

Chapter 1: Rational Numbers

Exercise 1.1

Q1. Add the following rational numbers:

(i) $\frac{-5}{7}$ and $\frac{3}{7}$

(ii) $\frac{-15}{4}$ and $\frac{7}{4}$

(iii) $\frac{-8}{11}$ and $\frac{-4}{11}$

(iv) $\frac{6}{13}$ and $\frac{-9}{13}$

Solution:

(i) $\frac{-5}{7} + \frac{3}{7} = \frac{-5+3}{7} = \frac{-2}{7}$

$$(ii) \frac{-15}{4} + \frac{7}{4} = \frac{-15+7}{4} = \frac{-8}{4} = -2$$

$$(iii) \frac{-8}{11} + \frac{-4}{11} = \frac{-8-4}{11} = \frac{-12}{11}$$

$$(iv) \frac{6}{13} + \frac{-9}{13} = \frac{6-9}{13} = \frac{-3}{13}$$

Q2: Add the following rational numbers:

$$(i) \frac{3}{4} \text{ and } \frac{-5}{8}$$

$$(ii) \frac{5}{-9} \text{ and } \frac{7}{3}$$

$$(iii) -3 \text{ and } \frac{3}{5}$$

$$(iv) \frac{-7}{27} \text{ and } \frac{11}{18}$$

$$(v) \frac{31}{-4} \text{ and } \frac{-5}{8}$$

$$(vi) \frac{5}{36} \text{ and } \frac{-7}{12}$$

$$(vii) \frac{-5}{16} \text{ and } \frac{-7}{24}$$

$$(viii) \frac{7}{-18} \text{ and } \frac{8}{27}$$

Solution:

(i) Clearly, denominators of the given numbers are positive.

The LCM of the denominators 4 and 8 is 8.

Now, we will express $\frac{3}{4}$ in the form in which it takes the denominator as 8.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8} = \frac{3}{4}$$

Now,

$$\begin{aligned} & \frac{-5}{8} + \frac{6}{8} \\ &= \frac{-5+6}{8} = \frac{1}{8} \end{aligned}$$

$$\begin{aligned} (ii) & \frac{5}{-9} + \frac{7}{3} \\ &= \frac{-5}{9} + \frac{7}{3} \end{aligned}$$

The LCM of the denominators 9 and 3 is 9.

Now,

We will express $\frac{7}{3}$ in the form in which it takes denominator as 9.

$$\frac{7 \times 3}{3 \times 3} = \frac{21}{9}$$

So,

$$\begin{aligned} & \frac{-5}{9} + \frac{21}{9} \\ &= \frac{-5+21}{9} = \frac{16}{9} \end{aligned}$$

$$\begin{aligned} (iii) & -3 + \frac{3}{5} \\ &= \frac{-3}{1} + \frac{3}{5} \end{aligned}$$

The LCM of the denominators 1 and 5 is 5.

Now,

We will express $\frac{-3}{1}$ in the form in which it takes denominator as 5.

$$\frac{-3}{1} = \frac{-3 \times 5}{1 \times 5} = \frac{-15}{5}$$

So,

$$\begin{aligned} & \frac{15}{5} + \frac{3}{5} \\ &= \frac{-15+3}{5} = \frac{-12}{5} \end{aligned}$$

$$(iv) \frac{-7}{27} + \frac{11}{18}$$

The LCM of the denominators 27 and 18 is 54.

Now,

We will express $\frac{-7}{27}$ and $\frac{11}{18}$ in the form in which it takes denominator as 54.

$$\frac{-7}{27} = \frac{-7 \times 2}{27 \times 2} = \frac{-14}{54}$$

$$\frac{11}{18} = \frac{11 \times 3}{18 \times 3} = \frac{33}{54}$$

So,

$$\frac{-14}{54} + \frac{33}{54}$$

$$= \frac{-14+33}{54} = \frac{19}{54}$$

$$(v) \frac{31}{-4} + \frac{-5}{8}$$

$$= \frac{31}{-4} = \frac{-31}{4}$$

The LCM of the denominators 4 and 8 is 8.

Now,

We will express $\frac{-31}{4}$ in the form in which it takes denominator as 8.

$$\frac{-31}{4} = \frac{-31 \times 2}{4 \times 2} = \frac{-62}{8}$$

So,

$$\frac{-62}{8} + \frac{-5}{8}$$

$$= \frac{-62-5}{8} = \frac{-67}{8}$$

$$(vi) \frac{5}{36} + \frac{-7}{12}$$

The LCM of the denominator 12 and 36 is 36.

Now,

We will express $\frac{-7}{12}$ in the form in which it takes denominator as 36.

$$\frac{-7}{12} = \frac{-7 \times 3}{12 \times 3} = \frac{-21}{36}$$

So,

$$\frac{-21}{36} + \frac{5}{36}$$

$$= \frac{-21+5}{36} = \frac{-16}{36} = \frac{-4}{9}$$

$$(vii) \frac{-5}{16} \text{ and } \frac{7}{24}$$

The LCM of the denominators 16 and 24 is 48.

Now,

We will express $\frac{-5}{16}$ and $\frac{7}{24}$ in the form in which it takes denominator as 48.

$$\frac{-5}{16} = \frac{-5 \times 3}{16 \times 3} = \frac{-15}{48}$$

$$\frac{7}{24} = \frac{7 \times 2}{24 \times 2} = \frac{14}{48}$$

So,

$$\frac{-15}{48} + \frac{14}{48}$$

$$= \frac{-15+14}{48} = \frac{-1}{48}$$

$$(viii) \frac{7}{-18} + \frac{8}{27}$$

$$\frac{7}{-18} = \frac{-7}{18}$$

The LCM of the denominator 18 and 27 is 54.

Now,

We will express $\frac{-7}{18}$ and $\frac{8}{27}$ in the form in which it takes denominator as 54.

$$\frac{-7}{18} = \frac{-7 \times 3}{18 \times 3} = \frac{-21}{54}$$

$$\frac{8}{27} = \frac{8 \times 2}{27 \times 2} = \frac{16}{54}$$

So,

$$\frac{-21}{54} + \frac{16}{54}$$

$$= \frac{-21+16}{54} = \frac{-5}{54}$$

Q-3. Simplify:

$$(i) \frac{8}{9} + \frac{-11}{6}$$

$$(ii) 3 + \frac{5}{-7}$$

$$(iii) \frac{1}{-12} + \frac{2}{-15}$$

$$(iv) \frac{-8}{19} + \frac{-4}{57}$$

$$(v) \frac{7}{9} + \frac{3}{-4}$$

$$(vi) \frac{5}{26} + \frac{11}{-39}$$

$$(vii) \frac{-16}{9} + \frac{-5}{12}$$

$$(viii) \frac{-13}{8} + \frac{5}{36}$$

$$(ix) 0 + \frac{-3}{5}$$

$$(x) 1 + \frac{-4}{5}$$

Solution:

$$(i) \frac{8}{9} + \frac{-11}{6}$$

The LCM of the denominator 9 and 6 is 18.

Now,

We will express $\frac{8}{9}$ and $\frac{-11}{6}$ in the form in which it takes denominator as 18.

$$\frac{8}{9} = \frac{8 \times 2}{9 \times 2} = \frac{16}{18}$$

$$\frac{-11}{6} = \frac{-11 \times 3}{6 \times 3} = \frac{-33}{18}$$

So,

$$\frac{16}{18} + \frac{-33}{18}$$

$$= \frac{16-33}{18} = \frac{-17}{18}$$

$$(ii) 3 + \frac{5}{-7}$$

$$\frac{5}{-7} = \frac{-5}{7}$$

The LCM of the denominator 1 and 7 is 7.

Now,

We will express $\frac{3}{1}$ in the form in which it takes denominator as 7.

$$\frac{3}{1} = \frac{3 \times 7}{1 \times 7} = \frac{21}{7}$$

So,

$$\frac{21}{7} + \frac{-5}{7}$$

$$= \frac{21-5}{7} = \frac{16}{7}$$

$$(iii) \frac{1}{-12} + \frac{2}{-15}$$

$$\frac{1}{-12} = \frac{-1}{12}$$

$$\frac{2}{-15} = \frac{-2}{15}$$

The LCM of the denominators 12 and 15 is 60.

Now,

We will express $\frac{-1}{12}$ and $\frac{-2}{15}$ in the form in which it takes denominator as 60.

$$\frac{-1}{12} = \frac{-1 \times 5}{12 \times 5} = \frac{-5}{60}$$

$$\frac{-2}{15} = \frac{-2 \times 4}{15 \times 4} = \frac{-8}{60}$$

So,

$$\frac{-5}{60} + \frac{-8}{60}$$

$$= \frac{-5-8}{60} = \frac{-13}{60}$$

$$(iv) \frac{-8}{19} + \frac{-4}{57}$$

The LCM of the denominator of 19 and 57 is 57.

Now,

We will express $\frac{-8}{19}$ in the form in which it takes denominator as 57.

$$\frac{-8}{19} = \frac{-8 \times 3}{19 \times 3} = \frac{-24}{57}$$

So,

$$\begin{aligned} & \frac{-24}{57} + \frac{-4}{57} \\ &= \frac{-24-4}{57} = \frac{-28}{57} \end{aligned}$$

$$(v) \frac{7}{9} + \frac{3}{-4}$$

$$\frac{3}{-4} = \frac{-3}{4}$$

The LCM of the denominator 9 and 4 is 36.

Now,

We will express $\frac{7}{9}$ and $\frac{-3}{4}$ in the form in which it takes denominator as 36.

$$\frac{7}{9} = \frac{7 \times 4}{9 \times 4} = \frac{28}{36}$$

$$\frac{-3}{4} = \frac{-3 \times 9}{4 \times 9} = \frac{-27}{36}$$

So,

$$\begin{aligned} & \frac{28}{36} + \frac{-27}{36} \\ &= \frac{28-27}{36} = \frac{1}{36} \end{aligned}$$

$$(vi) \frac{5}{26} + \frac{11}{-39}$$

$$\frac{11}{-39} = \frac{-11}{39}$$

The LCM of the denominator 26 and 39 is 78.

Now,

We will express $\frac{-3}{4}$ and $\frac{-11}{39}$ in the form in which it takes denominator as 78.

$$\frac{5}{26} = \frac{5 \times 3}{26 \times 3} = \frac{15}{78}$$

$$\frac{-11}{39} = \frac{-11 \times 2}{39 \times 2} = \frac{-22}{78}$$

So,

$$\begin{aligned} & \frac{15}{78} + \frac{-22}{78} \\ &= \frac{15-22}{78} = \frac{-7}{78} \end{aligned}$$

$$(vii) \frac{-16}{9} + \frac{-5}{12}$$

The LCM of the denominator 9 and 12 is 36.

Now,

We will express $\frac{-16}{9}$ and $\frac{-5}{12}$ in the form in which it takes denominator as 36.

$$\frac{-16}{9} = \frac{-16 \times 4}{9 \times 4} = \frac{-64}{36}$$

$$\frac{-5}{12} = \frac{-5 \times 3}{12 \times 3} = \frac{-15}{36}$$

So,

$$\begin{aligned} & \frac{-64}{36} + \frac{-15}{36} \\ &= \frac{-64-15}{36} = \frac{-79}{36} \end{aligned}$$

$$(viii) \frac{-13}{8} + \frac{5}{36}$$

The LCM of the denominator 8 and 36 is 72.

Now,

We will express $\frac{-13}{8}$ and $\frac{5}{36}$ in the form in which it takes denominator as 72.

$$\frac{-13}{8} = \frac{-13 \times 9}{8 \times 9} = \frac{-117}{72}$$

$$\frac{5}{36} = \frac{5 \times 2}{36 \times 2} = \frac{10}{72}$$

So,

$$\frac{-117}{72} + \frac{10}{72}$$

$$= \frac{-117+10}{72} = \frac{-107}{72}$$

$$(b) 0 + \frac{-3}{5}$$

$$= \frac{-3}{5}$$

$$(x) 1 + \frac{-4}{5}$$

The LCM of the denominator 1 and 5 is 5.

Now,

We need to express $\frac{1}{1}$ in the form in which it takes denominator as 5.

$$\frac{1}{1} = \frac{1 \times 5}{1 \times 5} = \frac{5}{5}$$

So,

$$\frac{5}{5} + \frac{-4}{5}$$

$$= \frac{5-4}{5} = \frac{1}{5}$$

Q-4. Add and express the sum as a mixed fraction:

$$(i) \frac{-12}{5} \text{ and } \frac{43}{10}$$

$$(ii) \frac{24}{7} \text{ and } \frac{-11}{4}$$

$$(iii) \frac{-31}{6} \text{ and } \frac{-27}{8}$$

$$(iv) \frac{101}{6} \text{ and } \frac{7}{8}$$

Solution:

(i) We have:

$$\frac{-12}{5} \text{ and } \frac{43}{10}$$

The LCM of the denominator 5 and 10 is 10.

Now,

We will express $\frac{-12}{5}$ in the form in which it takes denominator as 10.

$$\frac{-12}{5} = \frac{-12 \times 2}{5 \times 2} = \frac{-24}{10}$$

So,

$$\frac{-24}{10} + \frac{43}{10}$$

$$= \frac{-24+43}{10} = \frac{19}{10}$$

(ii) We have:

$$\frac{24}{7} \text{ and } \frac{-11}{4}$$

The LCM of the denominator 7 and 4 is 28.

Now,

We will express $\frac{24}{7}$ and $\frac{-11}{4}$ in the form in which it takes denominator as 28.

$$\frac{24}{7} = \frac{24 \times 4}{7 \times 4} = \frac{96}{28}$$

$$\frac{-11}{4} = \frac{-11 \times 7}{4 \times 7} = \frac{-77}{28}$$

So,

$$\frac{96}{28} + \frac{-77}{28}$$

$$= \frac{96-77}{28} = \frac{19}{28}$$

(iii) We have:

$$\frac{-31}{6} \text{ and } \frac{-27}{8}$$

The LCM of the denominator 6 and 8 is 24.

Now,

We will express $\frac{-31}{6}$ and $\frac{-27}{8}$ in the form in which it takes denominator as 24.

$$\frac{-31}{6} = \frac{-31 \times 4}{6 \times 4} = \frac{-124}{24}$$

$$\frac{-27}{8} = \frac{-27 \times 3}{8 \times 3} = \frac{-81}{24}$$

So,

$$\frac{-124}{24} + \frac{-81}{24}$$

$$= \frac{-124-81}{24}$$

$$= \frac{-205}{24} = -8 \frac{13}{24}$$

(iv) We have:

$$\frac{101}{6} \text{ and } \frac{7}{8}$$

The LCM of the denominator 6 and 8 is 24.

