

Ex 27.1

Direction Cosines and Direction Ratios Ex 27.1 Q1

Let l, m and n be the direction cosines of a line.

$$l = \cos 90^\circ = 0$$

$$m = \cos 60^\circ = \frac{1}{2}$$

$$n = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$\therefore$ The direction cosines of the line are $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$.

Direction Cosines and Direction Ratios Ex 27.1 Q2

Let the direction cosines of the line be l, m, n .

Here,

$a = 2, b = -1, c = -2$ are the direction ratios of the line.

$$l = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}}, n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$l = \pm \frac{2}{\sqrt{2^2 + (-1)^2 + (-2)^2}}, m = \pm \frac{-1}{\sqrt{2^2 + (-1)^2 + (-2)^2}}, n = \pm \frac{-2}{\sqrt{2^2 + (-1)^2 + (-2)^2}}$$

$$l = \pm \frac{2}{\sqrt{9}}, m = \pm \frac{-1}{\sqrt{9}}, n = \pm \frac{-2}{\sqrt{9}}$$

$$l = \pm \frac{2}{3}, m = \pm \frac{-1}{3}, n = \pm \frac{-2}{3}$$

$\therefore$ The direction ratios of the line are $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$.

Direction Cosines and Direction Ratios Ex 27.1 Q3

The direction ratios of the line joining $(-2, 4, -5)$ and $(1, 2, 3)$ are,

$$(1 + 2, 2 - 4, 3 + 5) = (3, -2, 8)$$

Here, $a = 3, b = -2, c = 8$

Direction cosines are

$$\frac{3}{\sqrt{3^2 + (-2)^2 + 8^2}}, \frac{-2}{\sqrt{3^2 + (-2)^2 + 8^2}}, \frac{8}{\sqrt{3^2 + (-2)^2 + 8^2}}$$
$$= \frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$$

Direction Cosines and Direction Ratios Ex 27.1 Q4

Here $A(2, 3, -4)$, $B(1, -2, 3)$ and $C(3, 8, -11)$.

Direction ratios of $AB = (1 - 2, -2 - 3, 3 + 4) = (-1, -5, 7)$

Direction ratios of $BC = (3 - 1, 8 + 2, -11 - 3) = (2, 10, -14)$

Here, the respective direction cosines of AB and AC ,

$$\frac{-1}{2} = \frac{-5}{10} = \frac{7}{-14} \text{ are proportional.}$$

Also, B is the common point between the two lines,

$\therefore$ The points $A(2, 3, -4)$, $B(1, -2, 3)$ and $C(3, 8, -11)$ are collinear.

Direction Cosines and Direction Ratios Ex 27.1 Q5

A(3, 5, -4), B(-1, 1, 2) and C(-5, -5, -2)

The direction ratios of the side AB = (-1 - 3, 1 - 5, 2 + 4)

= (-4, -4, 6)

Direction cosines of AB will be

$$\begin{aligned} & \frac{-4}{\sqrt{(-4)^2 + (-4)^2 + 6^2}}, \frac{-4}{\sqrt{(-4)^2 + (-4)^2 + 6^2}}, \frac{6}{\sqrt{(-4)^2 + (-4)^2 + 6^2}} \\ &= \frac{-4}{\sqrt{68}}, \frac{-4}{\sqrt{68}}, \frac{6}{\sqrt{68}} \\ &= \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}} \end{aligned}$$

The direction ratios of the side BC = (-5 + 1, -5 - 1, -2 - 2)

= (-4, -6, -4)

Direction cosines of BC will be

$$\begin{aligned} & \frac{-4}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}}, \frac{-6}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}}, \frac{-4}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}} \\ &= \frac{-4}{\sqrt{68}}, \frac{-6}{\sqrt{68}}, \frac{-4}{\sqrt{68}} \\ &= \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}} \end{aligned}$$

The direction ratios of the side AC = (-5 - 3, -5 - 5, -2 + 4)

= (-8, -10, 2)

