Chapter 2: Powers

Exercise 2.1

Question 1. Express each of the following as a rational number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:.

(i) 2^{-3}

(ii) $(-4)^{-2}$

(iii) $\frac{1}{3^{-2}}$

(iv) $\left(\frac{1}{2}\right)^{-5}$

(v) $\left(\frac{2}{3}\right)^{-2}$

Answer:

(i) $2^{-3}=\frac{1}{2^3}=\frac{1}{8}$

(ii) $(-4)^{-2}=\frac{1}{(-4)^2}=\frac{1}{16}$

(iii) $\frac{1}{3^{-2}}=3^2=9$

(iv) $\left(\frac{1}{2}\right)^{-5}=2^5=32$

(v) $\left(\frac{2}{3}\right)^{-2}=\left(\frac{3}{2}\right)^2=\frac{9}{4}$

Question 2. Find the values of the following:

(i) $3^{-1}+4^{-1}$

(ii) $\left(3^0+4^{-1}\right)\times 2^2$

(iii) $\left(3^{-1}+4^{-1}+5^{-1}\right)^0$

(iv) $\left(\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right)^{-1}$

Answer:

(i) We know from the property of powers that for every natural number a, $a^{-1}=\frac{1}{a}$, Then:

$3^{-1}+4^{-1}=\frac{1}{3}+\frac{1}{4}$

= $\frac{4+3}{12}$

= $\frac{7}{12}$

(ii) We know from the property of powers that for every natural number a, $a^{-1}=\frac{1}{a}$.

Moreover, a^0 is 1 for every natural number a not equal to 0. Then,

$\left(3^0+4^{-1}\right)\times 2^2$

= $\left(1+\frac{1}{4}\right)\times 4$

= $\frac{5}{4}\times 4$

= 5

(iii) We know from the property of powers that for every natural number a, $a^{-1}=\frac{1}{a}$.

Moreover, a^0 is 1 for every natural number a not equal to 0. Then,

$\left(3^{-1}+4^{-1}+5^{-1}\right)^0$ = 1 —> (Ignore the expression inside the bracket and use a^0 = 1)

(iv) We know from the property of powers that for every natural number a, $a^{-1}=\frac{1}{a}$.

Then:

$\left(\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right)^{-1}=(3-4)^{-1}$

= $(-1)^{-1}$

= -1

Question 3.Find the value of each of the following:

(i) $\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{3}\right)^{-1}+\left(\frac{1}{4}\right)^{-1}$

(ii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$

(iii) $\left(2^{-1} \times 4^{-4}\right) \div 2^{-2}$

(iv) $\left(5^{-1} \times 2^{-1}\right) \div 6^{-1}$

Answer:

(i) $\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{3}\right)^{-1}+\left(\frac{1}{4}\right)^{-1}$

= $\frac{1}{\frac{1}{2}}+\frac{1}{\frac{1}{3}}+\frac{1}{\frac{1}{4}}$

= 2 + 3 + 4 = 12

(ii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$

= $\frac{1}{\left(\frac{1}{2}\right)^{2}}+\frac{1}{\left(\frac{1}{3}\right)^{2}}+\frac{1}{\left(\frac{1}{4}\right)^{2}}$

= $\frac{1}{\frac{1}{4}}+\frac{1}{\frac{1}{9}}+\frac{1}{\frac{1}{16}}$

= 4 + 9 + 16 = 29

(iii) $\left(2^{-1} \times 4^{-4}\right) \div 2^{-2}$

= $\frac{1}{2} \times \frac{1}{4} \div \frac{1}{2^{2}}$

= $\frac{1}{8} \times 4$ = $\frac{1}{2}$

(iv) $\left(5^{-1} \times 2^{-1}\right) \div 6^{-1}$

= $\left(\frac{1}{5} \times \frac{1}{2}\right) \div \frac{1}{6}$

= $\frac{1}{10} \times 6$ = $\frac{3}{5}$

Question 4. Simplify:

(i) $\left(4^{-1} \times 3^{-1}\right)^{2}$

(ii) $\left(5^{-1} \div 6^{-1}\right)^{3}$

(iii) $\left(2^{-1}+3^{-1}\right)^{-1}$

(iv) $\left(3^{-1}+4^{-1}\right)^{-1} \times 5^{-1}$

Answer:

(i) $\left(4^{-1} \times 3^{-1}\right)^{2}$

= $\left(\frac{1}{4} \times \frac{1}{3}\right)^{2}$

= $\left(\frac{1}{12}\right)^{2}$

= $\left(\frac{1^{2}}{12^{2}}\right)$ = $\left(\frac{1}{24}\right)$

(ii) $\left(5^{-1} \div 6^{-1}\right)^{3}$

= $\left(\frac{1}{5} \div \frac{1}{6}\right)^{3}$

= $\left(\frac{1}{5} \times 6\right)^{3}$

= $\left(\frac{6}{5}\right)^3$ = $\frac{216}{125}$

(iii) $\left(2^{-1}+3^{-1}\right)^{-1}$

= $\left(\frac{1}{2}+\frac{1}{3}\right)^{-1}$

= $\left(\frac{5}{6}\right)^{-1}$

= $\left(\frac{1}{\frac{5}{6}}\right)$ = $\frac{6}{5}$

(iv) $\left(3^{-1}+4^{-1}\right)^{-1} \times 5^{-1}$

= $\left(\frac{1}{3} \times \frac{1}{4}\right)^{-1} \times \frac{1}{5}$

= $\left(\frac{1}{12}\right)^{-1} \times \frac{1}{5}$ = $\frac{12}{5}$

Question 5. Simplify:

(i) $\left(3^2+2^2\right) \times \left(\frac{1}{2}\right)^3$

(ii) $\left(3^2 – 2^2\right) \times \left(\frac{2}{3}\right)^{-3}$

(iii) $\left(\left(\frac{1}{3}^{-3}\right)-\left(\frac{1}{2}\right)^{-3}\right) \div \left(\frac{1}{4}\right)^{-3}$

(iv) $\left(2^2+3^2 – 4^2\right) \div \left(\frac{3}{2}\right)^2$

Answer:

(i) $\left(3^2+2^2\right) \times \left(\frac{1}{2}\right)^3$

= $(9+4) \times \frac{1}{8}$ = $\frac{13}{8}$

(ii) $\left(3^2 – 2^2\right) \times \left(\frac{2}{3}\right)^{-3}$

= $(9 – 4) \times \frac{1}{(2/3)^3}$

= $5 \times \frac{1}{(8/27)}$ = $\frac{135}{8}$

(iii) $\left(\left(\frac{1}{3}^{-3}\right)-\left(\frac{1}{2}\right)^{-3}\right) \div \left(\frac{1}{4}\right)^{-3}$

= $\left(3^3 – 2^3\right) \div 4^3$

= $(27 – 8) \div 64$

= $19 \times \frac{1}{64}$ = $\frac{19}{64}$

(iv) $\left(2^2+3^2 – 4^2\right) \div \left(\frac{3}{2}\right)^2$

= $(4+9 – 16) \div \left(\frac{9}{4}\right)$

= $-3 \times \frac{4}{9}$ = $-\frac{4}{3}$

Question 6. By what number should 5^{-1} be multiplies so that the product may be equal to -7^{-1}?

Answer:

Using the property $a^{-1}=\frac{1}{a}$ for every natural number a, we have $5^{-1}=\frac{1}{5}$ and $(-7)^{-1}=-\frac{1}{7}$. We have to find a number x such that

$\frac{1}{5} \times x=\frac{-1}{7}$

Multiply bith sides by 5, we get

$x=\frac{-5}{7}$

Hence, the required number is $\frac{-5}{7}$

Question 7. By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplies so that the product may be equal to $\left(-\frac{4}{7}\right)^{-1}$?