Now,

We will express $\frac{101}{6}$ and $\frac{7}{8}$ in the form in which it takes denominator as 24.

$$\frac{101}{6} = \frac{101 \times 4}{6 \times 4} = \frac{404}{24}$$

$$\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$$

So,

$$\frac{404}{24} + \frac{21}{24}$$

$$= \frac{404+21}{24}$$

$$= \frac{425}{24} = 17 \frac{17}{24}$$

Chapter 1: Rational Numbers

Exercise 1.2

Q-1. Verify commutativity of addition of rational numbers for each of the following pairs of rational numbers.

(i) $\frac{-11}{5}$ and $\frac{4}{7}$

(ii) $\frac{4}{9}$ and $\frac{7}{-12}$

(iii) $\frac{-3}{5}$ and $\frac{-2}{-15}$

(iv) $\frac{2}{-7}$ and $\frac{12}{-35}$

(v) 4 and $\frac{-3}{5}$

(vi) -4 and $\frac{4}{-7}$

Solution:

Commutativity of the addition of rational numbers means that if ab and cd are two rational numbers, then $ab + cd = cd + ab$.

(i) We have:

$$\frac{-11}{5} \text{ and } \frac{4}{7}$$

So,

$$\begin{aligned} & \frac{-11}{5} + \frac{4}{7} \\ &= \frac{-11 \times 7}{5 \times 7} + \frac{4 \times 5}{7 \times 5} \\ &= \frac{-77}{35} + \frac{20}{35} \\ &= \frac{-77+20}{35} \\ &= \frac{-57}{35} \end{aligned}$$

Now,

$$\begin{aligned} & \frac{4}{7} \text{ and } \frac{-11}{5} \\ &= \frac{4}{7} + \frac{-11}{5} \\ &= \frac{4 \times 5}{7 \times 5} + \frac{-11 \times 7}{5 \times 7} \\ &= \frac{20}{35} + \frac{-77}{35} \\ &= \frac{20-77}{35} = \frac{-57}{35} \end{aligned}$$

Hence, verified.

(ii) We have:

$$\frac{4}{9} \text{ and } \frac{7}{-12}$$

So,

$$\begin{aligned} & \frac{4}{9} + \frac{-7}{12} \\ &= \frac{4 \times 4}{9 \times 4} + \frac{-7 \times 3}{12 \times 3} \\ &= \frac{16}{36} + \frac{-21}{36} \\ &= \frac{16-21}{36} \\ &= \frac{-5}{36} \end{aligned}$$

Now,

$$\begin{aligned} & \frac{-7}{12} \text{ and } \frac{4}{9} \\ &= \frac{-7}{12} + \frac{4}{9} \\ &= \frac{-7 \times 3}{12 \times 3} + \frac{4 \times 4}{9 \times 4} \\ &= \frac{-21}{36} + \frac{16}{36} \\ &= \frac{-21+16}{36} = \frac{-5}{36} \end{aligned}$$

Hence, Verified.

(iii) We have:

$$\frac{-3}{5} \text{ and } \frac{-2}{-15}$$

So,

$$\begin{aligned} & \frac{-3}{5} + \frac{2}{15} \\ &= \frac{-3 \times 3}{5 \times 3} + \frac{2}{15} \\ &= \frac{-9}{15} + \frac{2}{15} \\ &= \frac{-9+2}{15} = \frac{-7}{15} \end{aligned}$$

Now,

$$\frac{2}{15} \text{ and } \frac{-3}{5}$$

$$\begin{aligned}
&= \frac{2}{15} + \frac{-3}{5} \\
&= \frac{2}{15} + \frac{-3 \times 3}{5 \times 3} \\
&= \frac{2}{15} + \frac{-9}{15} \\
&= \frac{2-9}{15} = \frac{-7}{15}
\end{aligned}$$

Hence, verified.

(iv) We have:

$$\frac{2}{-7} \text{ and } \frac{12}{-35}$$

So,

$$\begin{aligned}
&\frac{-2}{7} + \frac{-12}{35} \\
&= \frac{-2 \times 5}{7 \times 5} + \frac{-12}{35} \\
&= \frac{-10}{35} + \frac{-12}{35} \\
&= \frac{-10-12}{35} \\
&= \frac{-22}{35}
\end{aligned}$$

Now,

$$\begin{aligned}
&\frac{-12}{35} \text{ and } \frac{-2}{7} \\
&= \frac{-12}{35} + \frac{-2 \times 5}{7 \times 5} \\
&= \frac{-12}{35} + \frac{-10}{35} \\
&= \frac{-12-10}{35} = \frac{-22}{35}
\end{aligned}$$

Hence, verified.

(v) We have:

$$\frac{4}{1} \text{ and } \frac{-3}{5}$$

So,

$$\begin{aligned}
&\frac{4}{1} + \frac{-3}{5} \\
&= \frac{4 \times 5}{1 \times 5} + \frac{-3}{5} \\
&= \frac{20}{5} + \frac{-3}{5} \\
&= \frac{20-3}{5} = \frac{17}{5}
\end{aligned}$$

Now,

$$\begin{aligned}
&\frac{-3}{5} \text{ and } \frac{4}{1} \\
&= \frac{-3}{5} + \frac{4}{1} \\
&= \frac{-3}{5} + \frac{4 \times 5}{1 \times 5} \\
&= \frac{-3}{5} + \frac{20}{5} \\
&= \frac{-3+20}{5} = \frac{17}{5}
\end{aligned}$$

Hence, verified.

(vi) We have:

$$\frac{-4}{1} \text{ and } \frac{4}{-7}$$

So,

$$\begin{aligned}
&\frac{-4}{1} + \frac{-4}{7} \\
&= \frac{-4 \times 7}{1 \times 7} + \frac{-4}{7} \\
&= \frac{-28}{7} + \frac{-4}{7} \\
&= \frac{-28-4}{7} = \frac{-32}{7}
\end{aligned}$$

Now,

$$\frac{-4}{7} \text{ and } \frac{-4}{1}$$

$$\begin{aligned}
&= \frac{-4}{7} + \frac{-4}{1} \\
&= \frac{-4}{7} + \frac{-4 \times 7}{1 \times 7} \\
&= \frac{-4}{7} + \frac{-28}{7} \\
&= \frac{-4-28}{7} = \frac{-32}{7}
\end{aligned}$$

Hence, verified.

Q-2. Verify associativity of addition of the rational numbers i.e., $(x + y) + z = x + (y + z)$, when:

(i) $x = \frac{1}{2}, y = \frac{2}{3}, z = -\frac{1}{5}$

(ii) $x = -\frac{2}{5}, y = \frac{4}{3}, z = -\frac{7}{10}$

(iii) $x = -\frac{7}{11}, y = -\frac{2}{5}, z = -\frac{3}{22}$

(iv) $x = -2, y = \frac{3}{5}, z = -\frac{4}{3}$

Solution:

We have to verify that:

$$(x + y) + z = x + (y + z)$$

(i) $x = \frac{1}{2}, y = \frac{2}{3}, z = -\frac{1}{5}$

$$= \left(\frac{1}{2} + \frac{2}{3} \right) + \left(-\frac{1}{5} \right)$$

$$= \left(\frac{7}{6} \right) - \frac{1}{5}$$

$$= \left(\frac{7 \times 5}{6 \times 5} \right) - \frac{1 \times 7}{5 \times 7}$$

$$= \left(\frac{35}{30} \right) - \frac{7}{35}$$

$$= \frac{35-7}{30}$$

$$= \frac{28}{30}$$

Now,

$$\frac{1}{2} + \left(\frac{2}{3} + \frac{-1}{5} \right)$$

$$= \frac{1}{2} + \left(\frac{2 \times 5}{3 \times 5} + \frac{-1 \times 3}{5 \times 3} \right)$$

$$= \frac{1}{2} + \left(\frac{10}{15} + \frac{-3}{15} \right)$$

$$= \frac{1}{2} + \left(\frac{10-3}{15} \right)$$

$$= \frac{1}{2} + \left(\frac{7}{15} \right)$$

$$= \frac{1 \times 15}{2 \times 15} + \frac{7 \times 2}{15 \times 2}$$

$$= \frac{15}{30} + \frac{14}{30}$$

$$= \frac{15+14}{30} = \frac{29}{30}$$

Hence, verified.

(ii) $x = -\frac{2}{5}, y = \frac{4}{3}, z = -\frac{7}{10}$

$$= \left(-\frac{2}{5} + \frac{4}{3} \right) - \frac{7}{10}$$

$$= \left(\frac{-2 \times 3}{5 \times 3} + \frac{4 \times 5}{3 \times 5} \right) - \frac{7}{10}$$

$$= \left(\frac{-6}{15} + \frac{20}{15} \right) - \frac{7}{10}$$

$$= \left(\frac{-6+20}{15} \right) - \frac{7}{10}$$

$$= \left(\frac{14}{15} \right) - \frac{7}{10}$$

$$= \frac{14 \times 2}{15 \times 2} - \frac{7 \times 3}{10 \times 3}$$

$$= \frac{28}{30} - \frac{21}{30}$$

$$= \frac{28-21}{30} = \frac{7}{30}$$

Now,

$$\frac{-2}{5} + \left(\frac{4}{3} - \frac{7}{10} \right)$$

$$= \frac{-2}{5} + \left(\frac{4 \times 10}{3 \times 10} - \frac{7 \times 3}{10 \times 3} \right)$$

$$= \frac{-2}{5} + \left(\frac{40}{30} - \frac{21}{30} \right)$$

$$= \frac{-2}{5} + \left(\frac{40-21}{30} \right)$$

$$= \frac{-2}{5} + \left(\frac{19}{30} \right)$$

$$= \frac{-2 \times 6}{5 \times 6} + \left(\frac{19}{30} \right)$$

$$= \frac{-12}{30} + \left(\frac{19}{30} \right)$$

$$= \frac{-12+19}{30} = \frac{7}{30}$$

Hence, verified.

$$(iii) x = -\frac{7}{9}, y = \frac{2}{-5}, z = -\frac{3}{22}$$

$$= \left(\frac{-7}{11} + \frac{2}{-5} \right) - \frac{3}{22}$$

$$= \left(\frac{-7}{11} + \frac{-2}{5} \right) - \frac{3}{22}$$

$$= \left(\frac{-7 \times 5}{11 \times 5} + \frac{-2 \times 11}{5 \times 11} \right) - \frac{3}{22}$$

$$= \left(\frac{-35}{55} + \frac{-22}{55} \right) - \frac{3}{22}$$

$$= \left(\frac{-35-22}{55} \right) - \frac{3}{22}$$

$$= \left(\frac{-57}{55} \right) - \frac{3}{22}$$

$$= \frac{-57 \times 2}{55 \times 2} - \frac{3 \times 5}{22 \times 5}$$

$$= \frac{-114}{110} - \frac{15}{110}$$

$$= \frac{-114-15}{110} = \frac{-129}{110}$$

Now,

$$\frac{-7}{11} + \left(\frac{-2}{5} - \frac{3}{22} \right)$$

$$= \frac{-7}{11} + \left(\frac{-2 \times 22}{5 \times 22} - \frac{3 \times 5}{22 \times 5} \right)$$

$$= \frac{-7}{11} + \left(\frac{-44}{110} - \frac{15}{110} \right)$$

$$= \frac{-7}{11} + \left(\frac{-44-15}{110} \right)$$

$$= \frac{-7}{11} + \left(\frac{-59}{110} \right)$$

$$= \frac{-7 \times 10}{11 \times 10} + \left(\frac{-59}{110} \right)$$

$$= \frac{-70}{110} + \frac{-59}{110} = \frac{-129}{110}$$

Hence, verified.

$$(iv) x = -2, y = \frac{3}{5}, z = -\frac{4}{3}$$

$$= \left(-2 + \frac{3}{5} \right) - \frac{4}{3}$$

$$= \left(-2 \times 5 + \frac{3}{5} \right) - \frac{4}{3}$$

$$= \left(\frac{-10+3}{5} \right) - \frac{4}{3}$$

$$\begin{aligned}
&= \frac{-7}{5} - \frac{4}{3} \\
&= \frac{-7 \times 3}{5 \times 3} - \frac{4 \times 5}{3 \times 5} \\
&= \frac{-21}{15} - \frac{20}{15} \\
&= \frac{-21-20}{15} = \frac{-41}{15}
\end{aligned}$$

Now,

$$\begin{aligned}
&-2 + \left(\frac{3}{5} - \frac{4}{3} \right) \\
&= -2 + \left(\frac{3 \times 3}{5 \times 3} - \frac{4 \times 5}{3 \times 5} \right) \\
&= -2 + \left(\frac{9}{15} - \frac{20}{15} \right) \\
&= -2 + \left(\frac{9-20}{15} \right) \\
&= -2 + \left(\frac{-11}{15} \right) \\
&= -\frac{2 \times 15}{1 \times 15} + \frac{-11}{15} \\
&= -\frac{30}{15} + \frac{-11}{15} \\
&= \frac{-30-11}{15} = \frac{-41}{15}
\end{aligned}$$

Hence, verified.

Q-3. Write the additive inverse of each of the following rational numbers:

(i) $\frac{-2}{17}$

(ii) $\frac{3}{-11}$

(iii) $\frac{-17}{5}$

(iv) $\frac{-11}{-25}$

Solution:

(i) Additive inverse is the negative of the given number.

So, additive inverse of $\frac{-2}{17} = \frac{2}{17}$

(ii) Additive inverse is the negative of the given number.

So, additive inverse of $\frac{3}{-11} = \frac{3}{11}$

(iii) Additive inverse is the negative of the given number.

So, additive inverse of $\frac{-17}{5} = \frac{17}{5}$

(iv) Additive inverse is the negative of the given number.

