = A^{-1}A$

Therefore

$A^{-1} = B$

$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ inverse of A

5. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Answer

Given:- 2 x 2 square matrix

Tip:- Algorithm to find Inverse of a square matrix of ‘n’ order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write $A = I_nA$

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

$I_n = BA$

(iv) Write $A^{-1} = B$

Now,

We have,

$A = I_2A$

Where I_2 is 2 x 2 elementary matrix

$\Rightarrow \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_1 \rightarrow \frac{1}{3}r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{10}{3} \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $r_2 \rightarrow r_2 - 2r_1$

$\Rightarrow \begin{bmatrix} 1 & \frac{10}{3} \\ 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ -\frac{2}{3} & 1 \end{bmatrix} A$

Applying $r_2 \rightarrow 3r_2$

$\Rightarrow \begin{bmatrix} 1 & \frac{10}{3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ -2 & 3 \end{bmatrix} A$

Applying $r_1 \rightarrow r_1 - \frac{10}{3} r_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$

Hence, it is of the form

I = BA

So, as we know that

I = A^{-1}A

Therefore

A^{-1} = B

$\Rightarrow A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$ inverse of A

6. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of 'n' order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_3A

Where I_3 is 3 x 3 elementary matrix

$\Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $r_1 \leftrightarrow r_2$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $r_3 \rightarrow r_3 - 3r_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

Applying $r_1 \to r_1 - 2r_2$ and $r_3 \to r_3 + 5r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$

Applying $r_3 \to \frac{1}{2} r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A$$

Applying $r_1 \to r_1 + r_3$ and $r_2 \to r_2 - 2r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & 1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A$$

Hence, it is of the form

I = BA

So, as we know that

$I = A^{-1}A$

Therefore

$A^{-1} = B$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & 1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} \text{inverse of A}$$

7. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of 'n' order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write $A = I_n A$

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

$I_n = BA$

(iv) Write $A^{-1} = B$

Now,

We have,

A = I_3A

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \rightarrow \frac{1}{2} r_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \rightarrow r_2 - 5r_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_3 \rightarrow r_3 - r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{1}{2} & -1 & 1 \end{bmatrix} A$$

Applying $r_3 \rightarrow 2r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $r_1 \rightarrow r_1 + \frac{1}{2} r_3$ and $r_2 \rightarrow r_2 - \frac{5}{2} r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

Hence, it is of the form

I = BA

So, as we know that

I = A^{-1}A

Therefore

A^{-1} = B

$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ inverse of A

8. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of ‘n’ order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_3A

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to \frac{1}{2} r_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to r_2 - 2r_1 \; and \; r_3 \to r_3 - 3r_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & \frac{5}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to r_1 - \frac{3}{2} r_2$ and $r_3 \to r_3 - \frac{5}{2} r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & -\frac{3}{2} & 0 \\ -1 & 1 & 0 \\ 1 & -\frac{5}{2} & 1 \end{bmatrix} A$$

Applying $r_3 \to 2r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -\frac{3}{2} & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

Applying $r_1 \to r_1 - \frac{1}{2} r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

Hence , it is of the form

I = BA

So, as we know that

$I = A^{-1}A$

Therefore

$A^{-1} = B$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$ inverse of A

9. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of 'n' order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write $A = I_nA$

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

$I_n = BA$

(iv) Write $A^{-1} = B$

Now,

We have,

$A = I_3A$

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \rightarrow \frac{1}{3}r_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \rightarrow r_2 - 2r_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 0 & -1 & \frac{4}{3} \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to -r_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 0 & 1 & -\frac{4}{3} \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{2}{3} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to r_1 + r_2$ and $r_3 \to r_3 + r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{4}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ \frac{2}{3} & -1 & 1 \end{bmatrix} A$$

Applying $r_3 \to -3r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{4}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ -2 & 3 & -3 \end{bmatrix} A$$

Applying $r_2 \to r_2 + \frac{4}{3}r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} A$$

Hence , it is of the form

I = BA

So, as we know that

I = $A^{-1}A$

Therefore

A^{-1} = B

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ inverse of A

10. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of ‘n’ order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