Answer:

Using the property $a^{-1} = \frac{1}{a}$ for every natural number a, we have $\left(\frac{1}{2}\right)^{-1} = 2\ and\ \left(\frac{-4}{7}\right)^{-1} = \frac{-7}{4}$. We have to find the number x such that

$2x = \frac{-7}{4}$

Dividing both sides by 2, we get

$x = \frac{-7}{8}$

Hence, the required number is $\frac{-7}{8}$

Question 8. By what number should $(-15)^{-1}$ be multiplies so that the product may be equal to $(-5)^{-1}$

Answer:

Using the property $a^{-1} = \frac{1}{a}$ for every natural number a, we have $(-15)^{-1} = -\frac{1}{15}$ and $(-5)^{-1} = -\frac{1}{5}$. We have to find a number x such that

$\frac{-\frac{1}{15}}{\frac{x}{1}} = \frac{-1}{5}$

Or $\frac{1}{15} \times \frac{1}{x} =- \frac{-1}{5}$

Or $x = \frac{1}{3}$

Hence, $(-15)^{-1}$ should be divided by $\frac{1}{3}$ to obtain $(-5)^{-1}$..

Exercise 2.2

Q1. Write each of the following in exponential form:

(i) $\left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1}$

(ii) $\left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2}$

Solution:

(i) $\left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} = \left(\frac{3}{2}\right)^{-1+(-1)+(-1)+(-1)}$

$a^m \times a^n = a^{m+n}$ = $(\frac{3}{2})^{-4}$

(ii) $\left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} = \left(\frac{2}{5}\right)^{-1+(-2)+(-2)}$

$a^m \times a^n = a^{m+n}$ = $\left(\frac{2}{5}\right)^{-6}$

Q2. Evaluate:

(i) 5^{-2}

(ii) $(-3)^{-2}$

(iii) $\left(\frac{1}{3}\right)^{-4}$

(iv) $\left(\frac{-1}{2}\right)^{-1}$

Solution:

(i) $5^{-2} = \frac{1}{5^2}$ = $\frac{1}{25}$

(ii) $(-3)^{-2} = \frac{1}{(-3)^2}$ = $\frac{1}{9}$

(iii) $\left(\frac{1}{3}\right)^{-4} = \frac{1}{\left(\frac{1}{3}\right)^4}$ = $\frac{1}{\frac{1}{81}}$ = 81

(iv) $\left(\frac{-1}{2}\right)^{-1} = \left(\frac{1}{\frac{-1}{2}}\right)$ = -2

Q3. Express each of the following as a rational number in the form $\frac{p}{q}$:

(i) 6^{-1}

(ii) -7^{-1}

(iii) $\left(\frac{1}{4}\right)^{-1}$

(iv) $(-4)^{-1} \times \left(\frac{-3}{2}\right)^{-1}$

(v) $\left(\frac{3}{5}\right)^{-1} \times \left(\frac{5}{2}\right)^{-1}$

Solution:

(i) $6^{-1} = \frac{1}{6}$

(ii) $-7^{-1} = \frac{1}{-7}$ = $\frac{-1}{7}$

(iii) $\left(\frac{1}{4}\right)^{-1} = \frac{1}{\frac{1}{4}} = 4$

(iv) $(-4)^{-1} \times \left(\frac{-3}{2}\right)^{-1} = \frac{1}{-4} \times \frac{1}{\frac{-3}{2}}$

= $\frac{1}{-4} \times = \frac{2}{-3}$ = $\frac{1}{6}$

(v) $\left(\frac{3}{5}\right)^{-1} \times \left(\frac{5}{2}\right)^{-1} = \frac{1}{\frac{3}{5}} \times \frac{1}{\frac{5}{2}}$

= $\frac{5}{3} \times \frac{2}{5}$ = $\frac{2}{3}$

Q4. Simplify:

(i) $\{4^{-1} \times 3^{-1}\}^2$

(ii) $\{5^{-1} \div 6^{-1}\}^3$

(iii) $\{2^{-1} + 3^{-1}\}^{-1}$

(iv) $\{3^{-1} + 4^{-1}\}^{-1} \times 5^{-1}$

(v) $\{4^{-1} + 5^{-1}\}^{-1} + 3^{-1}$

Solution:

(i) $\{4^{-1} \times 3^{-1}\}^2 = \left(\frac{1}{4} \times \frac{1}{3}\right)^2$

= $\left(\frac{1}{12}\right)^2$ = $\left(\frac{1}{144}\right)$

(ii) $(5^{-1} \div 6^{-1})^3 = \left(\frac{1}{5} \div \frac{1}{6}\right)^3$

= $\left(\frac{6}{5}\right)^3$ = $\left(\frac{216}{125}\right)$

(iii) $\{2^{-1} + 3^{-1}\}^{-1} = \left(\frac{1}{2} + \frac{1}{3}\right)^{-1}$

= $\left(\frac{5}{6}\right)^{-1}$ = $\left(\frac{6}{5}\right)$

(iv) $\{3^{-1} + 4^{-1}\}^{-1} \times 5^{-1} = \left(\frac{1}{3} \times \frac{1}{4}\right)^{-1} \times \frac{1}{5}$

= $\left(\frac{1}{12}\right)^{-1} \times \frac{1}{5}$

= $12 \times \frac{1}{5}$ = $\frac{12}{5}$

(v) $\{4^{-1} + 5^{-1}\}^{-1} + 3^{-1} = \left(\frac{1}{4} - \frac{1}{5}\right) \div \frac{1}{3}$

= $\left(\frac{5-4}{20}\right) \times 3$

= $\frac{1}{20} \times 3$ = $\frac{3}{20}$

Q5. Express each of the following rational numbers with a negative exponent:

(i) $\left(\frac{1}{4}\right)^3$

(ii) $(3)^5$

(iii) $\left(\frac{3}{5}\right)^4$

(iv) $\left\{\left(\frac{3}{2}\right)^4\right\}^{-3}$

(v) $\left\{\left(\frac{7}{4}\right)^4\right\}^{-3}$

Solution:

(i) $\left(\frac{1}{4}\right)^3$

= $\left(\frac{4}{1}\right)^{-3}$

(ii) $(3)^5$

= $\left(\frac{1}{3}\right)^{-5}$

(iii) $\left(\frac{3}{5}\right)^4$

= $\left(\frac{5}{3}\right)^{-4}$

(iv) $\left\{\left(\frac{3}{2}\right)^4\right\}^{-3}$

= $\left(\frac{3}{2}\right)^{-12}$

(v) $\left\{\left(\frac{7}{4}\right)^4\right\}^{-3}$

= $\left(\frac{7}{3}\right)^{-12}$

Q6. Express each of the following rational numbers with a positive exponent.

(i) $\left(\frac{3}{4}\right)^{-2}$

(ii) $\left(\frac{5}{4}\right)^{-3}$

(iii) $4^3 \times 4^{-9}$

(iv) $\left\{\left(\frac{4}{3}\right)^{-3}\right\}^{-4}$

(v) $\left\{\left(\frac{3}{2}\right)^4\right\}^{-2}$

Solution:

(i) $\left(\frac{3}{4}\right)^{-2}$

= $\left(\frac{4}{3}\right)^2$

(ii) $\left(\frac{5}{4}\right)^{-3}$

= $\left(\frac{4}{5}\right)^3$

(iii) $4^3 \times 4^{-9}$

= $4^{3-9} = 4^{-6}$

= $\left(\frac{1}{4}\right)^6$

(iv) $\left\{\left(\frac{4}{3}\right)^{-3}\right\}^{-4}$

= $\left(\frac{4}{3}\right)^{-4\times-3}$

= $\left(\frac{4}{3}\right)^{12}$

(v) $\left\{\left(\frac{3}{2}\right)^4\right\}^{-2}$

$= \left(\frac{3}{2}\right)^{4\times-2}$

$= \left(\frac{3}{2}\right)^{-8}$

$= \left(\frac{2}{3}\right)^{8}$

Q7. Simplify:

(i) $\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3}$

(ii) $\left(3^2 - 2^2\right) \times \left(\frac{2}{3}\right)^{-3}$

(iii) $\left\{\left(\frac{1}{2}\right)^{-1} \times (-4)^{-1}\right\}^{-1}$

(iv) $\left[\left\{\left(\frac{-1}{4}\right)^{2}\right\}^{-2}\right]^{-1}$

(v) $\left\{\left(\frac{2}{3}\right)^{2}\right\}^{3} \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times 6^{-1}$