So, additive inverse of $\frac{-11}{-25} = \frac{-11}{25}$

Q-4. Write the negative (additive inverse) of each of the following:

(i) $\frac{-2}{5}$

(ii) $\frac{7}{-9}$

(iii) $\frac{-16}{13}$

(iv) $\frac{-5}{1}$

(v) 0

(vi) 1

(vii) -1

Solution:

(i) Additive inverse of $\frac{-2}{5} = \frac{2}{5}$

(ii) Additive inverse of $\frac{-7}{9} = \frac{7}{9}$

(iii) Additive inverse of $\frac{-16}{13} = \frac{16}{13}$

(iv) Additive inverse of $\frac{-5}{1} = \frac{5}{1}$

(v) Negative value of 0 is 0

(vi) Negative value of 1 is -1

(vii) Negative value of -1 is 1

Q-5. Using commutativity and associativity of addition of rational numbers, express each of the following as a rational number:

(i) $\frac{2}{5} + \frac{7}{3} + \frac{-4}{5} + \frac{-1}{3}$

(ii) $\frac{3}{7} + \frac{-4}{9} + \frac{-11}{7} + \frac{7}{9}$

(iii) $\frac{2}{5} + \frac{8}{3} + \frac{-11}{15} + \frac{4}{5} + \frac{-2}{3}$

(iv) $\frac{4}{7} + 0 + \frac{-8}{9} + \frac{-13}{7} + \frac{17}{21}$

Solution:

(i) We have:

$$\begin{aligned} & \frac{2}{5} + \frac{7}{3} + \frac{-4}{5} + \frac{-1}{3} \\ &= \left(\frac{2}{5} + \frac{-4}{5} \right) + \left(\frac{-1}{3} + \frac{7}{3} \right) \\ &= \frac{2-4}{5} + \frac{-1+7}{3} \\ &= \frac{-2}{5} + \frac{6}{3} \\ &= \frac{-2 \times 3}{5 \times 3} + \frac{6 \times 5}{3 \times 5} \\ &= \frac{-6}{15} + \frac{30}{15} \\ &= \frac{-6+30}{15} = \frac{24}{15} = \frac{8}{5} \end{aligned}$$

$$\begin{aligned} & \text{(ii) } \frac{3}{7} + \frac{-4}{9} + \frac{-11}{7} + \frac{7}{9} \\ &= \left(\frac{3}{7} + \frac{-11}{7} \right) + \left(\frac{-4}{9} + \frac{7}{9} \right) \\ &= \frac{3-11}{7} + \frac{-4+7}{9} \\ &= \frac{-8}{7} + \frac{3}{9} \\ &= \frac{-8 \times 9}{7 \times 9} + \frac{3 \times 7}{9 \times 7} \\ &= \frac{-72}{63} + \frac{21}{63} \\ &= \frac{-72+21}{63} \\ &= \frac{-51}{63} = \frac{-17}{21} \end{aligned}$$

$$\begin{aligned} & \text{(iii) } \frac{2}{5} + \frac{8}{3} + \frac{-11}{15} + \frac{4}{5} + \frac{-2}{3} \\ &= \left(\frac{2}{5} + \frac{8}{5} \right) + \left(\frac{-11}{15} + \frac{4}{5} + \frac{-2}{3} \right) \\ &= \frac{2+8}{5} + \frac{8-2}{3} + \frac{-11}{15} \\ &= \frac{6}{5} + \frac{6}{3} + \frac{-11}{15} \\ &= \frac{6 \times 3}{5 \times 3} + \frac{6 \times 5}{3 \times 5} + \frac{-11}{15} \\ &= \frac{18}{15} + \frac{30}{15} + \frac{-11}{15} \\ &= \frac{18+30}{15} + \frac{-11}{15} \\ &= \frac{48}{15} + \frac{-11}{15} \\ &= \frac{48-11}{15} = \frac{37}{15} \end{aligned}$$

$$\begin{aligned} & \text{(iv) } \frac{4}{7} + 0 + \frac{-8}{9} + \frac{-13}{7} + \frac{17}{21} \\ &= \left(\frac{4}{7} + \frac{-13}{7} \right) + \frac{-8}{9} + \frac{17}{21} \\ &= \frac{4-13}{7} + \frac{-8}{9} + \frac{17}{21} \\ &= \frac{-9}{7} + \frac{-8}{9} + \frac{17}{21} \\ &= \frac{-9 \times 9}{7 \times 9} + \frac{-8 \times 7}{9 \times 7} + \frac{17 \times 3}{21 \times 3} \end{aligned}$$

$$= \frac{-81}{63} + \frac{-56}{63} + \frac{51}{63}$$

$$= \frac{-81-56+51}{63}$$

$$= \frac{-86}{63}$$

Q-6. Re-arrange suitably and find the sum of each of the following:

(i) $\frac{11}{12} + \frac{-17}{3} + \frac{11}{2} + \frac{-25}{2}$

(ii) $\frac{-6}{7} + \frac{-5}{6} + \frac{-4}{9} + \frac{-15}{7}$

(iii) $\frac{3}{5} + \frac{7}{3} + \frac{9}{5} + \frac{-13}{15} + \frac{-7}{3}$

(iv) $\frac{4}{13} + \frac{-5}{8} + \frac{-8}{13} + \frac{9}{13}$

(v) $\frac{2}{3} + \frac{-4}{5} + \frac{1}{3} + \frac{2}{5}$

(vi) Extra close brace or missing open brace

Solution:

(i) $\frac{11}{12} + \frac{-17}{3} + \frac{11}{2} + \frac{-25}{2}$

$$= \left(\frac{11}{2} + \frac{-25}{2} \right) + \frac{11}{12} + \frac{-17}{3}$$

$$= \frac{11-25}{2} + \frac{11}{12} + \frac{-17}{3}$$

$$= \frac{-14}{2} + \frac{11}{12} + \frac{-17}{3}$$

$$= \frac{-14 \times 6}{2 \times 6} + \frac{11}{12} + \frac{-17 \times 4}{3 \times 4}$$

$$= \frac{-84}{12} + \frac{11}{12} + \frac{-68}{12}$$

$$= \frac{-84+11-68}{12} = \frac{-141}{12} = \frac{-47}{4}$$

(ii) $\frac{-6}{7} + \frac{-5}{6} + \frac{-4}{9} + \frac{-15}{7}$

$$= \left(\frac{-6}{7} + \frac{-15}{7} \right) + \frac{-5}{6} + \frac{-4}{9}$$

$$= \frac{-6-15}{7} + \frac{-5}{6} + \frac{-4}{9}$$

$$= \frac{-21}{7} + \frac{-5}{6} + \frac{-4}{9}$$

$$= \frac{-21 \times 18}{7 \times 18} + \frac{-5 \times 21}{6 \times 21} + \frac{-4 \times 14}{9 \times 14}$$

$$= \frac{-378}{126} + \frac{-105}{126} + \frac{-56}{126}$$

$$= \frac{-378-105-56}{126} = \frac{-539}{126} = \frac{-77}{18}$$

(iii) $\frac{3}{5} + \frac{7}{3} + \frac{9}{5} + \frac{-13}{15} + \frac{-7}{3}$

$$= \left(\frac{3}{5} + \frac{9}{5} \right) + \frac{7}{3} + \frac{-13}{15} + \frac{-7}{3}$$

$$= \frac{3+9}{5} + \frac{7}{3} + \frac{-13}{15} + \frac{-7}{3}$$

$$= \frac{12}{5} + \frac{7}{3} + \frac{-13}{15} + \frac{-7}{3}$$

$$= \frac{12 \times 3}{5 \times 3} + \frac{7 \times 5}{3 \times 5} + \frac{-13}{15} + \frac{-7 \times 5}{3 \times 5}$$

$$= \frac{36}{15} + \frac{35}{15} + \frac{-13}{15} + \frac{-35}{15} = \frac{36+35-13-35}{15} = \frac{23}{15}$$

(iv) $\frac{4}{13} + \frac{-5}{8} + \frac{-8}{13} + \frac{9}{13}$

$$= \left(\frac{4}{13} + \frac{9}{13} + \frac{-8}{13} \right) + \frac{-5}{8}$$

$$= \frac{4+9-8}{13} + \frac{-5}{8} = \frac{5}{13} + \frac{-5}{8}$$

$$= \frac{5 \times 8}{13 \times 8} + \frac{-5 \times 13}{8 \times 13}$$

$$= \frac{40}{104} + \frac{-65}{104} = \frac{40-65}{104} = \frac{-25}{104}$$

(v) $\frac{2}{3} + \frac{-4}{5} + \frac{1}{3} + \frac{2}{5}$

$$= \left(\frac{2}{3} + \frac{1}{3} \right) + \left(\frac{2}{5} + \frac{-4}{5} \right)$$

$$= \frac{2+1}{3} + \frac{2-4}{5}$$

$$= \frac{3}{3} + \frac{-2}{5}$$

$$= \frac{3 \times 5}{3 \times 5} + \frac{-2 \times 3}{5 \times 3}$$

$$= \frac{15}{15} + \frac{-6}{15}$$

$$= \frac{15-6}{15} = \frac{9}{15} = \frac{3}{5}$$

$$\text{(vi)} \quad \frac{1}{8} + \frac{5}{12} + \frac{2}{7} + \frac{7}{12} + \frac{9}{7} + \frac{-5}{16}$$

$$= \frac{1}{8} + \left(\frac{5}{12} + \frac{7}{12} \right) + \left(\frac{2}{7} + \frac{9}{7} \right) + \frac{-5}{16}$$

$$= \frac{1}{8} + \frac{5+7}{12} + \frac{2+9}{7} + \frac{-5}{16}$$

$$= \frac{1}{8} + \frac{12}{12} + \frac{11}{7} + \frac{-5}{16}$$

$$= \frac{1 \times 42}{8 \times 42} + \frac{12 \times 28}{12 \times 28} + \frac{11 \times 48}{7 \times 48} + \frac{-5 \times 21}{16 \times 21}$$

$$= \frac{42}{336} + \frac{336}{336} + \frac{528}{336} + \frac{-105}{336}$$

$$= \frac{42+336+528-105}{336} = \frac{801}{336} = \frac{267}{112}$$

Chapter 1: Rational Numbers

Exercise 1.3

Q-1. Subtract the first rational number from the second in each of the following:

(i) $\frac{3}{8}, \frac{5}{8}$

(ii) $\frac{-7}{9}, \frac{4}{9}$

(iii) $\frac{-2}{11}, \frac{-9}{11}$

(iv) $\frac{11}{13}, \frac{-4}{13}$

(v) $\frac{1}{4}, \frac{-3}{8}$

(vi) $\frac{-2}{3}, \frac{5}{6}$

$$(vii) \frac{-6}{7}, \frac{-13}{14}$$

$$(viii) \frac{-8}{33}, \frac{-7}{22}$$

Solution:

$$(i) \frac{3}{8}, \frac{5}{8}$$

$$= \frac{5}{8} - \frac{3}{8}$$

$$= \frac{5-3}{8} = \frac{2}{8} = \frac{1}{4}$$

$$(ii) \frac{-7}{9}, \frac{4}{9}$$

$$= \frac{4}{9} - \frac{-7}{9}$$

$$= \frac{4+7}{9} = \frac{11}{9}$$

$$(iii) \frac{-2}{11}, \frac{-9}{11}$$

$$= \frac{-9}{11} - \frac{-2}{11} = \frac{-9+2}{11} = \frac{-7}{11}$$

$$(iv) \frac{11}{13}, \frac{-4}{13}$$

$$= \frac{-4}{13} - \frac{11}{13} = \frac{-4-11}{13} = \frac{-15}{13}$$

$$(v) \frac{1}{4}, \frac{-3}{8}$$

$$= \frac{-3}{8} - \frac{1}{4}$$

$$= \frac{-3}{8} - \frac{1 \times 2}{4 \times 2}$$

$$= \frac{-3}{8} - \frac{2}{8} = \frac{-3-2}{8} = \frac{-5}{8}$$

$$(vi) \frac{-2}{3}, \frac{5}{6}$$

$$= \frac{5}{6} - \frac{-2}{3}$$

$$= \frac{5}{6} - \frac{-2 \times 2}{3 \times 2}$$

$$= \frac{5}{6} - \frac{-4}{6} = \frac{5+4}{6} = \frac{9}{6} = \frac{3}{2}$$

$$(vii) \frac{-6}{7}, \frac{-13}{14}$$

$$= \frac{-13}{14} - \frac{-6}{7}$$

$$= \frac{-13}{14} - \frac{-6 \times 2}{7 \times 2}$$

$$= \frac{-13}{14} - \frac{-12}{14} = \frac{-13+12}{14} = \frac{-1}{14}$$

$$(viii) \frac{-8}{33}, \frac{-7}{22}$$

$$= \frac{-7}{22} - \frac{-8}{33}$$

$$= \frac{-7 \times 3}{22 \times 3} - \frac{-8 \times 2}{33 \times 2}$$

$$= \frac{-21}{66} - \frac{-16}{66} = \frac{-21+16}{66} = \frac{-5}{66}$$

Q-2. Evaluate each of the following:

$$(i) \frac{2}{3} - \frac{3}{5}$$

$$(ii) \frac{-4}{7} - \frac{2}{-3}$$

$$(iii) \frac{4}{7} - \frac{-5}{-7}$$

$$(iv) \frac{-2}{1} - \frac{5}{9}$$

$$(v) \frac{-3}{-8} - \frac{-2}{7}$$

$$(vi) \frac{-4}{13} - \frac{-5}{26}$$

$$(vii) \frac{-5}{14} - \frac{-2}{7}$$

$$(viii) \frac{13}{15} - \frac{12}{25}$$

$$(ix) \frac{-6}{13} - \frac{-7}{13}$$

$$(x) \frac{7}{24} - \frac{19}{36}$$

$$(xi) \frac{5}{63} - \frac{-8}{21}$$

Solution:

$$\begin{aligned} (i) \quad & \frac{2}{3} - \frac{3}{5} \\ &= \frac{2 \times 5}{3 \times 5} - \frac{3 \times 3}{5 \times 3} \\ &= \frac{10}{15} - \frac{9}{15} \\ &= \frac{10-9}{15} = \frac{1}{15} \end{aligned}$$

$$\begin{aligned} (ii) \quad & \frac{-4}{7} - \frac{2}{-3} \\ &= \frac{-4 \times 3}{7 \times 3} - \frac{-2 \times 7}{3 \times 7} \\ &= \frac{-12}{21} - \frac{-14}{21} \\ &= \frac{-12+14}{21} = \frac{2}{21} \end{aligned}$$

$$\begin{aligned} (iii) \quad & \frac{4}{7} - \frac{-5}{-7} \\ &= \frac{4}{7} - \frac{5}{7} \\ &= \frac{4-5}{7} = \frac{-1}{7} \end{aligned}$$

$$\begin{aligned} (iv) \quad & \frac{-2}{1} - \frac{5}{9} \\ &= \frac{-2 \times 9}{1 \times 9} - \frac{5}{9} \\ &= \frac{-18}{9} - \frac{5}{9} \\ &= \frac{-18-5}{9} = \frac{-23}{9} \end{aligned}$$

$$\begin{aligned} (v) \quad & \frac{-3}{-8} - \frac{-2}{7} \\ &= \frac{3 \times 7}{8 \times 7} - \frac{-2 \times 8}{7 \times 8} \\ &= \frac{21}{56} - \frac{-16}{56} \\ &= \frac{21+16}{56} = \frac{37}{56} \end{aligned}$$

$$\begin{aligned} (vi) \quad & \frac{-4}{13} - \frac{-5}{26} \\ &= \frac{-4 \times 2}{13 \times 2} - \frac{-5}{26} \\ &= \frac{-8}{26} - \frac{-5}{26} \\ &= \frac{-8+5}{26} = \frac{-3}{26} \end{aligned}$$

$$\begin{aligned} (vii) \quad & \frac{-5}{14} - \frac{-2}{7} \\ &= \frac{-5}{14} - \frac{-2 \times 2}{7 \times 2} \\ &= \frac{-5}{14} - \frac{-4}{14} \\ &= \frac{-5+4}{14} = \frac{-1}{14} \end{aligned}$$

$$\begin{aligned} (viii) \quad & \frac{13}{15} - \frac{12}{25} \\ &= \frac{13 \times 5}{15 \times 5} - \frac{12 \times 3}{12 \times 3} \\ &= \frac{65}{75} - \frac{36}{75} \\ &= \frac{65-36}{75} = \frac{29}{75} \end{aligned}$$

$$\begin{aligned} (ix) \quad & \frac{-6}{13} - \frac{-7}{13} \\ &= \frac{-6+7}{13} = \frac{1}{13} \end{aligned}$$

$$\begin{aligned} (x) \quad & \frac{7}{24} - \frac{19}{36} \\ &= \frac{7 \times 3}{24 \times 3} - \frac{19 \times 2}{36 \times 2} \\ &= \frac{21}{72} - \frac{38}{72} \\ &= \frac{21-38}{72} = \frac{-17}{72} \end{aligned}$$

$$(xi) \frac{5}{63} - \frac{-8}{21}$$

$$\begin{aligned}
 &= \frac{5}{63} - \frac{-8 \times 3}{21 \times 3} \\
 &= \frac{5}{63} - \frac{-24}{63} \\
 &= \frac{5+24}{63} = \frac{29}{63}
 \end{aligned}$$

Q-3. The sum of the two numbers is $\frac{5}{9}$. If one of the numbers is $\frac{1}{3}$, Find the others.