Direction cosines of AC will be

$$\begin{aligned} & \frac{-8}{\sqrt{(-8)^2 + (-10)^2 + 2^2}}, \frac{-10}{\sqrt{(-8)^2 + (-10)^2 + 2^2}}, \frac{2}{\sqrt{(-8)^2 + (-10)^2 + 2^2}} \\ &= \frac{-8}{\sqrt{168}}, \frac{-10}{\sqrt{168}}, \frac{2}{\sqrt{168}} \\ &= \frac{-4}{\sqrt{42}}, \frac{-5}{\sqrt{42}}, \frac{1}{\sqrt{42}} \end{aligned}$$

Direction Cosines and Direction Ratios Ex 27.1 Q6

Let, θ be the angle between the vectors with direction ratios a, b, c and a_2, b_2, c_2 then.

$$\begin{aligned} \cos \theta &= \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \\ &= \frac{(1)(4) + (-2)(3) + (1)(2)}{\sqrt{(1)^2 + (-2)^2 + (1)^2} \sqrt{(4)^2 + (3)^2 + (2)^2}} \\ &= \frac{4 - 6 + 2}{\sqrt{1 + 4 + 1} \sqrt{16 + 9 + 4}} \\ &= \frac{6 - 6}{\sqrt{6} \sqrt{29}} \\ &= \frac{0}{\sqrt{174}} \end{aligned}$$

$$\cos \theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

Direction Cosines and Direction Ratios Ex 27.1 Q7

Here, given that the direction cosines of the vectors are proportional to 2, 3, -6 and 3, -4, 5.

Therefore, 2, 3, -6 and 3, -4, 5 are the direction ratios of two vectors.

Let, θ be the angle between two vectors having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is given by

$$\begin{aligned}\cos \theta &= \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \\ &= \frac{(2)(3) + (3)(-4) + (-6)(5)}{\sqrt{(2)^2 + (3)^2 + (-6)^2} \sqrt{(3)^2 + (-4)^2 + (5)^2}}\end{aligned}$$

$$\begin{aligned}\cos \theta &= \frac{6 - 12 - 30}{\sqrt{4 + 9 + 36} \sqrt{9 + 16 + 25}} \\ &= \frac{6 - 42}{\sqrt{49} \sqrt{50}} \\ &= \frac{-36 \times \sqrt{2}}{7 \times 5 \times \sqrt{2} \times \sqrt{2}} \quad (\text{Rationalizing the denominator}) \\ &= \frac{-36\sqrt{2}}{70} \\ \cos \theta &= \frac{-18\sqrt{2}}{35}\end{aligned}$$

$$\theta = \cos^{-1} \left(\frac{-18\sqrt{2}}{35} \right)$$

Direction Cosines and Direction Ratios Ex 27.1 Q8

The vectors, represented by these are

$$\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\text{and } \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$$

Let, θ be the angle between the lines,
then,

$$\begin{aligned}\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{(2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{(2)^2 + (3)^2 + (6)^2} \sqrt{(1)^2 + (2)^2 + (2)^2}} \\ &= \frac{(2)(1) + (3)(2) + (6)(2)}{\sqrt{4 + 9 + 36} \sqrt{1 + 4 + 4}} \\ &= \frac{2 + 6 + 12}{\sqrt{49} \sqrt{9}} \\ &= \frac{20}{7 \times 3}\end{aligned}$$

$$\cos \theta = \frac{20}{21}$$

$$\theta = \cos^{-1} \left(\frac{20}{21} \right)$$

$$\text{Angle between the lines} = \cos^{-1} \left(\frac{20}{21} \right)$$

Direction Cosines and Direction Ratios Ex 27.1 Q9

We have, $(2, 3, 4)$, $(-1, -2, 1)$ and $(5, 8, 7)$

Let the points are A , B , C respectively.

Position vector of $A = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Position vector of $B = -\hat{i} - 2\hat{j} + \hat{k}$

Position vector of $C = 5\hat{i} + 8\hat{j} + 7\hat{k}$

$\overrightarrow{AB}$ = Position vector of B - Position vector of A

$$= (-\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$= -\hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - 3\hat{j} - 4\hat{k}$$

$$\overrightarrow{AB} = -3\hat{i} - 5\hat{j} - 3\hat{k}$$

$\overrightarrow{BC}$ = Position vector of C - Position vector of B

$$= (5\hat{i} + 8\hat{j} + 7\hat{k}) - (-\hat{i} - 2\hat{j} + \hat{k})$$

$$= 5\hat{i} + 8\hat{j} + 7\hat{k} + \hat{i} + 2\hat{j} - \hat{k}$$

$$\overrightarrow{BC} = 6\hat{i} + 10\hat{j} + 6\hat{k}$$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$$\overrightarrow{BC} = -2 \overrightarrow{AB}$$