$I_n = BA$

(iv) Write $A^{-1} = B$

Now,

We have,

$A = I_3A$

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to r_2 - 2r_1$ and $r_3 \to r_3 - r_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to -r_2$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to r_1 - 2r_2$ and $r_3 \to r_3 + 3r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$

Applying $r_3 \to \frac{1}{6}r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ \frac{5}{6} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix} A$$

Applying $r_1 \to r_1 + 2r_3$ and $r_2 \to r_2 - r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & -\frac{1}{2} & -\frac{1}{6} \\ \frac{5}{6} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix} A$$

Hence , it is of the form

I = BA

So, as we know that

$I = A^{-1}A$

Therefore

$A^{-1} = B$

$$\Rightarrow A^{-1} = \begin{bmatrix} -\frac{4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & -\frac{1}{2} & -\frac{1}{6} \\ \frac{5}{6} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix} \text{inverse of A}$$

11. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of 'n' order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_3A

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to \frac{1}{2} r_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{2} & \frac{3}{2} \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to r_2 - r_1 \; and \; r_3 \to r_3 - 3r_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{2} & \frac{3}{2} \\ 0 & \frac{5}{2} & \frac{5}{2} \\ 0 & \frac{5}{2} & -\frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to \frac{2}{5} r_2$

$$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{2} & \frac{3}{2} \\ 0 & 1 & 1 \\ 0 & \frac{5}{2} & -\frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to r_1 + \frac{1}{2}r_2$ and $r_3 \to r_3 - \frac{5}{2}r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ -1 & -1 & 1 \end{bmatrix} A$$

Applying $r_3 \to -\frac{1}{6}r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \end{bmatrix} A$$

Applying $r_2 \to r_2 - r_3\ and\ r_1 \to r_1 - 2r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{15} & \frac{-2}{15} & \frac{-1}{3} \\ -\frac{11}{30} & \frac{7}{30} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \end{bmatrix} A$$

Hence , it is of the form

I = BA

So, as we know that

I = A^{-1}A

Therefore

A^{-1} = B

$$A^{-1} = \begin{bmatrix} \frac{1}{15} & \frac{-2}{15} & \frac{-1}{3} \\ -\frac{11}{30} & \frac{7}{30} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \end{bmatrix} \text{inverse of A}$$

12. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of ‘n’ order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_3A

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to r_2 - 3r_1 \; and \; r_3 \to r_3 - 2r_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to \frac{-1}{2} r_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & -\frac{1}{2} & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to r_1 - r_2$ and $r_3 \to r_3 - r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & -\frac{11}{2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & -\frac{1}{2} & 0 \\ -\frac{7}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} A$$

Applying $r_3 \to -\frac{2}{11} r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & -\frac{1}{2} & 0 \\ \frac{7}{11} & -\frac{1}{11} & -\frac{2}{11} \end{bmatrix} A$$

Applying $r_1 \to r_1 + \frac{1}{2} r_3$ and $r_2 \to r_2 - \frac{5}{2} r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{11} & \frac{5}{11} & -\frac{1}{11} \\ -\frac{1}{11} & -\frac{3}{11} & \frac{5}{11} \\ \frac{7}{11} & -\frac{1}{11} & -\frac{2}{11} \end{bmatrix} A$$

Hence , it is of the form

I = BA

So, as we know that

I = A^{-1}A

Therefore

A^{-1} = B

$$\Rightarrow A^{-1} = \begin{bmatrix} -\frac{2}{11} & \frac{5}{11} & -\frac{1}{11} \\ -\frac{1}{11} & -\frac{3}{11} & \frac{5}{11} \\ \frac{7}{11} & -\frac{1}{11} & -\frac{2}{11} \end{bmatrix} \text{inverse of A}$$

13. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of 'n' order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_3A