Solution:

It is given that:

The sum of the two numbers is $\frac{5}{9}$.

One of the number is $\frac{1}{3}$

Let, the other number be x.

$$\therefore x + \frac{1}{3} = \frac{5}{9}$$

$$\Rightarrow x = \frac{5}{9} - \frac{1}{3}$$

$$\Rightarrow x = \frac{5}{9} - \frac{1 \times 3}{3 \times 3}$$

$$\Rightarrow x = \frac{5-3}{9}$$

$$\Rightarrow x = \frac{2}{9}$$

Hence, the other number is $\frac{2}{9}$.

Q-4. The sum of the two numbers is $\frac{-1}{3}$. If one of the number is $\frac{-12}{3}$, Find the others.

Solution: It is given that:

The sum of the two numbers is $\frac{-1}{3}$.

One of the number is $\frac{-12}{3}$

Let, the other number be x.

$$\therefore x + \frac{-12}{3} = \frac{-1}{3}$$

$$\Rightarrow x = \frac{-1}{3} - \frac{-12}{3}$$

$$\Rightarrow x = \frac{-1+12}{3}$$

$$\Rightarrow x = \frac{11}{3}$$

Hence, the other number is $\frac{11}{3}$.

Q-5. The sum of the two numbers is $\frac{-4}{3}$. If one of the number is -5, Find the others.

Solution: It is given that:

The sum of the two numbers is $\frac{-4}{3}$.

One of the number is -5

Let, the other number be x.

$$\therefore x + (-5) = \frac{-4}{3}$$

$$\Rightarrow x = 5 - \frac{-4}{3}$$

$$\Rightarrow x = \frac{-4}{3} - \frac{5 \times 3}{1 \times 3}$$

$$\Rightarrow x = \frac{-4+15}{3}$$

$$\Rightarrow x = \frac{11}{3}$$

Hence, the other number is $\frac{11}{3}$.

Q-6. The sum of the two rational numbers is -8. If one of the number is $\frac{-15}{7}$, Find the others.

Solution: It is given that:

The sum of the two numbers is -8.

One of the number is $\frac{-15}{7}$

Let, the other number be x.

$$\therefore x + \frac{-15}{7} = (-8)$$

$$\Rightarrow x = -8 - \frac{-15}{7}$$

$$\Rightarrow x = \frac{-8 \times 7}{1 \times 7} - \frac{-15}{7}$$

$$\Rightarrow x = \frac{-56+15}{7}$$

$$\Rightarrow x = \frac{-41}{7}$$

Hence, the other number is $\frac{-41}{7}$.

Q-7. What should be added to $\frac{-7}{8}$ so as to get $\frac{5}{9}$?

Solution:

It is given that:

The sum of the two numbers is $\frac{5}{9}$

One of the number is $\frac{-7}{8}$

Let, the other number be x.

$$\therefore x + \frac{-7}{8} = \frac{5}{9}$$

$$\Rightarrow x = \frac{5}{9} - \frac{-7}{8}$$

$$\Rightarrow x = \frac{5 \times 8}{9 \times 8} - \frac{-7 \times 9}{8 \times 9}$$

$$\Rightarrow x = \frac{40}{72} - \frac{-63}{72}$$

$$\Rightarrow x = \frac{40+63}{72}$$

$$\Rightarrow x = \frac{103}{72}$$

Hence, the other number is $\frac{103}{72}$.

Q-8. What number should be added to $\frac{-5}{11}$ so as to get $\frac{26}{33}$?

Solution:

It is given that:

The sum of the two numbers is $\frac{26}{33}$

One of the number is $\frac{-5}{11}$

Let, the other number be x.

$$\therefore x + \frac{-5}{11} = \frac{26}{33}$$

$$\Rightarrow x = \frac{26}{33} - \frac{-5}{11}$$

$$\Rightarrow x = \frac{26}{33} - \frac{-5 \times 3}{11 \times 3}$$

$$\Rightarrow x = \frac{26}{33} - \frac{-15}{33}$$

$$\Rightarrow x = \frac{26+15}{33}$$

$$\Rightarrow x = \frac{41}{33}$$

Hence, the other number is $\frac{41}{33}$.

Q-9. What number should be added to $\frac{-5}{7}$ to get $\frac{-2}{3}$?

Solution:

It is given that:

The sum of the two numbers is $\frac{-2}{3}$

One of the number is $\frac{-5}{7}$

Let, the other number be x.

$$\begin{aligned}\therefore x + \frac{-5}{7} &= \frac{-2}{3} \\ \Rightarrow x &= \frac{-2}{3} - \frac{-5}{7} \\ \Rightarrow x &= \frac{-2 \times 7}{3 \times 7} - \frac{-5 \times 3}{7 \times 3} \\ \Rightarrow x &= \frac{-14}{21} - \frac{-15}{21} \\ \Rightarrow x &= \frac{-14+15}{21} \\ \Rightarrow x &= \frac{1}{21}\end{aligned}$$

Hence, the other number is $\frac{1}{21}$.

Q-10. What number should be subtracted from $\frac{-5}{3}$ to get $\frac{5}{6}$?

Solution:

It is given that:

The sum of the two numbers is $\frac{5}{6}$

One of the number is $\frac{-5}{3}$

Let, the other number be x.

$$\begin{aligned}\therefore x + \frac{-5}{3} &= \frac{5}{6} \\ \Rightarrow x &= \frac{5}{6} - \frac{-5}{3} \\ \Rightarrow x &= \frac{5}{6} - \frac{-5 \times 2}{3 \times 2} \\ \Rightarrow x &= \frac{5}{6} - \frac{-63}{72} \\ \Rightarrow x &= \frac{40+63}{72} \\ \Rightarrow x &= \frac{103}{72}\end{aligned}$$

Hence, the other number is $\frac{103}{72}$.

Q-11. What number must be subtracted from $\frac{3}{7}$ to get $\frac{5}{4}$?

Solution:

It is given that:

The sum of the two numbers is $\frac{5}{4}$

One of the number is $\frac{3}{7}$

Let, the other number be x.

$$\begin{aligned}\therefore \frac{3}{7} - x &= \frac{5}{4} \\ \Rightarrow x &= \frac{3}{7} - \frac{5}{4} \\ \Rightarrow x &= \frac{3 \times 4}{7 \times 4} - \frac{5 \times 7}{4 \times 7} \\ \Rightarrow x &= \frac{12}{28} - \frac{35}{28} \\ \Rightarrow x &= \frac{12-35}{28} \\ \Rightarrow x &= \frac{-23}{28}\end{aligned}$$

Hence, the other number is $\frac{-23}{28}$.

Q-12. What should be added to $\left(\frac{2}{3} + \frac{3}{5}\right)$ to get $\frac{-2}{15}$?

Solution:

It is given that:

The sum of the numbers is $\frac{-2}{15}$

One of the number is $\left(\frac{2}{3} + \frac{3}{5}\right)$

Let, the other number be x.

$$\therefore \frac{2}{3} + \frac{3}{5} + x = \frac{-2}{15}$$

$$\Rightarrow x = \frac{-2}{15} - \frac{2}{3} - \frac{3}{5}$$

$$\Rightarrow x = \frac{-2}{15} - \frac{2 \times 5}{3 \times 5} - \frac{3 \times 3}{5 \times 3}$$

$$\Rightarrow x = \frac{-2}{15} - \frac{10}{15} - \frac{9}{15}$$

$$\Rightarrow x = \frac{-2-10-9}{15}$$

$$\Rightarrow x = \frac{-21}{15}$$

$$\Rightarrow x = \frac{-7}{5}$$

Hence, the other number is $\frac{-7}{5}$.

Q-13. What should be added to $\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5}\right)$ to get 3?

Solution:

It is given that:

The sum of the numbers is 3

One of the number is $\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5}\right)$

Let, the other number be x.

$$\therefore \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + x = 3$$

$$\Rightarrow x = 3 - \frac{1}{2} - \frac{1}{3} - \frac{1}{5}$$

$$\Rightarrow x = \frac{3 \times 30}{1 \times 30} - \frac{1 \times 15}{2 \times 15} - \frac{1 \times 10}{3 \times 10} - \frac{1 \times 6}{5 \times 6}$$

$$\Rightarrow x = \frac{90}{30} - \frac{15}{30} - \frac{10}{30} - \frac{6}{30}$$

$$\Rightarrow x = \frac{90-15-10-6}{30}$$

$$\Rightarrow x = \frac{59}{30}$$

Hence, the other number is $\frac{59}{30}$.

Q-14. What should be subtracted from $\left(\frac{3}{4} - \frac{2}{3}\right)$ to get $\frac{-1}{6}$?

Solution:

It is given that:

The numbers is $\frac{-1}{6}$

One of the number is $\left(\frac{3}{4} - \frac{2}{3}\right)$

Let, the other number be x.

$$\therefore \frac{3}{4} - \frac{2}{3} - x = \frac{-1}{6}$$

$$\Rightarrow x = \frac{3}{4} - \frac{2}{3} - \frac{-1}{6}$$

$$\Rightarrow x = \frac{3 \times 3}{4 \times 3} - \frac{2 \times 4}{3 \times 4} - \frac{-1 \times 2}{6 \times 2}$$

$$\Rightarrow x = \frac{9}{12} - \frac{8}{12} + \frac{2}{12}$$

$$\Rightarrow x = \frac{2-8+9}{12}$$

$$\Rightarrow x = \frac{3}{12}$$

$$\Rightarrow x = \frac{1}{4}$$

Hence, the other number is $\frac{1}{4}$.

Q-15. Fill in the blanks:

$$(i) \frac{-4}{13} - \frac{-3}{26} = \dots\dots\dots$$

$$(ii) \frac{-9}{14} + \dots\dots\dots = -1$$

$$(iii) \frac{-7}{9} + \dots\dots\dots = 3$$

$$(iv) \dots\dots\dots + \frac{15}{23} = 4$$

Solution:

$$(i) \frac{-4}{13} - \frac{-3}{26} = \dots\dots\dots$$

Let the required number be x.

$$\frac{-4}{13} - \frac{-3}{26} = x$$

$$\Rightarrow \frac{-4 \times 2}{13 \times 2} - \frac{-3}{26} = x$$

$$\Rightarrow \frac{-8}{26} - \frac{-3}{26} = x$$

$$\Rightarrow x = \frac{-8+3}{26}$$

$$\Rightarrow x = \frac{-5}{26}$$

$$\frac{-4}{13} - \frac{-3}{26} = \frac{-5}{26}$$

$$(ii) \frac{-9}{14} + \dots\dots\dots = -1$$

Let, the required number be x.

$$\frac{-9}{14} + x = -1$$

$$\Rightarrow x = -1 - \frac{-9}{14}$$

$$\Rightarrow x = \frac{-1 \times 14}{1 \times 14} - \frac{-9}{14}$$

$$\Rightarrow x = \frac{-14+9}{14}$$

$$\Rightarrow x = \frac{-5}{14}$$

$$\frac{-9}{14} + \frac{-5}{14} = -1$$

$$(iii) \frac{-7}{9} + \dots\dots\dots = 3$$

$$\Rightarrow \frac{-7}{9} + x = 3$$

$$\Rightarrow x = 3 - \frac{-7}{9}$$

$$\Rightarrow x = \frac{3 \times 9}{1 \times 9} - \frac{-7}{9}$$

$$\Rightarrow x = \frac{27}{9} - \frac{-7}{9}$$

$$\Rightarrow x = \frac{27+7}{9}$$

$$\Rightarrow x = \frac{34}{9}$$

$$\text{Hence, } \frac{-7}{9} + \frac{34}{9} = 3$$

$$(iv) \dots\dots\dots + \frac{15}{13} = 4$$

Let, the required number be x.

$$\Rightarrow x + \frac{15}{13} = 4$$

$$\Rightarrow x = 4 - \frac{15}{13}$$

$$\Rightarrow x = \frac{4 \times 13}{1 \times 13} - \frac{15}{13}$$

$$\Rightarrow x = \frac{52}{13} - \frac{15}{13}$$

$$\Rightarrow x = \frac{52-15}{13}$$

$$\Rightarrow x = \frac{37}{13}$$

$$\text{Hence, } \frac{37}{13} + \frac{15}{13} = 4$$

Chapter 1: Rational Numbers

Exercise 1.4

Q-1. Simplify each of the following and write as a rational number of the form $\frac{p}{q}$:

(i) $\frac{3}{4} + \frac{5}{6} + \frac{-7}{8}$

(ii) $\frac{2}{3} + \frac{-5}{6} + \frac{-7}{9}$

(iii) $\frac{-11}{2} + \frac{7}{6} + \frac{-5}{8}$

(iv) $\frac{-4}{5} + \frac{-7}{10} + \frac{-8}{15}$

(v) $\frac{-9}{10} + \frac{22}{15} + \frac{13}{-20}$

(vi) $\frac{5}{3} + \frac{3}{-2} + \frac{-7}{3} + 3$

Solution:

$$(i) \frac{3}{4} + \frac{5}{6} + \frac{-7}{8}$$

$$= \frac{3}{4} + \frac{5}{6} - \frac{7}{8}$$

$$= \frac{3 \times 6}{4 \times 6} + \frac{5 \times 4}{6 \times 4} - \frac{7 \times 3}{8 \times 3}$$

$$= \frac{18}{24} + \frac{20}{24} - \frac{21}{24}$$

$$= \frac{18+20-21}{24}$$

$$= \frac{17}{24}$$

$$(ii) \frac{2}{3} + \frac{-5}{6} + \frac{-7}{9}$$

$$= \frac{2}{3} + \frac{-5}{6} - \frac{7}{9}$$

$$= \frac{2 \times 6}{3 \times 6} - \frac{5 \times 3}{6 \times 3} - \frac{7 \times 2}{9 \times 2}$$

$$= \frac{12}{18} - \frac{15}{18} - \frac{14}{18}$$

$$= \frac{12-15-14}{18} = \frac{-17}{18}$$

$$(iii) \frac{-11}{2} + \frac{7}{6} + \frac{-5}{8}$$

$$= \frac{-11}{2} + \frac{7}{6} - \frac{5}{8}$$

$$= \frac{-11 \times 12}{2 \times 12} + \frac{7 \times 4}{6 \times 4} - \frac{5 \times 3}{8 \times 3}$$

$$= \frac{-132}{24} + \frac{28}{24} - \frac{15}{24}$$

$$= \frac{-132+28-15}{24} = \frac{-119}{24}$$

$$(iv) \frac{-4}{5} + \frac{-7}{10} + \frac{-8}{15}$$

$$= \frac{-4}{5} - \frac{7}{10} - \frac{8}{15}$$

$$= \frac{-4 \times 6}{5 \times 6} - \frac{7 \times 3}{10 \times 3} - \frac{8 \times 2}{15 \times 2}$$

$$= \frac{-24}{30} - \frac{21}{30} - \frac{16}{30}$$

$$= \frac{-24-21-16}{30} = \frac{-61}{30}$$

$$(v) \frac{-9}{10} + \frac{22}{15} + \frac{13}{-20}$$

$$= \frac{-9}{10} + \frac{22}{15} - \frac{13}{20}$$

$$= \frac{-9 \times 6}{10 \times 6} + \frac{22 \times 4}{15 \times 4} - \frac{13 \times 3}{20 \times 3}$$

$$= \frac{-54}{60} + \frac{88}{60} - \frac{39}{60}$$

$$= \frac{-54+88-39}{60} = \frac{-5}{60} = \frac{-1}{60}$$

$$(vi) \frac{5}{3} + \frac{3}{-2} + \frac{-7}{3} + 3$$

$$= \frac{5}{3} - \frac{3}{2} - \frac{7}{3} + \frac{3}{1}$$

$$= \frac{5 \times 2}{3 \times 2} - \frac{3 \times 3}{2 \times 3} - \frac{7 \times 2}{3 \times 2} + \frac{3 \times 6}{1 \times 6}$$

$$= \frac{10}{6} - \frac{9}{6} - \frac{21}{6} + \frac{9}{3}$$

$$= \frac{10-9-21+18}{6} = \frac{5}{6}$$

Q-2. Express each of the following as a rational number of the form $\frac{p}{q}$:

$$(i) \frac{-8}{3} + \frac{-1}{4} + \frac{-11}{6} + \frac{3}{8} - 3$$

$$(ii) \frac{6}{7} + 1 + \frac{-7}{9} + \frac{19}{21} + \frac{-12}{7}$$

$$(iii) \frac{15}{2} + \frac{9}{8} + \frac{-11}{3} + 6 + \frac{-7}{6}$$

$$(iv) \frac{-7}{4} + 0 + \frac{-9}{5} + \frac{19}{10} + \frac{11}{14}$$

$$(v) \frac{-7}{4} + \frac{5}{3} + \frac{-1}{2} + \frac{-5}{6} + 2$$

Solution:

$$(i) \frac{-8}{3} + \frac{-1}{4} + \frac{-11}{6} + \frac{3}{8} - 3$$

$$\begin{aligned}
&= \frac{-8}{3} - \frac{1}{4} - \frac{11}{6} + \frac{3}{8} - \frac{3}{1} \\
&= \left(\frac{-8}{3} - \frac{11}{6} \right) - \left(\frac{1}{4} - \frac{3}{8} \right) - \frac{3}{1} \\
&= \left(\frac{-8 \times 2}{3 \times 2} - \frac{11}{6} \right) - \left(\frac{1 \times 2}{4 \times 2} - \frac{3}{8} \right) - \frac{3}{1} \\
&= \left(\frac{-16}{6} - \frac{11}{6} \right) - \left(\frac{2}{8} - \frac{3}{8} \right) - \frac{3}{1} \\
&= \frac{-27}{6} - \frac{-1}{8} - \frac{3}{1} \\
&= \frac{-27 \times 4}{6 \times 4} - \frac{-1 \times 3}{8 \times 3} - \frac{3 \times 24}{1 \times 24} \\
&= \frac{-108}{24} + \frac{3}{24} - \frac{72}{24} \\
&= \frac{-108+3-72}{24} \\
&= \frac{-177}{24} = \frac{-59}{8}
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad &\frac{6}{7} + 1 + \frac{-7}{9} + \frac{19}{21} + \frac{-12}{7} \\
&= \frac{6}{7} + \frac{1}{1} - \frac{7}{9} + \frac{19}{21} - \frac{12}{7} \\
&= \left(\frac{6}{7} + \frac{19}{21} - \frac{12}{7} \right) + \frac{1}{1} - \frac{7}{9} \\
&= \left(\frac{6 \times 3}{7 \times 3} + \frac{19}{21} - \frac{12 \times 3}{7 \times 3} \right) + \frac{1}{1} - \frac{7}{9} \\
&= \left(\frac{18}{21} + \frac{19}{21} - \frac{36}{21} \right) + \frac{1}{1} - \frac{7}{9} \\
&= \left(\frac{18+19-36}{21} \right) + \frac{1}{1} - \frac{7}{9} \\
&= \frac{1}{21} + \frac{1}{1} - \frac{7}{9} \\
&= \frac{1 \times 3}{21 \times 3} + \frac{1 \times 63}{1 \times 63} - \frac{7 \times 7}{9 \times 7} \\
&= \frac{3}{63} + \frac{63}{63} - \frac{49}{63} \\
&= \frac{3+63-49}{63} = \frac{17}{63}
\end{aligned}$$

$$\begin{aligned}
\text{(iii)} \quad &\frac{15}{2} + \frac{9}{8} + \frac{-11}{3} + 6 + \frac{-7}{6} \\
&= \frac{15}{2} + \frac{9}{8} - \frac{11}{3} + \frac{6}{1} - \frac{7}{6} \\
&= \left(\frac{15}{2} + \frac{9}{8} \right) - \left(\frac{11}{3} + \frac{7}{6} \right) + \frac{6}{1} \\
&= \left(\frac{15 \times 4}{2 \times 4} + \frac{9}{8} \right) - \left(\frac{11 \times 2}{3 \times 2} + \frac{7}{6} \right) + \frac{6}{1} \\
&= \left(\frac{60}{8} + \frac{9}{8} \right) - \left(\frac{22}{6} + \frac{7}{6} \right) + \frac{6}{1} \\
&= \frac{60+9}{8} - \frac{22+7}{6} + \frac{6}{1} \\
&= \frac{69}{8} - \frac{29}{6} + \frac{6}{1} \\
&= \frac{69 \times 3}{8 \times 3} - \frac{29 \times 4}{6 \times 4} + \frac{6 \times 24}{1 \times 24} \\
&= \frac{207}{24} - \frac{116}{24} + \frac{144}{24} \\
&= \frac{207-116+144}{24} = \frac{235}{24}
\end{aligned}$$

$$\begin{aligned}
\text{(iv)} \quad &\frac{-7}{4} + 0 + \frac{-9}{5} + \frac{19}{10} + \frac{11}{14} \\
&= \frac{-7}{4} - \frac{9}{5} + \frac{19}{10} + \frac{11}{14} \\
&= \frac{-7}{4} + \frac{11}{14} - \left(\frac{9}{5} + \frac{19}{10} \right) \\
&= \left(\frac{-7}{4} + \frac{11}{14} \right) - \left(\frac{9 \times 2}{5 \times 2} + \frac{19}{10} \right) \\
&= \frac{-7}{4} + \frac{11}{14} - \left(\frac{18}{10} + \frac{19}{10} \right) \\
&= \frac{-7}{4} + \frac{11}{14} - \frac{18+19}{10} \\
&= \frac{-7}{4} + \frac{11}{14} - \frac{37}{10} \\
&= \frac{-7 \times 35}{4 \times 35} + \frac{11 \times 10}{14 \times 10} - \frac{37 \times 14}{10 \times 14} \\
&= \frac{245}{140} + \frac{110}{140} - \frac{518}{140}
\end{aligned}$$

$$\begin{aligned}
&= \frac{245+110-518}{140} = \frac{-121}{24} \\
\text{(v)} \quad &\frac{-7}{4} + \frac{5}{3} + \frac{-1}{2} + \frac{-5}{6} + 2 \\
&= \frac{-7}{4} + \frac{5}{3} - \frac{1}{2} - \frac{5}{6} + \frac{2}{1} \\
&= \left(\frac{-7}{4} - \frac{1}{2} \right) + \left(\frac{5}{3} - \frac{5}{6} \right) + \frac{2}{1} \\
&= \left(\frac{-7}{4} - \frac{1 \times 2}{2 \times 2} \right) + \left(\frac{5 \times 2}{3 \times 2} - \frac{5}{6} \right) + \frac{2}{1} \\
&= \left(\frac{-7}{4} - \frac{2}{4} \right) + \left(\frac{10}{6} - \frac{5}{6} \right) + \frac{2}{1} \\
&= \frac{-7-2}{4} + \frac{10-5}{6} + \frac{2}{1} \\
&= \frac{-9}{4} + \frac{5}{6} + \frac{2}{1} \\
&= \frac{-9 \times 3}{4 \times 3} + \frac{5 \times 2}{6 \times 2} + \frac{2 \times 12}{1 \times 12} \\
&= \frac{-27}{12} + \frac{10}{12} + \frac{24}{12} \\
&= \frac{-27+10+24}{12} = \frac{7}{12}
\end{aligned}$$

Q-3. Simplify:

$$\begin{aligned}
\text{(i)} \quad &\frac{-3}{2} + \frac{5}{4} - \frac{7}{4} \\
\text{(ii)} \quad &\frac{5}{3} - \frac{7}{6} + \frac{-2}{3} \\
\text{(iii)} \quad &\frac{5}{4} - \frac{7}{6} - \frac{-2}{3} \\
\text{(iv)} \quad &\frac{-2}{5} - \frac{-3}{10} - \frac{-4}{7} \\
\text{(v)} \quad &\frac{5}{6} + \frac{-2}{5} - \frac{-2}{15} \\
\text{(vi)} \quad &\frac{3}{8} - \frac{-2}{9} + \frac{-5}{36}
\end{aligned}$$

Solution:

$$\text{(i)} \quad \frac{-3}{2} + \frac{5}{4} - \frac{7}{4}$$

Taking the LCM of the denominators:

$$\begin{aligned}
&\frac{-3 \times 2}{2 \times 2} + \frac{5}{4} - \frac{7}{4} \\
&= \frac{-6}{4} + \frac{5}{4} - \frac{7}{4} \\
&= \frac{-6+5-7}{4} \\
&= \frac{-8}{4} = -2
\end{aligned}$$

$$\text{(ii)} \quad \frac{5}{3} - \frac{7}{6} + \frac{-2}{3}$$

Taking the LCM of the denominators:

$$\begin{aligned}
&\frac{5 \times 2}{3 \times 2} - \frac{7}{6} + \frac{-2 \times 2}{3 \times 2} \\
&= \frac{10}{6} - \frac{7}{6} - \frac{4}{6} \\
&= \frac{10-7-4}{6} = \frac{-1}{6}
\end{aligned}$$

$$\text{(iii)} \quad \frac{5}{4} - \frac{7}{6} - \frac{-2}{3}$$

Taking the LCM of the denominators:

$$\begin{aligned}
&\frac{5 \times 3}{4 \times 3} - \frac{7 \times 2}{6 \times 2} + \frac{2 \times 4}{3 \times 4} \\
&= \frac{15}{12} - \frac{14}{12} + \frac{8}{12} \\
&= \frac{15-14+8}{12} = \frac{-5}{12}
\end{aligned}$$

$$\text{(iv)} \quad \frac{-2}{5} - \frac{-3}{10} - \frac{-4}{7}$$

Taking the LCM of the denominators:

$$\begin{aligned}
&\frac{-2 \times 14}{5 \times 14} + \frac{3 \times 7}{10 \times 7} + \frac{4 \times 10}{7 \times 10} \\
&= \frac{-28}{70} + \frac{21}{70} + \frac{40}{70}
\end{aligned}$$

$$= \frac{-28+21+40}{70} = \frac{33}{70}$$

$$(v) \frac{5}{6} + \frac{-2}{5} - \frac{-2}{15}$$

Taking the LCM of the denominators:

$$\frac{5 \times 5}{6 \times 5} - \frac{2 \times 6}{5 \times 6} + \frac{2 \times 2}{15 \times 2}$$

$$= \frac{25}{30} - \frac{12}{30} + \frac{4}{30}$$

$$= \frac{25-12+4}{30} = \frac{17}{30}$$

$$(vi) \frac{3}{8} - \frac{-2}{9} + \frac{-5}{36}$$

Taking the LCM of the denominators:

$$\frac{3 \times 9}{8 \times 9} + \frac{2 \times 8}{9 \times 6} - \frac{5 \times 2}{36 \times 2}$$

$$= \frac{27}{72} + \frac{16}{72} - \frac{10}{72}$$

$$= \frac{27+16-10}{72}$$

$$= \frac{33}{72} = \frac{11}{24}$$

Chapter 1: Rational Numbers

Exercise 1.5

Q-1. Multiply:

(i) $\frac{7}{11}$ *by* $\frac{5}{4}$

(ii) $\frac{5}{7}$ *by* $\frac{-3}{4}$

(iii) $\frac{-2}{9}$ *by* $\frac{5}{11}$

(iv) $\frac{-3}{17}$ *by* $\frac{-5}{-4}$

(v) $\frac{9}{-7}$ *by* $\frac{36}{-11}$

(vi) $\frac{-11}{13}$ *by* $\frac{-21}{7}$

$$(vii) \frac{-3}{5} \text{ by } \frac{-4}{7}$$

$$(viii) \frac{-15}{11} \text{ by } 7$$

Solution:

$$(i) \frac{7}{11} \times \frac{5}{4} = \frac{7 \times 5}{11 \times 4} = \frac{35}{44}$$

$$(ii) \frac{5}{7} \times \frac{-3}{4} = \frac{5 \times -3}{7 \times 4} = \frac{-15}{28}$$

$$(iii) \frac{-2}{9} \times \frac{5}{11} = \frac{-2 \times 5}{11 \times 9} = \frac{-10}{99}$$

$$(iv) \frac{-3}{17} \times \frac{-5}{-4} = \frac{-3 \times -5}{17 \times -4} = \frac{15}{-68} = \frac{15}{-68}$$

$$(v) \frac{9}{-7} \times \frac{36}{-11} = \frac{9 \times 36}{-7 \times -11} = \frac{324}{77}$$

$$(vi) \frac{-11}{13} \times \frac{-21}{7} = \frac{-11 \times -21}{13 \times 7} = \frac{33}{13}$$

$$(vii) \frac{-3}{5} \times \frac{-4}{7} = \frac{-3 \times -4}{5 \times 7} = \frac{12}{35}$$

$$(viii) \frac{-15}{11} \times 7 = \frac{-15 \times 7}{11} = \frac{-105}{11}$$

Q-2. Multiply:

$$(i) \frac{-5}{17} \text{ by } \frac{51}{-60}$$

$$(ii) \frac{-6}{11} \text{ by } \frac{-55}{36}$$

$$(iii) \frac{-8}{25} \text{ by } \frac{-5}{16}$$

$$(iv) \frac{6}{7} \text{ by } \frac{-49}{36}$$

$$(v) \frac{8}{-9} \text{ by } \frac{-7}{-16}$$

$$(vi) \frac{-8}{9} \text{ by } \frac{3}{64}$$

Solution:

$$(i) \frac{-5}{17} \times \frac{51}{-60} = \frac{-5 \times 51}{17 \times -60} = \frac{1}{4}$$

$$(ii) \frac{-6}{11} \times \frac{-55}{36} = \frac{-6 \times -55}{11 \times 36} = \frac{5}{6}$$

$$(iii) \frac{-8}{25} \times \frac{-5}{16} = \frac{-8 \times -5}{25 \times 16} = \frac{1}{10}$$

$$(iv) \frac{6}{7} \times \frac{-49}{36} = \frac{6 \times -49}{7 \times 36} = \frac{-7}{6}$$

$$(v) \frac{8}{-9} \times \frac{-7}{-16} = \frac{8 \times -7}{-9 \times -16} = \frac{-7}{18}$$

$$(vi) \frac{-8}{9} \times \frac{3}{64} = \frac{-8 \times 3}{9 \times 64} = \frac{-1}{24}$$

Q-3. Simplify each of the following and express the result as a rational number in standard form:

$$(i) \frac{-16}{21} \times \frac{14}{5}$$

$$(ii) \frac{7}{6} \times \frac{-3}{28}$$

$$(iii) \frac{-19}{36} \times 16$$

$$(iv) \frac{-13}{9} \times \frac{27}{-26}$$

$$(v) \frac{-9}{16} \times \frac{-64}{-27}$$

$$(vi) \frac{-50}{7} \times \frac{14}{3}$$

$$(vii) \frac{-11}{9} \times \frac{-81}{-88}$$

$$(viii) \frac{-5}{9} \times \frac{72}{-25}$$

Solution:

$$(i) \frac{-16}{21} \times \frac{14}{5} = \frac{-2 \times 2 \times 2 \times 2}{3 \times 7} \times \frac{2 \times 7}{5} = \frac{-32}{15}$$

$$(ii) \frac{7}{6} \times \frac{-3}{28} = \frac{7}{2 \times 3} \times \frac{-3}{2 \times 2 \times 7} = \frac{-1}{8}$$

$$(iii) \frac{-19}{36} \times 16 = \frac{-19}{2 \times 2 \times 3 \times 3} \times 2 \times 2 \times 2 \times 2 = \frac{-76}{9}$$

$$(iv) \frac{-13}{9} \times \frac{27}{-26} = \frac{-13}{3 \times 3} \times \frac{3 \times 3 \times 3}{-2 \times 13} = \frac{-3}{2}$$

$$(v) \frac{-9}{16} \times \frac{-64}{27} = \frac{-9}{16} \times \frac{-4 \times 16}{-3 \times 9} = \frac{-4}{3}$$

$$(vi) \frac{-50}{7} \times \frac{14}{3} = \frac{-50}{7} \times \frac{2 \times 7}{3} = \frac{-100}{3}$$

$$(vii) \frac{-11}{9} \times \frac{-81}{-88} = \frac{-11}{3 \times 3} \times \frac{-9 \times 9}{-8 \times 11} = \frac{-9}{8}$$

$$(viii) \frac{-5}{9} \times \frac{72}{-25} = \frac{-5}{3 \times 3} \times \frac{8 \times 9}{-5 \times 5} = \frac{8}{5}$$

Q-4. Simplify:

$$(i) \left(\frac{25}{8} \times \frac{2}{5} \right) - \left(\frac{3}{5} \times \frac{-10}{9} \right)$$

$$(ii) \left(\frac{1}{2} \times \frac{1}{4} \right) + \left(\frac{1}{2} \times 6 \right)$$

$$(iii) \left(-5 \times \frac{2}{15} \right) - \left(-6 \times \frac{2}{9} \right)$$

$$(iv) \left(\frac{-9}{4} \times \frac{5}{3} \right) + \left(\frac{13}{2} \times \frac{5}{6} \right)$$

$$(v) \left(\frac{-4}{3} \times \frac{12}{-5} \right) + \left(\frac{3}{7} \times \frac{21}{15} \right)$$

$$(vi) \left(\frac{13}{5} \times \frac{8}{3} \right) - \left(\frac{-5}{2} \times \frac{11}{3} \right)$$

$$(vii) \left(\frac{13}{7} \times \frac{11}{26} \right) - \left(\frac{-4}{3} \times \frac{5}{6} \right)$$

$$(viii) \left(\frac{8}{5} \times \frac{-3}{2} \right) + \left(\frac{-3}{10} \times \frac{-11}{16} \right)$$

Solution:

$$(i) \left(\frac{25}{8} \times \frac{2}{5} \right) - \left(\frac{3}{5} \times \frac{-10}{9} \right)$$

$$= \frac{5}{4} - \frac{-2}{3}$$

$$= \frac{5 \times 3 + 2 \times 4}{12} = \frac{23}{12}$$

$$(ii) \left(\frac{1}{2} \times \frac{1}{4} \right) + \left(\frac{1}{2} \times 6 \right)$$

$$= \frac{1}{8} + 3$$

$$= \frac{1 + 3 \times 8}{8} = \frac{25}{8}$$

$$(iii) \left(-5 \times \frac{2}{15} \right) - \left(-6 \times \frac{2}{9} \right)$$

$$= \frac{-2}{3} - \frac{-4}{3}$$

$$= \frac{-2 + 4}{3} = \frac{2}{3}$$

$$(iv) \left(\frac{-9}{4} \times \frac{5}{3} \right) + \left(\frac{13}{2} \times \frac{5}{6} \right)$$

$$= \frac{-3 \times 5}{4} + \frac{13 \times 5}{12}$$

$$= \frac{-15}{4} + \frac{65}{12}$$

$$= \frac{-15 \times 3 + 65}{12}$$

$$= \frac{20}{12} = \frac{5}{3}$$

$$(v) \left(\frac{-4}{3} \times \frac{12}{-5} \right) + \left(\frac{3}{7} \times \frac{21}{15} \right)$$

$$= \frac{4 \times 4}{5} + \frac{1 \times 3}{5}$$

$$= \frac{16}{5} + \frac{3}{5}$$

$$= \frac{16 + 3}{5} = \frac{19}{5}$$

$$(vi) \left(\frac{13}{5} \times \frac{8}{3} \right) - \left(\frac{-5}{2} \times \frac{11}{3} \right)$$

$$= \frac{13 \times 8}{15} - \frac{-5 \times 11}{6}$$

$$= \frac{104}{15} - \frac{-55}{6}$$

$$= \frac{104 \times 2 + 55 \times 5}{30} = \frac{483}{30}$$

$$(vii) \left(\frac{13}{7} \times \frac{11}{26} \right) - \left(\frac{-4}{3} \times \frac{5}{6} \right)$$

$$= \frac{1 \times 11}{7 \times 2} - \frac{-2 \times 5}{3 \times 3}$$

$$= \frac{11}{14} - \frac{-10}{9}$$

$$= \frac{11 \times 9 + 10 \times 14}{126} = \frac{239}{126}$$

$$(viii) \left(\frac{8}{5} \times \frac{-3}{2} \right) + \left(\frac{-3}{10} \times \frac{11}{16} \right)$$

$$= \frac{4 \times (-3)}{5} + \frac{-3 \times 11}{10 \times 16}$$

$$= \frac{-12}{5} + \frac{-33}{160}$$

$$= \frac{-12 \times 32 - 33}{160} = \frac{-417}{160}$$

Q-5. Simplify:

$$(i) \left(\frac{3}{2} \times \frac{1}{6} \right) + \left(\frac{5}{3} \times \frac{7}{2} \right) - \left(\frac{13}{8} \times \frac{4}{3} \right)$$

$$(ii) \left(\frac{1}{4} \times \frac{2}{7} \right) - \left(\frac{5}{14} \times \frac{-2}{3} \right) + \left(\frac{3}{7} \times \frac{9}{2} \right)$$

$$(iii) \left(\frac{13}{9} \times \frac{-15}{2} \right) + \left(\frac{7}{3} \times \frac{8}{5} \right) + \left(\frac{3}{5} \times \frac{1}{2} \right)$$

$$(iv) \left(\frac{3}{11} \times \frac{5}{6} \right) - \left(\frac{9}{12} \times \frac{4}{3} \right) + \left(\frac{5}{13} \times \frac{6}{15} \right)$$

Solution:

$$(i) \left(\frac{3}{2} \times \frac{1}{6} \right) + \left(\frac{5}{3} \times \frac{7}{2} \right) - \left(\frac{13}{8} \times \frac{4}{3} \right)$$

$$= \frac{1}{4} + \frac{35}{6} - \frac{13}{6}$$

$$= \frac{1 \times 3 + 35 \times 2 - 13 \times 6}{12}$$

$$= \frac{3 + 70 - 26}{12} = \frac{47}{12}$$

$$(ii) \left(\frac{1}{4} \times \frac{2}{7} \right) - \left(\frac{5}{14} \times \frac{-2}{3} \right) + \left(\frac{3}{7} \times \frac{9}{2} \right)$$

$$= \frac{1}{14} - \frac{-5}{21} + \frac{27}{14}$$

$$= \frac{1 \times 3 + 5 \times 2 + 27 \times 3}{21}$$

$$= \frac{3 + 10 + 81}{21} = \frac{94}{21}$$

$$(iii) \left(\frac{13}{9} \times \frac{-15}{2} \right) + \left(\frac{7}{3} \times \frac{8}{5} \right) + \left(\frac{3}{5} \times \frac{1}{2} \right)$$

$$= \frac{-13 \times 5}{6} + \frac{7 \times 8}{15} + \frac{3}{10}$$

$$= \frac{-65}{6} + \frac{56}{15} + \frac{3}{10}$$

$$= \frac{-65 \times 5 + 56 \times 2 + 3 \times 3}{30}$$

$$= \frac{-204}{30} = \frac{-34}{5}$$

$$(iv) \left(\frac{3}{11} \times \frac{5}{6} \right) - \left(\frac{9}{12} \times \frac{4}{3} \right) + \left(\frac{5}{13} \times \frac{6}{15} \right)$$

$$= \frac{5}{22} - 1 + \frac{2}{13}$$

$$= \frac{5 \times 13 - 286 + 2 \times 22}{286}$$

$$= \frac{65 - 286 + 44}{286} = \frac{-177}{286}$$

Chapter 1: Rational Numbers

Exercise 1.6

Q-1. Verify the property $x \times y = y \times x$ by taking:

(i) $x = -\frac{1}{3}, y = \frac{2}{7}$

(ii) $x = -\frac{3}{5}, y = -\frac{11}{13}$

(iii) $x = 2, y = -\frac{7}{8}$

(iv) $x = 0, y = -\frac{15}{8}$

Solution. We have to verify that, $x \times y = y \times x$

(i) $x = -\frac{1}{3}, y = \frac{2}{7}$

$$x \times y = \frac{-1}{3} \times \frac{2}{7} = \frac{-2}{21}$$

$$y \times x = \frac{2}{7} \times \frac{-1}{3} = \frac{-2}{21}$$

$$\therefore \frac{-1}{3} \times \frac{2}{7} = \frac{2}{7} \times \frac{-1}{3}$$

Hence, verified.

$$(ii) x = \frac{-3}{5}, y = \frac{-11}{13}$$

$$x \times y = \frac{-3}{5} \times \frac{-11}{13} = \frac{33}{65}$$

$$y \times x = \frac{-11}{13} \times \frac{-3}{5} = \frac{33}{65}$$

$$\therefore \frac{-3}{5} \times \frac{-11}{13} = \frac{-11}{13} \times \frac{-3}{5}$$

Hence, verified.

$$(iii) x = 2, y = \frac{7}{-8}$$

$$x \times y = 2 \times \frac{7}{-8} = \frac{7}{-4}$$

$$y \times x = \frac{7}{-8} \times 2 = \frac{7}{-4}$$

$$\therefore 2 \times \frac{7}{-8} = \frac{7}{-8} \times 2$$

Hence, verified.

$$(iv) x = 0, y = \frac{-15}{8}$$

$$x \times y = 0 \times \frac{-15}{8} = 0$$

$$y \times x = \frac{-15}{8} \times 0 = 0$$

$$\therefore 0 \times \frac{-15}{8} = \frac{-15}{8} \times 0 = 0$$

Hence, verified.