So, $\overrightarrow{BC}$ is parallel to $\overrightarrow{AB}$ but $\vec{B}$ is the common vector,

Hence, A , B , C are collinear

Direction Cosines and Direction Ratios Ex 27.1 Q10

line through points $(4, 7, 8)$ and $(2, 3, 4)$

$$\frac{x-4}{2} = \frac{y-7}{4} = \frac{z-8}{4} \rightarrow \frac{x-4}{1} = \frac{y-7}{2} = \frac{z-8}{2}$$

line through the points $(-1, -2, 1)$ and $(1, 2, 5)$

$$\frac{x+1}{-2} = \frac{y+2}{-4} = \frac{z-1}{-4} \rightarrow \frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$$

the direction ratios are same for both the lines

$\therefore$ they are parallel to each other

Direction Cosines and Direction Ratios Ex 27.1 Q11

Given,

$$A(1, -1, 2) \text{ and } B(3, 4, -2)$$

$$C(0, 3, 2) \text{ and } D(3, 5, 6)$$

Direction ratios of line AB

$$a_1 = 2, \quad b_1 = 5, \quad c_1 = -4$$

Direction ratios of line CD

$$a_2 = 3, \quad b_2 = 2, \quad c_2 = 4$$

We know that, lines are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$LHS = (2)(3) + (5)(2) + (-4)(4)$$

$$6 + 10 - 16$$

$$16 - 16$$

$$0$$

$$\therefore LHS = RHS$$

Lines are perpendicular

Direction Cosines and Direction Ratios Ex 27.1 Q12

Here,

$$A(0, 0, 0) \text{ and } B(2, 1, 1)$$

$$C(3, 5, -1) \text{ and } D(4, 3, -1)$$

Direction ratios of line AB

$$a_1 = 2, \quad b_1 = 1, \quad c_1 = 1$$

Direction ratios of line CD

$$a_2 = 1, \quad b_2 = -2, \quad c_2 = 0$$

Now,

$$a_1a_2 + b_1b_2 + c_1c_2$$

$$= (2)(1) + (1)(-2) + (1)(0)$$

$$= 2 - 2 + 0$$

$$= 0$$

Since, $a_1a_2 + b_1b_2 + c_1c_2 = 0$, lines are perpendicular

Direction Cosines and Direction Ratios Ex 27.1 Q13

Given, that the direction ratios of lines are proportional to a, b, c and $b - c, c - a, a - b$.

Let, $\vec{x}$ and $\vec{y}$ be the vector parallel to these lines respectively, so

$$\vec{x} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$\text{And, } \vec{y} = (b - c)\hat{i} + (c - a)\hat{j} + (a - b)\hat{k}$$

Let, θ be the angle between $\vec{x}$ and $\vec{y}$, so,

$$\begin{aligned}\cos\theta &= \frac{\vec{x} \cdot \vec{y}}{|\vec{x}| |\vec{y}|} \\ &= \frac{(a\hat{i} + b\hat{j} + c\hat{k}) \cdot [(b - c)\hat{i} + (c - a)\hat{j} + (a - b)\hat{k}]}{\sqrt{a^2 + b^2 + c^2} \sqrt{(b - c)^2 + (c - a)^2 + (a - b)^2}} \\ &= \frac{a(b - c) + b(c - a) + c(a - b)}{\sqrt{a^2 + b^2 + c^2} \sqrt{b^2 + c^2 - 2bc + c^2 + a^2 - 2ca + a^2 + b^2 - 2ab}}\end{aligned}$$

$$\cos\theta = \frac{ab - ac + bc - ba + ca - bc}{\sqrt{a^2 + b^2 + c^2} \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca}}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