Solution:

(i)

$\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3} = \left(\frac{1}{(1/3)^3} - \frac{1}{(1/2)^3}\right) \div \frac{1}{(1/4)^3}$

$= \left(\frac{1}{(1/27)} - \frac{1}{(1/8)}\right) \div \frac{1}{(1/64)}$

$= \left(\frac{27}{1} - \frac{8}{1}\right) \div 64$

$= (19) \times \frac{1}{64}$

$= \frac{19}{64}$

(ii)

$\left(3^2 - 2^2\right) \times \left(\frac{2}{3}\right)^{-3} = (9-4) \times \frac{1}{(2/3)^2}$

$= 5 \times \frac{27}{8}$

$= \frac{135}{8}$

(iii)

$\left(\left(\frac{1}{2}\right)^{-1} \times (-4)^{-1}\right)^{-1} = \left(\left(\frac{1}{1/2}\right) \times \left(\frac{1}{-4}\right)\right)^{-1}$

$= \left(2 \times \left(\frac{1}{-4}\right)\right)^{-1}$

$= \left(\frac{1}{-2}\right)$

$= \frac{1}{1/(-2)}$

$= -2$

(iv) $\left(\left(\left(\frac{-1}{4}\right)^{2}\right)^{-2}\right)^{-1} = \left(\left(\frac{(-1)^2}{4^2}\right)^{-2}\right)^{-1}$

$= \left(\left(\frac{1}{16}\right)^{-2}\right)^{-1}$

$= \left(\left(\frac{1}{(1/16)^2}\right)\right)^{-1}$

$= \left(\frac{1}{(1/256)}\right)^{-1}$

$= 256^{-1} = \frac{1}{256}$

(v)

$\left\{\left(\frac{2}{3}\right)^2\right\}^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times 6^{-1}$

$= \left(\frac{2^2}{3^2}\right)^3 \times \frac{1}{(1/3)^4} \times \frac{1}{3} \times \frac{1}{6}$

$= \frac{4^3}{9^3} \times 81 \times \frac{1}{18}$

$= \frac{64}{729} \times 81 \times \frac{1}{18}$

$= \frac{64}{9} \times \frac{1}{18}$

$= 64 \times \frac{1}{162}$

$= \frac{64}{162}$

$= \frac{32}{81}$

Q8. By what number should 5^{-1} be multiplies so that the product may be equal to $(-7)^{-1}$?

Solution:

Expressing in fraction form, we get:

$5^{-1} = \frac{1}{5}$

And $(-7)^{-1} = \frac{1}{-7}$

We have to find a number x such that

$\frac{1}{5}x = \frac{-1}{7}$

Multiplying both side by 5, we get:

x = $-\frac{5}{7}$

Hence, 5^{-1} be multiplied by $-\frac{5}{7}$ to obtain $(-7)^{-1}$.

Q9. By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplies so that the product may be equal to $\left(\frac{-4}{7}\right)^{-1}$?

Solution:

Expressing in fraction form, we get

$\left(\frac{1}{2}\right)^{-1} = 2,$

And $\left(\frac{-4}{7}\right)^{-1} = -\frac{7}{4}$

We have to find a number x such that:

$2x = -\frac{7}{4}$

Dividing both side by 2, we get

$x = -\frac{7}{8}$

Hence, $\left(\frac{1}{2}\right)^{-1}$ should be multiplies by $-\frac{7}{8}$ to obtain $\left(\frac{-4}{7}\right)^{-1}$.

Q10. By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$

Solution:

Expressing in fraction form, we get:

$(-15)^{-1} =-\frac{1}{15}$ (using $a^{-1} = \frac{1}{a}$)

And

$(-5)^{-1} =-\frac{1}{5}$ (using $a^{-1} = \frac{1}{a}$)

We have to find a number xx such that

$-\frac{1}{15} \div x = -\frac{1}{5}$

Solving this equation, we get:

$-\frac{1}{15} \times \frac{1}{x} =-\frac{1}{5}$

$-\frac{1}{15} =-\frac{x}{5}$

$\frac{-5}{-15} = x$

$x = \frac{1}{3}$

Hence, $(-15)^{-1}$ should be divided by $\frac{1}{3}$ to obtain $(-5)^{-1}$

Q11. By what number should $\left(\frac{5}{3}\right)^{-2}$ be multiplies so that the product may be $\left(\frac{7}{3}\right)^{-1}$?