Q-2. Verify the property: $x \times (y \times z) = (x \times y) \times z$

$$(i) x = \frac{-7}{3}, y = \frac{12}{5}, z = \frac{4}{9}$$

$$(ii) x = 0, y = \frac{-3}{5}, z = \frac{-9}{4}$$

$$(iii) x = \frac{1}{2}, y = \frac{5}{-4}, z = \frac{-7}{5}$$

$$(iv) x = \frac{5}{7}, y = \frac{-12}{13}, z = \frac{-7}{18}$$

Solution: We have to verify that, $x \times (y \times z) = (x \times y) \times z$

$$(i) x = \frac{-7}{3}, y = \frac{12}{5}, z = \frac{4}{9}$$

$$\Rightarrow x \times (y \times z) = \frac{-7}{3} \times \left(\frac{12}{5} \times \frac{4}{9} \right) = \frac{-7}{3} \times \frac{16}{15} = \frac{-112}{45}$$

$$\Rightarrow (x \times y) \times z = \left(\frac{-7}{3} \times \frac{12}{5} \right) \times \frac{4}{9} = \frac{-28}{5} \times \frac{4}{9} = \frac{-112}{45}$$

$$\therefore \frac{-7}{3} \times \left(\frac{12}{5} \times \frac{4}{9} \right) = \left(\frac{-7}{3} \times \frac{12}{5} \right) \times \frac{4}{9}$$

$$(ii) x = 0, y = \frac{-3}{5}, z = \frac{-9}{4}$$

$$\Rightarrow x \times (y \times z) = 0 \times \left(\frac{-3}{5} \times \frac{-9}{4} \right) = 0 \times \frac{27}{20} = 0$$

$$\Rightarrow (x \times y) \times z = \left(0 \times \frac{-3}{5} \right) \times \frac{-9}{4} = 0 \times \frac{-9}{4} = 0$$

$$\therefore 0 \times \left(\frac{-3}{5} \times \frac{-9}{4} \right) = \left(0 \times \frac{-3}{5} \right) \times \frac{-9}{4}$$

$$(iii) x = \frac{1}{2}, y = \frac{5}{-4}, z = \frac{-7}{5}$$

$$\Rightarrow x \times (y \times z) = \frac{1}{2} \times \left(\frac{5}{-4} \times \frac{-7}{5} \right) = \frac{-1}{2} \times \frac{35}{16} = \frac{35}{32}$$

$$\Rightarrow (x \times y) \times z = \left(\frac{1}{2} \times \frac{5}{-4} \right) \times \frac{-7}{5} = \frac{-5}{8} \times \frac{-7}{4} = \frac{35}{32}$$

$$\therefore \frac{1}{2} \times \left(\frac{5}{-4} \times \frac{-7}{5} \right) = \left(\frac{1}{2} \times \frac{5}{-4} \right) \times \frac{-7}{5}$$

$$(iv) x = \frac{5}{7}, y = \frac{-12}{13}, z = \frac{-7}{18}$$

$$\Rightarrow x \times (y \times z) = \frac{5}{7} \times \left(\frac{-12}{13} \times \frac{-7}{18} \right) = \frac{5}{7} \times \frac{14}{39} = \frac{10}{39}$$

$$\Rightarrow (x \times y) \times z = \left(\frac{5}{7} \times \frac{-12}{13} \right) \times \frac{-7}{18} = \frac{-60}{91} \times \frac{-7}{18} = \frac{10}{39}$$

$$\therefore \frac{5}{7} \times \left(\frac{-12}{13} \times \frac{-7}{18} \right) = \left(\frac{5}{7} \times \frac{-12}{13} \right) \times \frac{-7}{18}$$

Q-3. Verify the property: $x \times (y \times z) = x \times y + x \times z$:

$$(i) x = \frac{-3}{7}, y = \frac{12}{13}, z = \frac{-5}{6}$$

$$(ii) x = \frac{-12}{5}, y = \frac{-15}{4}, z = \frac{8}{3}$$

$$(iii) x = \frac{-8}{3}, y = \frac{5}{6}, z = \frac{-13}{12}$$

$$(iv) x = \frac{-3}{4}, y = \frac{-5}{2}, z = \frac{7}{6}$$

Solution: We have to verify that, $x \times (y \times z) = x \times y + x \times z$

$$(i) x = \frac{-3}{7}, y = \frac{12}{13}, z = \frac{-5}{6}$$

$$x \times (y + z) = \frac{-3}{7} \times \left(\frac{12}{13} + \frac{-5}{6} \right) = \frac{-3}{7} \times \frac{72-65}{78} = \frac{-3}{7} \times \frac{7}{78} = \frac{-1}{26}$$

$$x \times y + x \times z = \frac{-3}{7} \times \frac{12}{13} + \frac{-3}{7} \times \frac{-5}{6} = \frac{-36}{91} + \frac{5}{14} = \frac{-36 \times 2 + 5 \times 13}{182} = \frac{-1}{26}$$

$$\therefore \frac{-3}{7} \times \left(\frac{12}{13} + \frac{-5}{6} \right) = \frac{-3}{7} \times \frac{12}{13} + \frac{-3}{7} \times \frac{-5}{6}$$

Hence, verified.

$$(ii) x = \frac{-12}{5}, y = \frac{-15}{4}, z = \frac{8}{3}$$

$$x \times (y + z) = \frac{-12}{5} \times \left(\frac{-15}{4} + \frac{8}{3} \right) = \frac{-12}{5} \times \frac{-45+32}{12} = \frac{-12}{5} \times \frac{-13}{12} = \frac{13}{5}$$

$$x \times y + x \times z = \frac{-12}{5} \times \frac{-15}{4} + \frac{-12}{5} \times \frac{8}{3} = \frac{9}{1} + \frac{-32}{5} = \frac{45-32}{5} = \frac{13}{5}$$

$$\therefore \frac{-12}{5} \times \left(\frac{-15}{4} + \frac{8}{3} \right) = \frac{-12}{5} \times \frac{-15}{4} + \frac{-12}{5} \times \frac{8}{3}$$

Hence, verified.

$$(iii) x = \frac{-8}{3}, y = \frac{5}{6}, z = \frac{-13}{12}$$

$$x \times (y + z) = \frac{-8}{3} \times \left(\frac{5}{6} + \frac{-13}{12} \right) = \frac{-8}{3} \times \frac{10-13}{12} = \frac{-8}{3} \times \frac{-3}{12} = \frac{2}{3}$$

$$x \times y + x \times z = \frac{-8}{3} \times \frac{5}{6} + \frac{-8}{3} \times \frac{-13}{12} = \frac{-20}{9} + \frac{26}{9} = \frac{-20+26}{9} = \frac{6}{9} = \frac{2}{3}$$

$$\therefore \frac{-8}{3} \times \left(\frac{5}{6} + \frac{-13}{12} \right) = \frac{-8}{3} \times \frac{5}{6} + \frac{-8}{3} \times \frac{-13}{12}$$

Hence, verified.

$$(iv) x = \frac{-3}{4}, y = \frac{-5}{2}, z = \frac{7}{6}$$

$$x \times (y + z) = \frac{-3}{4} \times \left(\frac{-5}{2} + \frac{7}{6} \right) = \frac{-3}{4} \times \frac{-15+7}{6} = \frac{-3}{4} \times \frac{-8}{6} = 1$$

$$x \times y + x \times z = \frac{-3}{4} \times \frac{-5}{2} + \frac{-3}{4} \times \frac{7}{6} = \frac{15}{8} + \frac{-7}{8} = \frac{15-7}{8} = 1$$

$$\therefore \frac{-3}{4} \times \left(\frac{-5}{2} + \frac{7}{6} \right) = \frac{-3}{4} \times \frac{-5}{2} + \frac{-3}{4} \times \frac{7}{6}$$

Hence, verified.

Q-4. Use the distributivity of multiplication of rational numbers over their addition to simplify:

$$(i) \frac{3}{5} \times \left(\frac{35}{24} + \frac{10}{1} \right)$$

$$(ii) \frac{-5}{4} \times \left(\frac{8}{5} + \frac{16}{5} \right)$$

$$(iii) \frac{2}{7} \times \left(\frac{7}{16} - \frac{21}{4} \right)$$

$$(iv) \frac{3}{4} \times \left(\frac{8}{9} - 40 \right)$$

Solution:

$$(i) \frac{3}{5} \times \left(\frac{35}{24} + \frac{10}{1} \right) = \frac{3}{5} \times \frac{35}{24} + \frac{3}{5} \times \frac{10}{1} = \frac{7+48}{8} = \frac{55}{8}$$

$$(ii) \frac{-5}{4} \times \left(\frac{8}{5} + \frac{16}{5} \right) = \frac{-5}{4} \times \frac{8}{5} + \frac{-5}{4} \times \frac{16}{5} = \frac{-2}{2} \times \frac{-4}{1} = -6$$

$$(iii) \frac{2}{7} \times \left(\frac{7}{16} - \frac{21}{4} \right) = \frac{2}{7} \times \frac{7}{16} - \frac{2}{7} \times \frac{21}{4} = \frac{1}{8} - \frac{3}{2} = \frac{1-12}{8} = \frac{-11}{8}$$

$$(iv) \frac{3}{4} \times \left(\frac{8}{9} - 40 \right) = \frac{3}{4} \times \frac{8}{9} - \frac{3}{4} \times 40 = \frac{2}{3} - 30 = \frac{2-90}{3} = \frac{-88}{3}$$

Q-5. Find the multiplicative inverse (reciprocal) of each of the following rational numbers:

(i) 9

(ii) -7

(iii) $\frac{12}{5}$

(iv) $\frac{-7}{9}$

(v) $\frac{-3}{-5}$

(vi) $\frac{2}{3} \times \frac{9}{4}$

(vii) $\frac{-5}{16} \times 15 \times \frac{-3}{5}$

(viii) $-2 \times \frac{-3}{5}$

(ix) -1

(x) $\frac{0}{3}$

(xi) 1

Solution:

(i) Multiplicative inverse (reciprocal) of 9 = $\frac{1}{9}$.

(ii) Multiplicative inverse (reciprocal) of -7 = $\frac{1}{-7}$

(iii) Multiplicative inverse (reciprocal) of $\frac{12}{5}$ = $\frac{5}{12}$

(iv) Multiplicative inverse (reciprocal) of $\frac{-7}{9}$ = $\frac{-9}{7}$

(v) Multiplicative inverse (reciprocal) of $\frac{-3}{-5}$ = $\frac{-5}{-3}$ or $\frac{5}{3}$

(vi) Multiplicative inverse (reciprocal) of $\frac{2}{3} \times \frac{9}{4} = \frac{3}{2} \times \frac{4}{9} = \frac{2}{3}$

(vii) Multiplicative inverse (reciprocal) of $\frac{-5}{8} \times \frac{16}{15} = \frac{8}{-5} \times \frac{15}{16} = \frac{-3}{2}$

(viii) Multiplicative inverse (reciprocal) of $-2 \times \frac{-3}{5} = \frac{1}{-2} \times \frac{5}{-3} = \frac{5}{6}$

(ix) Multiplicative inverse (reciprocal) of -1 = $\frac{1}{-1} = -1$

(x) Multiplicative inverse (reciprocal) of $\frac{0}{3} = \frac{3}{0}$ Undefined

(xi) Multiplicative inverse (reciprocal) of 1 = $\frac{1}{1} = 1$

Q-6. Name the property of multiplication of rational numbers illustrated by the following statements:

(i) $\frac{-5}{16} \times \frac{8}{15} = \frac{8}{15} \times \frac{-5}{16}$

(ii) $\frac{-17}{5} \times 9 = 9 \times \frac{-17}{5}$

(iii) $\frac{7}{4} \times \left(\frac{-8}{3} + \frac{-13}{12} \right) = \frac{7}{4} \times \frac{-8}{3} + \frac{7}{4} \times \frac{-13}{12}$

(iv) $\frac{-5}{9} \times \left(\frac{4}{15} + \frac{-9}{8} \right) = \left(\frac{-5}{9} \times \frac{4}{15} \right) + \frac{-9}{8}$

(v) $\frac{13}{-17} \times 1 = \frac{13}{-17} = 1 \times \frac{13}{-17}$

(vi) $\frac{-11}{16} \times \frac{16}{-11} = 1$

(vii) $\frac{2}{13} \times 0 = 0 = 0 \times \frac{2}{13}$

(viii) $\frac{3}{-2} \times \frac{5}{4} + \frac{-3}{2} \times \frac{-7}{6} = \frac{-3}{2} \times \left(\frac{5}{4} + \frac{-7}{6} \right)$

Solution:

(i) Commutative property

(ii) Commutative Property

(iii) Distributivity of multiplication over addition

(iv) Associativity of multiplication.

(v) The existence of identity for multiplication.

(vi) Existence of multiplicative inverse

(vii) Multiplication by 0

(viii) Distributive property

Q-7. Fill in the blanks:

(i) The product of two positive rational numbers is always

(ii) The product of a positive rational number and a negative rational number is always

(iii) The product of two negative rational numbers is always

(iv) The reciprocal of a positive rational number is

(v) The reciprocal of a negative rational number is

(vi) Zero has reciprocal.

(vii) The product of a rational number and its reciprocal is

(viii) The numbers and are their own reciprocals.

(ix) If a is reciprocal of b , then the reciprocal of b is

(x) The number 0 is The reciprocal of any number.

(xi) Reciprocal of $\frac{1}{a}$, $a \neq 0$ is

(xii) $(17 \times 12)^{-1} = (17)^{-1} \times \dots\dots\dots$

Solution:

(i) Positive

(ii) Negative

(iii) Positive

(iv) Positive

(v) Negative

(vi) No

(vii) 1

(viii) -1 and 1

(ix) a

(x) not

(xi) a

(xii) 12^{-1}

Q-8. Fill in the blanks:

(i) $-4 \times \frac{7}{9} = \frac{7}{9} \times \dots\dots\dots$

(ii) $\frac{5}{11} \times \frac{-3}{8} = \frac{-3}{8} \times \dots\dots\dots$

(iii) $\frac{1}{2} \times \left(\frac{3}{4} + \frac{-5}{12} \right) = \frac{1}{2} \times \dots\dots\dots + \dots\dots\dots \times \frac{-5}{12}$

(iv) $\frac{-4}{5} \times \left(\frac{5}{7} + \frac{-8}{9} \right) = \left(\frac{-4}{5} \times \dots\dots\dots \right) + \frac{-4}{5} \times \frac{-8}{9}$

Solution:

(i) As, $x \times y = y \times x$ that is commutativity

So, -4

(ii) As, $x \times y = y \times x$ that is commutativity

So, $\frac{5}{11}$

(iii) As, $x \times (y + z) = x \times y + x \times z$ that is distributivity of multiplication over addition

So, $\frac{3}{4}, \frac{1}{2}$

(iv) As, $x \times (y \times z) = (x \times y) \times z$ that is associativity of multiplication

So, $\frac{5}{7}$

Chapter 1: Rational Numbers

Exercise 1.7

Q-1. Divide:

(i) 1 by $\frac{1}{2}$

(ii) 5 by $\frac{-5}{7}$

(iii) $\frac{-3}{4}$ by $\frac{1}{2}$

(iv) $\frac{-7}{8}$ by $\frac{-21}{16}$

(v) $\frac{7}{-4}$ by $\frac{63}{64}$

(vi) 0 by $\frac{-7}{5}$

$$(vii) \frac{-3}{4} \text{ by } 6$$

$$(viii) \frac{2}{3} \text{ by } \frac{-7}{12}$$

$$(ix) -4 \text{ by } \frac{-3}{5}$$

$$(x) \frac{-3}{13} \text{ by } \frac{-4}{65}$$

Solution:

$$(i) 1 \div \frac{1}{2} = 1 \times \frac{2}{1} = 2$$

$$(ii) 5 \div \frac{-5}{7} = 5 \times \frac{7}{-5} = -7$$

$$(iii) \frac{-3}{4} \div \frac{9}{-16} = \frac{-3}{4} \times \frac{-16}{9} = \frac{4}{3}$$

$$(iv) \frac{-7}{8} \div \frac{-21}{16} = \frac{-7}{8} \times \frac{-16}{21} = \frac{2}{3}$$

$$(v) \frac{-7}{4} \div \frac{63}{64} = \frac{-7}{4} \times \frac{64}{63} = \frac{-16}{9}$$

$$(vi) 0 \div \frac{-7}{5} = 0 \times \frac{-5}{7} = 0$$

$$(vii) \frac{-3}{4} \div -6 = \frac{-3}{4} \times \frac{1}{-6} = \frac{1}{8}$$

$$(viii) \frac{2}{3} \div \frac{-7}{12} = \frac{2}{3} \times \frac{12}{-7} = \frac{-8}{7}$$

$$(ix) -4 \div \frac{-3}{5} = -4 \times \frac{5}{-3} = \frac{20}{3}$$

$$(x) \frac{-3}{13} \div \frac{-4}{65} = \frac{-3}{13} \times \frac{65}{-4} = \frac{15}{4}$$

Q-2. Find the value and express as a rational number in standard form:

$$(i) \frac{2}{5} \div \frac{26}{15}$$

$$(ii) \frac{10}{3} \div \frac{-35}{12}$$

$$(iii) -6 \div \frac{-8}{17}$$

$$(iv) \frac{-40}{99} \div (-20)$$

$$(v) \frac{-22}{27} \div \frac{-110}{18}$$

$$(vi) \frac{-36}{125} \div \frac{-3}{75}$$

Solution:

$$(i) \frac{2}{5} \div \frac{26}{15} = \frac{2}{5} \times \frac{15}{26} = \frac{3}{13}$$

$$(ii) \frac{10}{3} \div \frac{-35}{12} = \frac{10}{3} \times \frac{12}{-35} = \frac{-8}{7}$$

$$(iii) -6 \div \frac{-8}{17} = -6 \times \frac{17}{-8} = \frac{51}{4}$$

$$(iv) \frac{-40}{99} \div (-20) = \frac{-40}{99} \times \frac{1}{-20} = \frac{2}{99}$$

$$(v) \frac{-22}{27} \div \frac{-110}{18} = \frac{-22}{27} \times \frac{18}{-110} = \frac{2}{15}$$

$$(vi) \frac{-36}{125} \div \frac{-3}{75} = \frac{-36}{125} \times \frac{75}{-3} = \frac{36}{5}$$

Q-3. The product of two rational numbers is 15. If one of the numbers is -10. Find the other number.