$$\text{Angle between the lines} = \frac{\pi}{2}$$

Direction Cosines and Direction Ratios Ex 27.1 Q14

Here we have,

$$A(1, 2, 3), B(4, 5, 7), C(-4, 3, -6), D(2, 9, 2)$$

Direction ratios of AB

$$a_1 = 3, \quad b_1 = 3, \quad c_1 = 4$$

Direction ratios of CD

$$a_2 = 6, \quad b_2 = 6, \quad c_2 = 8$$

Let, θ be the angle between AB and CD , so,

$$\begin{aligned}\cos\theta &= \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \\ \cos\theta &= \frac{(3)(6) + (3)(6) + (4)(8)}{\sqrt{(3)^2 + (3)^2 + (4)^2} \sqrt{(6)^2 + (6)^2 + (8)^2}} \\ &= \frac{18 + 18 + 32}{\sqrt{9 + 9 + 16} \sqrt{36 + 36 + 64}} \\ &= \frac{68}{\sqrt{34} \sqrt{136}} \\ &= \frac{68}{\sqrt{34} \cdot 2\sqrt{34}} \\ &= \frac{68}{34 \times 2}\end{aligned}$$

$$\cos\theta = 1$$

$$\theta = \cos^{-1}(1)$$

$$\theta = 0^\circ$$

Therefore,

$$\text{Angle between } AB \text{ and } CD = 0^\circ$$

Direction Cosines and Direction Ratios Ex 27.1 Q15

The given equations are

$$2lm + 2ln - mn = 0$$

$$l + m + n = 0$$

$$\rightarrow l = -(m+n) \dots\dots\dots (1)$$

$$2l(m+n) = mn \rightarrow l = \frac{mn}{2(m+n)} \dots\dots\dots (2)$$

put $l = -(m+n)$ in (2)

$$\rightarrow -(m+n) = \frac{mn}{2(m+n)} \rightarrow -2(m+n)^2 = mn$$

$$\rightarrow -2(m^2 + n^2 + 2mn) = mn \rightarrow (m^2 + n^2 + 2mn) = -\frac{mn}{2}$$

$$\rightarrow \left(m^2 + n^2 + 2mn + \frac{mn}{2}\right) = 0 \rightarrow \left(m^2 + n^2 + \frac{5mn}{2}\right) = 0$$

$$\rightarrow (2m^2 + 2n^2 + 5mn) = 0 \rightarrow (2m+n)(m+2n) = 0$$

$$\rightarrow m = -\frac{n}{2} \rightarrow l = -\left(n - \frac{n}{2}\right) = -\frac{n}{2}$$

$$\rightarrow m = -2n \rightarrow l = -(-2n + n) = n$$

Thus the direction ratios of two lines are proportional to $-\frac{n}{2}, -\frac{n}{2}, n$

and $n, -2n, n$

i.e. $-\frac{1}{2}, -\frac{1}{2}, 1$ and $1, -2, 1$

Hence the direction cosines are

$$-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}} \text{ and } 1, -2, 1$$

Direction Cosines and Direction Ratios Ex 27.1 Q16(i)

$$\text{Given that, } l + m + n = 0 \quad \text{--- (i)}$$

$$l^2 + m^2 - n^2 = 0 \quad \text{--- (ii)}$$

From equation (i),

$$l = -(m + n)$$

Put the value of l in equation (ii),

$$[-(m + n)]^2 + m^2 - n^2 = 0$$

$$(m + n)^2 + m^2 - n^2 = 0$$

$$m^2 + n^2 + 2mn + m^2 - n^2 = 0$$

$$2m^2 + 2mn = 0$$

$$2m(m + n) = 0$$

$$m = 0, \quad m + n = 0$$

$$m = -n \text{ and } m = 0$$

Put the value of $m = -n$ in equation (i)

$$l = -(-n + n)$$

$$l = 0$$

Again put the value of $m = 0$ in equation (i)

$$l = -(m + n)$$

$$= -(0 + n)$$

$$l = -n$$

Thus the direction ratios are proportional to

$$0, -n, n \text{ and } -n, 0, n$$

$$\Rightarrow \quad 0, -1, 1 \text{ and } -1, 0, 1$$

So, vectors parallel to these lines are

$$\vec{a} = 0\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{b} = -\hat{i} + 0\hat{j} + \hat{k} \text{ respectively.}$$

Let, θ be the angle between the $\vec{a}$ and $\vec{b}$

$$\text{So, } \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$\vec{a} = 0\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 0\hat{j} + \hat{k}$ respectively.