Solution:

Expressing as a positive exponent, we have:

$\left(\frac{5}{3}\right)^{-2} = \frac{1}{(5/3)^2}$

$= \frac{1}{25/9}$

$= \frac{9}{25}$

and

$= \left(\frac{7}{3}\right)^{-1} = \frac{3}{7}$

$We\ have\ to\ find\ a\ number\ x\ such\ that$

$\frac{9}{25} \times x = \frac{3}{7}$

Multiplying both sides by 25/9, we get:

$x = \frac{3}{7} \times \frac{25}{9} = \frac{1}{7} \times \frac{25}{3} = \frac{25}{21}$

Hence, $\left(\frac{5}{3}\right)^{-2}$ should be multiplies by $\frac{25}{21}$ top obtain $\left(\frac{7}{3}\right)^{-1}$.

Q12. Find x, if:

(i) $\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} = \left(\frac{1}{4}\right)^{-4x}$

(ii) $\left(\frac{-1}{2}\right)^{-19} \times \left(\frac{-1}{2}\right)^{8} = \left(\frac{-1}{2}\right)^{-2x+1}$

(iii) $\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{5} = \left(\frac{3}{2}\right)^{2x+1}$

(iv) $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$

(v) $\left(\frac{5}{4}\right)^{-x} \div \left(\frac{5}{4}\right)^{-4} = \left(\frac{5}{4}\right)^{5}$

(vi) $\left(\frac{8}{3}\right)^{2x+1} \times \left(\frac{8}{3}\right)^{5} = \left(\frac{8}{3}\right)^{x+2}$

Answer:

(i) We have:

$\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} = \left(\frac{1}{4}\right)^{-4x}$

$\left(\frac{1}{4}\right)^{-12} = \left(\frac{1}{4}\right)^{-4x}$

$-12 = -4x$

$3 = x$

Therefore, x = 3

(ii) We have:

$\left(\frac{-1}{2}\right)^{-19} \times \left(\frac{-1}{2}\right)^{8} = \left(\frac{-1}{2}\right)^{-2x+1}$

$\left(\frac{-1}{2}\right)^{-11} = \left(\frac{-1}{2}\right)^{-2x+1}$

$-11 = -2x + 1$

$-12 = -2x$

$6 = x$

Therefore, x = 6

(iii) We have:

$\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{5} = \left(\frac{3}{2}\right)^{2x+1}$
$\left(\frac{3}{2}\right)^{2} = \left(\frac{3}{2}\right)^{2x+1}$
$2 = 2x + 1$
$1 = 2x$
$\frac{1}{2} = x$

Therefore, $x = \frac{1}{2}$

(iv) We have:

$\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$
$\left(\frac{2}{5}\right)^{12} = \left(\frac{2}{5}\right)^{2x+1}$
$12 = 2 + 3x$
$10 = 3x$
$\frac{10}{3} = x$

Therefore, $x = \frac{10}{3}$

(v) We have:

$\left(\frac{5}{4}\right)^{-x} \div \left(\frac{5}{4}\right)^{-4} = \left(\frac{5}{4}\right)^{5}$
$\left(\frac{5}{4}\right)^{-x+4} = \left(\frac{5}{4}\right)^{5}$
$-x + 4 = 5$
$-x = 1$
$x = -1$
Therefore, x = -1

(vi) We have:

$\left(\frac{8}{3}\right)^{2x+1} \times \left(\frac{8}{3}\right)^{5} = \left(\frac{8}{3}\right)^{x+2}$
$\left(\frac{8}{3}\right)^{2x+6} = \left(\frac{8}{3}\right)^{x+2}$
$2x + 6 = x + 2$
$x = -4$

Therefore, x = -4

Q13.

(i) if $x = \left(\frac{3}{2}\right)^{2} \times \left(\frac{2}{3}\right)^{-4}$, find the value of x^{-2}.