Solution:

Let, the other number be x.

$$\text{So, } x \times (-10) = 15$$

$$\Rightarrow x = \frac{15}{-10} = \frac{-3}{2}$$

So, the other number is $\frac{-3}{2}$.

Q-4. The product of two rational numbers is $\frac{-8}{9}$. If one of the number is $\frac{-4}{15}$, Find the other number.

Solution: Let, the other number be x.

$$\text{So, } x \times \frac{-4}{15} = \frac{-8}{9}$$

$$\Rightarrow x = \frac{-8}{9} \div \frac{-4}{15}$$

$$\Rightarrow x = \frac{-8}{9} \times \frac{15}{-4}$$

$$\Rightarrow x = \frac{10}{3}$$

Thus, the other number is $\frac{10}{3}$

Q-5. By what number should we multiply $\frac{-1}{6}$ so that the product may be $\frac{-23}{9}$?

Solution:

Let, the number be x .

$$x \times \frac{-1}{6} = \frac{-23}{9}$$

$$\Rightarrow x = \frac{-23}{9} \div \frac{-1}{6}$$

$$\Rightarrow x = \frac{-23}{9} \times \frac{6}{-1}$$

$$\Rightarrow x = \frac{46}{3}$$

Thus, the other number is $\frac{46}{3}$

Q-6. By what number should we multiply $\frac{-15}{28}$ so that the product may be $\frac{-5}{7}$?

Solution:

Let, the number be x .

$$x \times \frac{-15}{28} = \frac{-5}{7}$$

$$\Rightarrow x = \frac{-5}{7} \div \frac{-15}{28}$$

$$\Rightarrow x = \frac{-5}{7} \times \frac{28}{-15}$$

$$\Rightarrow x = \frac{4}{3}$$

Thus, the other number is $\frac{4}{3}$

Q-7. By what number should we multiply $\frac{-8}{13}$ so that the product may be 24?

Solution:

Let, the number be x .

$$x \times \frac{-8}{13} = 24$$

$$\Rightarrow x = 24 \div \frac{-8}{13}$$

$$\Rightarrow x = 24 \times \frac{13}{-8}$$

$$\Rightarrow x = -39$$

Thus, the other number is -39.

Q-8. By what number should $\frac{-3}{4}$ be multiplied in order to produce $\frac{2}{3}$?

Solution:

Let, the other number that should be multiplied with $\frac{-3}{4}$ to produce $\frac{2}{3}$ be x .

$$x \times \frac{-3}{4} = \frac{2}{3}$$

$$\Rightarrow x = \frac{2}{3} \div \frac{-3}{4}$$

$$\Rightarrow x = \frac{2}{3} \times \frac{4}{-3}$$

$$\Rightarrow x = \frac{-8}{9}$$

Thus, the other number is $\frac{-8}{9}$

Q-9. Find $(x + y) \div (x - y)$, if

$$(i) x = \frac{2}{3}, y = \frac{3}{2}$$

$$(ii) x = \frac{2}{5}, y = \frac{1}{2}$$

$$(iii) x = \frac{5}{4}, y = \frac{-1}{3}$$

$$(iv) x = \frac{2}{7}, y = \frac{4}{3}$$

$$(v) x = \frac{1}{4}, y = \frac{3}{2}$$

Solution:

$$(i) (x + y) \div (x - y)$$

$$= \left(\frac{2}{3} + \frac{3}{2} \right) \div \left(\frac{2}{3} - \frac{3}{2} \right)$$

$$= \frac{13}{6} \times \frac{6}{-5} = \frac{-13}{5}$$

$$\text{Thus, } (x + y) \div (x - y) = \frac{-13}{5}$$

$$(ii) (x + y) \div (x - y)$$

$$= \left(\frac{2}{5} + \frac{1}{2} \right) \div \left(\frac{2}{5} - \frac{1}{2} \right)$$

$$= \frac{9}{10} \times \frac{10}{-1} = -9$$

$$\text{Thus, } (x + y) \div (x - y) = -9$$

$$(iii) (x + y) \div (x - y)$$

$$= \left(\frac{5}{4} + \frac{-1}{3} \right) \div \left(\frac{5}{4} - \frac{-1}{3} \right)$$

$$= \frac{11}{12} \times \frac{12}{11} = \frac{11}{19}$$

$$\text{Thus, } (x + y) \div (x - y) = \frac{11}{19}$$

$$(iv) (x + y) \div (x - y)$$

$$= \left(\frac{2}{7} + \frac{4}{3} \right) \div \left(\frac{2}{7} - \frac{4}{3} \right)$$

$$= \frac{34}{21} \times \frac{21}{-22} = \frac{-17}{11}$$

$$\text{Thus, } (x + y) \div (x - y) = \frac{-17}{11}$$

$$(v) (x + y) \div (x - y)$$

$$= \left(\frac{1}{4} + \frac{3}{2} \right) \div \left(\frac{1}{4} - \frac{3}{2} \right)$$

$$= \frac{7}{4} \times \frac{4}{-5} = \frac{-7}{5}$$

$$\text{Thus, } (x + y) \div (x - y) = \frac{-7}{5}$$

Q-10: The cost of $7\frac{2}{3}$ metres of rope is Rs $12\frac{3}{4}$. Find its cost per metres.

Solution: The cost of $7\frac{2}{3}$ metres of rope is Rs. $12\frac{3}{4}$.

Therefore,

$$\text{Cost per metre} = 12\frac{3}{4} \div 7\frac{2}{3}$$

$$= \frac{51}{4} \div \frac{23}{3}$$

$$= \frac{51}{4} \times \frac{3}{23}$$

$$= \frac{153}{92} = \text{Rs. } 1\frac{61}{92}$$

Hence, the cost of rope per metres = Rs. $1\frac{61}{92}$

Q-11. The cost of $2\frac{1}{3}$ metres of cloth is Rs. $75\frac{1}{4}$. Find the cost of cloth per metres.

Solution: The cost of $2\frac{1}{3}$ metres of cloth is Rs. $75\frac{1}{4}$.

Therefore,

$$\text{Cost per metre} = 75\frac{1}{4} \div 2\frac{1}{3}$$

$$= \frac{301}{4} \div \frac{7}{3}$$

$$= \frac{301}{4} \times \frac{3}{7}$$

$$= \frac{129}{4} = \text{Rs. } 32\frac{1}{4}$$

Thus, Rs. $32\frac{1}{4}$ or Rs. 32.25 is the cost of cloth per metre.

Q-12. By what number should $\frac{-33}{16}$ be divided to get $\frac{-11}{4}$?

Solution:

Let, the other number be x.

$$\frac{-33}{16} \div x = \frac{-11}{4}$$

$$\Rightarrow \frac{-33}{16} \times \frac{1}{x} = \frac{-11}{4}$$

$$\Rightarrow \frac{1}{x} = \frac{-11}{4} \times \frac{16}{-33}$$

$$\Rightarrow \frac{1}{x} = \frac{4}{3}$$

$$\Rightarrow x = \frac{3}{4}$$

Thus, the other number is $\frac{3}{4}$

Q-13. Divide the sum of $\frac{-13}{5}$ and $\frac{12}{7}$ by the product of $\frac{-31}{7}$ and $\frac{-1}{2}$?

Solution:

$$\begin{aligned} & \left(\frac{-13}{5} + \frac{12}{7} \right) \div \left(\frac{-31}{7} \times \frac{-1}{2} \right) \\ &= \frac{-13 \times 7 + 12 \times 5}{35} \div \frac{31}{14} \\ &= \frac{-91 + 60}{35} \div \frac{31}{14} \\ &= \frac{-31}{35} \times \frac{14}{31} = \frac{-2}{5} \end{aligned}$$

Q-14. Divide the sum of $\frac{65}{12}$ and $\frac{12}{7}$ by their differences.

Solution:

$$\begin{aligned} & \left(\frac{65}{12} + \frac{12}{7} \right) \div \left(\frac{65}{12} - \frac{12}{7} \right) \\ &= \frac{65 \times 7 + 12 \times 12}{84} \div \frac{65 \times 7 - 12 \times 12}{84} \\ &= \frac{455 + 144}{84} \div \frac{455 - 144}{84} \\ &= \frac{599}{84} \div \frac{311}{84} \\ &= \frac{599}{84} \times \frac{84}{311} = \frac{599}{311} \end{aligned}$$

Q-15. If 24 trousers of equal size can be prepared in 54 meters of cloth, what length of cloth is required for each trouser?

Solution:

Cloth needed to prepare 24 trousers = 54 m

So,

Length of the cloth required for each trousers = $54 \div 24 = \frac{54}{24} = \frac{9}{4} \text{ m} = 2\frac{1}{4} \text{ metres.}$

Chapter 1: Rational Numbers

Exercise 1.8

Q-1: Find a rational number between -3 and 1.

Solution:

Rational number between -3 and 1 is $\frac{-3+1}{2} = \frac{-2}{2} = -1$

Q-2. Find any five rational numbers less than 2.

Solution.

We can write:

$$2 = \frac{2}{1} = \frac{2 \times 5}{1 \times 5} = \frac{10}{5}$$

Integers less than 10 are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Hence, five rational numbers less than 2 are $\frac{0}{5}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}$ and $\frac{4}{5}$.

Q-3. Find two rational numbers between $-\frac{2}{9}$ and $\frac{5}{9}$.

Solution:

Since, both the fractions $\left(-\frac{2}{9} \text{ and } \frac{5}{9}\right)$ have the same denominator, the integers between the numerators (-2 and 5) are -1, 0, 1, 2, 3, 4.

Hence, two rational numbers between $\left(-\frac{2}{9} \text{ and } \frac{5}{9}\right)$ are $\left(\frac{0}{9} \text{ or } 0 \text{ and } \frac{1}{9}\right)$

Q-4. Find two rational numbers between $\frac{1}{5}$ and $\frac{1}{2}$.

Solution:

$$\text{Rational number between } \frac{1}{5} \text{ and } \frac{1}{2} = \frac{\left(\frac{1}{5} + \frac{1}{2}\right)}{2} = \frac{\frac{2+5}{10}}{2} = \frac{7}{20}$$

$$\text{Rational number between } \frac{1}{5} \text{ and } \frac{7}{20} = \frac{\left(\frac{1}{5} + \frac{7}{20}\right)}{2} = \frac{\frac{4+7}{20}}{2} = \frac{11}{40}$$

Therefore, two rational numbers between $\frac{1}{5}$ and $\frac{1}{2}$ are $\frac{7}{20}$ and $\frac{11}{40}$.

Q-5. Find ten rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

Solution:

The LCM of the denominators (2 and 4) is 4.

So, we can write $\frac{1}{4}$ as it is.

$$\text{Also, } \frac{1}{2} = \frac{1 \times 2}{2 \times 2}$$

As the integers between the numerators 1 and 2 of both the fractions are not sufficient, we will multiply the fractions by 20.

$$\therefore \frac{1}{4} = \frac{1 \times 20}{4 \times 20} = \frac{20}{80}$$

$$\frac{2}{4} = \frac{2 \times 20}{4 \times 20} = \frac{40}{80}$$

Between 20 and 40, there are 19 integers. They are 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40.

Thus, $\frac{21}{40}, \frac{22}{40}, \frac{23}{40}, \frac{24}{40}, \frac{25}{40}, \dots, \frac{38}{40}$ and $\frac{39}{40}$ are the fractions.

We can take any 10 of these.

Q-6. Find ten rational numbers between $-\frac{2}{5}$ and $\frac{1}{2}$.

Solution:

LCM of the denominators (2 and 5) of both the fractions is 10.

We can write:

$$-\frac{2}{5} = \frac{-2 \times 2}{5 \times 2} = \frac{-4}{10}$$

$$\text{And } \frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

Since, the integers between the numerators (-4 and 5) of both the fractions are not sufficient, we will multiply the fractions by 2.

$$\therefore \frac{-4}{10} = \frac{-4 \times 2}{10 \times 2} = \frac{-8}{20}$$

$$\frac{5}{10} = \frac{5 \times 2}{10 \times 2} = \frac{10}{20}$$

There are 17 integers between -8 and 10, which are -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

These can be written as:

$$\frac{-7}{20}, \frac{-6}{20}, \frac{-5}{20}, \frac{-4}{20}, \dots, \frac{8}{20} \text{ and } \frac{9}{20}$$

We can take any 10 of these.

Q-7. Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$.

Solution:

LCM of the denominators (5 and 4) of both the fractions is 20.

We can write:

$$\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

$$\text{And } \frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

Since, the integers between the numerators (12 and 15) of both the fractions are not sufficient, we will multiply the fractions by 5.

$$\therefore \frac{12}{20} = \frac{12 \times 5}{20 \times 5} = \frac{60}{100}$$

$$\frac{15}{20} = \frac{15 \times 5}{20 \times 5} = \frac{75}{100}$$

There are 14 integers between 60 and 75, which are 61, 62, 63, 64, 65,, 73 and 74. These can be written as:

$$\frac{60}{100}, \frac{61}{100}, \frac{62}{100}, \frac{63}{100}, \dots, \frac{73}{100} \text{ and } \frac{74}{100}$$

We can take any 10 of these.