$$\cos \theta = \frac{(0\hat{i} - \hat{j} + \hat{k}) \cdot (-\hat{i} + 0\hat{j} + \hat{k})}{\sqrt{0^2 + (-1)^2 + (1)^2} \sqrt{(-1)^2 + (0)^2 + (1)^2}}$$

$$= \frac{(0)(-1) + (-1)(0) + (1)(1)}{\sqrt{1+1}\sqrt{1+1}}$$

$$= \frac{0+0+1}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{3}$$

So, angle between the lines = $\frac{\pi}{3}$

Direction Cosines and Direction Ratios Ex 27.1 Q16(ii)

Given that,

$$2l - m + 2n = 0 \quad \text{--- (i)}$$

$$mn + nl + lm = 0 \quad \text{--- (ii)}$$

From equation (i),

$$2l - m + 2n = 0$$

$$m = 2l + 2n$$

Put the value of m in equation (ii),

$$mn + nl + lm = 0$$

$$(2l + 2n)n + nl + l(2l + 2n) = 0$$

$$2ln + 2n^2 + nl + 2l^2 + 2ln = 0$$

$$2l^2 + 5ln + 2n^2 = 0$$

$$2l^2 + 4ln + ln + 2n^2 = 0$$

$$2l(l + 2n) + n(l + 2n) = 0$$

$$(l + 2n)(2l + n) = 0$$

$$l + 2n = 0 \quad \text{or} \quad 2l + n = 0$$

$$l = -2n \quad \text{or} \quad l = -\frac{n}{2}$$

Put the value of $l = -2n$ in equation (i)

$$2l - m + 2n = 0$$

$$2(-2n) - m + 2n = 0$$

$$-4n - m + 2n = 0$$

$$-2n - m = 0$$

$$-2n = m$$

$$m = -2n$$

Again, put the value of $l = -\frac{1}{2}n$ in equation (i)

$$2l - m + 2n = 0$$

$$2\left(-\frac{1}{2}n\right) - m + 2n = 0$$

$$-n - m + 2n = 0$$

$$-m + n = 0$$

$$-m = -n$$

$$m = n$$

So, direction cosines of the lines are given by,

$$-2n, -2n, n \quad \text{or} \quad -\frac{1}{2}n, n, n$$

$$-2, -2, 1 \quad \text{or} \quad -\frac{1}{2}, 1, 1$$

So, vectors parallel to these lines

$$\vec{a} = -2\hat{i} - 2\hat{j} + \hat{k} \text{ and } \vec{b} = -\frac{1}{2}\hat{i} + \hat{j} + \hat{k} \text{ respectively.}$$

Let, θ be the angle between $\vec{a}$ and $\vec{b}$,

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{(-2\hat{i} - 2\hat{j} + \hat{k}) \cdot \left(-\frac{1}{2}\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{(-2)^2 + (-2)^2 + (1)^2} \sqrt{\left(-\frac{1}{2}\right)^2 + (1)^2 + (1)^2}} \\ &= \frac{(-2)\left(-\frac{1}{2}\right) + (-2)(1) + (1)(1)}{\sqrt{4+4+1} \sqrt{\frac{1}{4}+1+1}} \\ &= \frac{1-2+1}{\sqrt{9} \sqrt{\frac{9}{4}}} \end{aligned}$$

$$\cos \theta = \frac{0}{3 \times \frac{3}{2}}$$

$$\cos \theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

$$\text{Angle between the lines} = \frac{\pi}{2}$$

Direction Cosines and Direction Ratios Ex 27.1 Q16(iii)

Here,

$$l + 2m + 3n = 0 \quad \text{--- (i)}$$

$$3lm - 4ln + mn = 0 \quad \text{--- (ii)}$$

From equation (i),

$$l + 2m + 3n = 0$$

$$l = -2m - 3n$$

Put the value of l in equation (ii),

$$3lm - 4ln + mn = 0$$

$$3(-2m - 3n)m - 4(-2m - 3n)n + mn = 0$$

$$-6m^2 - 9nm + 8mn + 12n^2 + mn = 0$$

$$-6m^2 + 12n^2 = 0$$

$$-6m^2 = -12n^2$$

$$m^2 = 2n^2$$

$$m = \pm\sqrt{2n^2}$$

$$m = n\sqrt{2} \quad \text{or} \quad m = -n\sqrt{2}$$

Put $m = n\sqrt{2}$ in equation (i)