(ii) If $x = \left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^{2}$, find the value of x^{-1}.

Answer:

(i) First, we have to find x.

$x = \left(\frac{3}{2}\right)^{2} \times \left(\frac{2}{3}\right)^{-4}$
$= \left(\frac{3}{2}\right)^{2} \times \left(\frac{3}{2}\right)^{4}$
$= \left(\frac{3}{2}\right)^{6}$

Hence, x^{-2} is:

$x^{-2} = \left(\left(\frac{3}{2}\right)^{6}\right)^{-2}$
$= \left(\frac{3}{2}\right)^{-12}$
$= \left(\frac{2}{3}\right)^{12}$

(ii) First we will have to find x.

$x = \left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^{2}$
$= \left(\frac{4^{-2}}{5^{-2}}\right) \times 4^{2}$
$= \frac{4^{0}}{5^{-2}}$
$\left(5^{2}\right)^{-1}$
$= \frac{1}{5^{2}}$

Q14. Find the value of x for which $5^{2x} \div 5^{-3} = 5^{5}$.

Answer: We have:

$5^{2x} \div 5^{-3} = 5^{5}$
$5^{2x+3} = 5^{5}$
$2x + 3 = 5$
$2x = 2$
$x = 1$

Hence, x is 1.

Exercise 2.3

1. Express the following numbers in standard form:

(i) 6020000000000000

(ii) 0.0000000000943

(iii) 0.00000000085

(iv) 846×10^7

(v) 3759×10^{-4}

(vi) 0.00072984

(vii) 0.000437×10^4

(viii) $4 \div 100000$

Answers:

To express a number in the standard for, move the decimal point such that there is only one digit to the left of the decimal point.

(i) 6020000000000000 = 6.02×10^{15} (The decimal point is moved 15 places to the left.)

(ii) 0.0000000000943 = 9.43×10^{-12} (The decimal point is moved 12 places to the right.)

(iii) 0.00000000085 = 8.5×10^{-10} (The decimal point is moved 10 places to the right.)

(iv) $846 \times 10^7 = 8.46 \times 10^2 \times 10^7 = 8.46 \times 10^9$ (The decimal point is moved two places to the left.)

(v) $3759 \times 10^{-4} = 3.759 \times 10^3 \times 10^{-4} = 3.759 \times 10^{-1}$ (The decimal point is moved three places to the left.)

(vi) $0.00072984 = 7.984 \times 10^{-4}$ (The decimal point is moved four places to the right.)

(vii) $0.000437 \times 10^4 = 4.37 \times 10^{-4} \times 10^4 = 4.37 \times 10^0 = 4.37$ (The decimal point is moved four places to the right.)

(viii) $4 \div 100000 = 4 \times 100000^{-1} = 4 \times 10^{-5}$ (Just count the number of zeros in 1,00,000 to determine the exponent of 10.)

2. Write the following numbers in the usual form:

(i) 4.83×10^7

(ii) 3.02×10^{-6}

(iii) 4.5×10^4

(iv) 3×10^{-8}

(v) 1.0001×10^9

(vi) 5.8×10^2

(vii) 3.61492×10^6

(viii) 3.25×10^{-7}

Answers:

(i) $4.83 \times 10^7 = 4.83 \times 1,00,00,000 = 4,83,00,000$

(ii) $3.02 \times 10^{-6} = \frac{3.02}{10^6} = \frac{3.02}{10,00,000} = 0.00000302$

(iii) $4.5 \times 10^4 = 4.5 \times 10,000 = 45,000$

(iv) $3 \times 10^{-8} = \frac{3}{8} = \frac{3}{10,00,00,000} = 0.00000003$

(v) $1.0001 \times 10^9 = 1.0001 \times 1,00,00,00,000 = 1,00,01,00,000$

(vi) $5.8 \times 10^2 = 5.8 \times 100 = 580$

(vii) $3.61492x10^6 = 3.61492x10,00,000 = 3614920$

(viii) $3.25x10^{-7} = \frac{3.25}{10^7} = \frac{3.25}{1,00,00,000} = 0.000000325$