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \rightarrow \frac{1}{2} r_1$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & 2 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \rightarrow r_2 - 4r_1 \; and \; r_3 \rightarrow r_3 - 3r_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{2} & 2 \\ 0 & 2 & -6 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -2 & 1 & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \rightarrow \frac{1}{2} r_2$

$$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{2} & 2 \\ 0 & 1 & -3 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & \frac{1}{2} & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \rightarrow r_1 + \frac{1}{2} r_2$ and $r_3 \rightarrow r_3 - \frac{5}{2} r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -3 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ -1 & \frac{1}{2} & 0 \\ -2 & \frac{1}{4} & 1 \end{bmatrix} A$$

Applying $r_3 \to -2r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ -1 & \frac{1}{2} & 0 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix} AA$$

Applying $r_1 \to r_1 - \frac{1}{2}r_3\ and\ r_2 \to r_2 + 3r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix} A$$

Hence , it is of the form

I = BA

So, as we know that

I = A^{-1}A

Therefore

A^{-1} = B

$$\Rightarrow A^{-1} = \begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix}$$ inverse of A

14. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of ‘n’ order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_3A

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to \frac{1}{3} r_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to r_2 - 2r_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 3 & \frac{2}{3} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to \frac{1}{3} r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_3 \to r_3 - 4r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ \frac{8}{9} & -\frac{4}{3} & 1 \end{bmatrix} A$$

Applying $r_3 \to 9r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ 8 & -12 & 9 \end{bmatrix} A$$

Applying $r_1 \to r_1 + \frac{1}{3} r_3 \; and \; r_2 \to r_2 - \frac{2}{9} r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A$$

Hence , it is of the form

I = BA

So, as we know that

I = A^{-1}A

Therefore

A^{-1} = B

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} \text{ inverse of A}$$

15. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of 'n' order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_3A

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{A}$$

Applying $r_2 \to r_2 + 3r_1 \; and \; r_3 \to r_3 - 2r_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -5 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \mathrm{A}$$

Applying $r_2 \to \frac{-1}{2} r_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & -\frac{1}{2} & 0 \\ -2 & 0 & 1 \end{bmatrix} \mathrm{A}$$

Applying $r_1 \to r_1 - 3r_2$ and $r_3 \to r_3 + 5r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & -\frac{5}{9} \\ 0 & 0 & \frac{11}{9} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{1}{3} & \frac{5}{9} & 1 \end{bmatrix} \mathrm{A}$$

Applying $r_1 \to \frac{9}{11} r_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & -\frac{5}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix} \mathrm{A}$$

Applying $r_1 \to r_1 + \frac{1}{3} r_3$ and $r_2 \to r_2 + \frac{5}{9} r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{11} & -\frac{2}{11} & \frac{3}{11} \\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11} \\ -\frac{3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix} A$$

Hence , it is of the form

I = BA

So, as we know that

I = A^{-1}A

Therefore

A^{-1} = B

$$\Rightarrow A^{-1} = \begin{bmatrix} -\frac{1}{11} & -\frac{2}{11} & \frac{3}{11} \\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11} \\ -\frac{3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix}$$ inverse of A

16. Question

Find the inverse of each of the following matrices by using elementary row transformations:

$$\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Answer

Given:- 3 x 3 square matrix

Tip:- Algorithm to find Inverse of a square matrix of ‘n’ order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write A = I_nA

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre-factor I_n on the RHS till we obtain the result

I_n = BA

(iv) Write A^{-1} = B

Now,

We have,

A = I_3A

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \rightarrow -1r_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \rightarrow r_2 - r_1 \; and \; r_3 \rightarrow r_3 - 3r_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to \frac{1}{3} r_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & \frac{5}{3} \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to r_1 + r_2$ and $r_3 \to r_3 - 4r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{5}{3} & -\frac{4}{3} & 1 \end{bmatrix} A$$

Applying $r_3 \to 3r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -4 & 3 \end{bmatrix} A$$

Applying $r_1 \to r_1 + \frac{1}{3} r_3 \; and \; r_2 \to r_2 - \frac{5}{3} r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

Hence , it is of the form

I = BA

So, as we know that

I = $A^{-1}A$

Therefore

A^{-1} = B

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$ inverse of A