$$l + 2m + 3n = 0$$

$$l + 2(n\sqrt{2}) + 3n = 0$$

$$l + n(2\sqrt{2} + 3) = 0$$

$$l = -(2\sqrt{2} + 3)n$$

Again, $m = -n\sqrt{2}$ in equation (i)

$$l + 2m + 3n = 0$$

$$l + 2(-n\sqrt{2}) + 3n = 0$$

$$l - 2\sqrt{2}n + 3n = 0$$

$$l + n(-2\sqrt{2} + 3) = 0$$

$$l = (2\sqrt{2} - 3)n$$

Thus, direction cosines of the lines are given by,

$$\begin{aligned} &-(2\sqrt{2}+3)n, \sqrt{2}n, n \quad \text{or} \quad (2\sqrt{2}-3)n, -\sqrt{2}n, n \\ &-(2\sqrt{2}+3), \sqrt{2}, 1 \quad \text{or} \quad (2\sqrt{2}-3), -\sqrt{2}, 1 \end{aligned}$$

So, vectors parallel to these lines are

$$\vec{a} = -(2\sqrt{2}+3)\hat{i} + \sqrt{2}\hat{j} + \hat{k} \quad \text{and} \quad \vec{b} = (2\sqrt{2}-3)\hat{i} - \sqrt{2}\hat{j} + \hat{k}$$

Let, θ be the angle between the lines,
then,

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{-(2\sqrt{2}+3) \times (2\sqrt{2}-3) + (\sqrt{2}) \times (-\sqrt{2}) + (1)(1)}{\sqrt{(2\sqrt{2}+3)^2 + (\sqrt{2})^2 + (1)^2} \sqrt{(2\sqrt{2}-3)^2 + (-\sqrt{2})^2 + (1)^2}} \\ &= \frac{-(8-9) - 2 + 1}{\sqrt{8+9+12\sqrt{2}+2+1} \sqrt{8+9-12\sqrt{2}+2+1}} \\ &= \frac{-(-1) - 2 + 1}{\sqrt{20+12\sqrt{2}} \sqrt{20-12\sqrt{2}}} \\ &= \frac{1-2+1}{\sqrt{20+12\sqrt{2}} \sqrt{20-12\sqrt{2}}} \end{aligned}$$

$$\cos \theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

$$\text{Angle between the lines} = \frac{\pi}{2}$$

Direction Cosines and Direction Ratios Ex 27.1 Q16(iv)

The given equations are,

$$2l + 2m - n = 0 \dots\dots\dots (i)$$

$$mn + ln + lm = 0 \dots\dots\dots (ii)$$

From (i), we get $n = 2l + 2m$.

Putting $n = 2l + 2m$ in (ii), we get

$$m(2l + 2m) + l(2l + 2m) + lm = 0$$

$$\Rightarrow 2lm + 2m^2 + 2l^2 + 2ml + lm = 0$$

$$\Rightarrow 2m^2 + 5lm + 2l^2 = 0$$

$$\Rightarrow 2m^2 + 4lm + lm + 2l^2 = 0$$

$$\Rightarrow (2m+l)(m+2l) = 0$$

$$\Rightarrow m = -\frac{l}{2} \quad \text{or} \quad m = -2l$$

By putting $m = -\frac{l}{2}$ in (i) we get $n = l$

By putting $m = -2l$ in (i) we get $n = -2l$

So direction ratios of two lines are proportional to

$$l, -\frac{l}{2}, l \quad \text{and} \quad l, -2l, -2l \quad \text{or} \quad 1, -\frac{1}{2}, 1 \quad \text{and} \quad 1, -2, -2$$

So, vectors parallel to these lines are

$$\vec{a} = \hat{i} - \frac{1}{2}\hat{j} + \hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$$

If θ is the angle between the lines, then θ is also the angle between $\vec{a}$ and $\vec{b}$.

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{1 + 1 - 2}{\sqrt{1 + \frac{1}{4} + 1} \sqrt{1 + 4 + 9}} = 0$$

$$\Rightarrow \theta = \cos^{-1}(0) = \frac{\pi}{2}$$