Chapter 22: Mensuration III (Surface Area and Volume of a Right Circular Cylinder)

Exercise 22.1

Q1. Find the curved surface area and total surface area of a cylinder, the diameter of whose base is 7 cm and height is 60 cm.

Soln:

Let r and h be the radius and the height of the cylinder,

Given: r = $\frac{7}{2}cm$ h = 60 cm

Curved area of the cylinder = $2\Pi \times r \times h$

= $2 \times \frac{22}{7} \times \frac{7}{2} \times 60$

= $22 \times 60 = 1320\ cm^2$

Total surface area of the cylinder = $2\Pi \times r \times (r + h)$

= $2 \times \frac{22}{7} \times \frac{7}{2} \times (\frac{7}{2} + 60) = 22$

= $22 \times \frac{127}{2}$

= $11 \times 127 = 1397\ cm^2$

Q2. The curved surface area of a cylindrical rod is 132 cm^2. Find its length if the radius is 0.35 cm.

Soln:

Consider h to be the height of the cylindrical rod

Given: Radius, r = 0.35 cm

Curved surface area = 132 cm^2

We know; curved surface area $2\Pi \times r \times h$

132 = $2 \times \frac{22}{7} \times 0.35 \times h$

h = $h = \frac{132 \times 7}{2 \times 22 \times 0.35}$

h = 60

Therefore, the length of the cylindrical rod is 60 cm.

Q3. The area of the base of a right circular cylinder is 616 cm2 and its height is 2.5 cm. Find the curved surface area of the cylinder.

Soln:

Given: Area of the base of a right circular = 616 cm^2

Height = 2.5 cm

Let r be the radius of the base of a right cylinder, $\Pi r^2 = 616$

=> $r^2 = 616 \times \frac{7}{22}$

=> $r^2 = 196$

=> r = 14 cm

Curved surface area of the circular cylinder = $2\Pi rh = 2 \times \frac{22}{7} \times 14 \times 2.5 = 220\ cm^2$

Q4. The circumference of the base of a cylinder is 88 cm and its height is 15 cm. Find its curved surface area and total surface area.

Soln: Given: Height, h = 15 cm

Circumference of the base of the cylinder = 88 cm^2

Let r be the radius of the radius of the cylinder. The circumference of the base of the cylinder = $2\Pi r 88 = 2 \times \frac{22}{7} \times rr = \frac{88 \times 7}{2 \times 22} = 14cm$

Curved surface area = $2 \times \Pi \times r \times h$

= $2 \times \frac{22}{7} \times 14 \times 15 = 1320cm^2$

Total surface area = $2 \times \Pi \times r \times (r + h) = 2 \times \frac{22}{7} \times 14 \times (14 + 15) = 2552cm^2$

Q5. A rectangular strip 25 cm x 7 cm is rotated on the longer side. Find the total surface area of the solid thus generated.

Soln: Since the rectangle strip of 25 cm x 7 cm is rotated about the longer side, we have:

Height, h = 25 cm

Radius, r = 7cm

∴ Total surface area = $2\Pi r(r + h) = 2\Pi(7)(25 + 7) = 14\Pi(32) = 448\Pi cm^2 = 448 \times \frac{22}{7} = 1408cm^2$

Q6. A rectangular sheet of paper, 44 cm x 20 cm, is rolled along its length to form a cylinder. Find the total surface area of the cylinder thus generated.

Soln:

The rectangular sheet of paper 44 cm x 20 cm is rolled along its length to from a cylinder. The height of the cylinder is 20 cm and circumference is 44 cm.

We have: Height, h = 20 cm

Circumference = $2 \times \Pi \times r = 44cm$

∴ Total surface area is $S = 2\Pi rh = 44 \times 20cm^2 = 880cm^2$

Q7. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Calculate the ratio of their curved surface areas.

Soln:

Let the radii of two cylinders of 2r and 3r, respectively, and their heights be 5h and 3h, respectively. Let S_1 and S_2 be the curved surface areas of the two cylinders. S_1 = Curved surface area of the cylinder of height 5h and radius 2rS_2 = curved surface area of the cylinder of height 3h and radius 3r.

∴ $S_1 : S_2$ = $2\Pi rh = \frac{2\times\Pi\times2r\times5h}{2\times\Pi\times3r\times3h}10:9$

Q8.The ratio between the curved surface area and the total surface area of a right circular cylinder is 1:2. Prove that its height and radius are equal.

Soln:

Let Si and S, be the curved surface area and total surface area of the circular cylinder, respectively.

Then, $S_1 = 2\Pi rh, S_2 = 2\Pi r(r+h)$

According to the question: $S_1 : S_2 = 1 : 22\Pi rh : 2\Pi r(r+h) = 1 : 2$

$\frac{h}{r+h} = \frac{1}{2}$

h : (r + h) = 1 : 2

2h = r + h

h = r Therefore, the height and the radius are equal.

Q9. The curved surface area of a cylinder is 1320 cm^2 and its base has diameter 21 cm. Find the height of the cylinder.

Soln:

Let h be the height of the cylinder.

Given: Curved surface area, S = 1320 cm^2

Diameter, d = 21 cm Radius, r = 10.5

$S = 2\Pi rh 1320 = 2\Pi \times 10.5 \times h$

$h = \frac{1320}{2\Pi\times10.5}$

h = 20 cm

Q10. The height of a right circular cylinder is 10.5 cm. If three times the sum of the areas of its two circular faces is twice the area of the curved surface area. Find the radius of its base

Soln:

Let r be the radius of the circular cylinder.

Height, h = 10.5 cm

Area of the curved surface, $S_1 = 2\Pi rh$ sum of the areas of its two circular faces, $S_1 = 2\Pi r^2$

According to question: $3S_2 = 2S_1 3 \times 2\Pi r^2 = 2 \times 2\Pi rh$

6r = 4h

3r = 2h

$r = \frac{2}{3} \times 10.5cm$

= 7 cm

Q11. Find the cost of plastering the inner surface of a well at Its 9.50 per m^2, if it is 21 m deep and diameter of its top is 6 m.

Soln:

Given: Height, h = 21 m

Diameter, d = 6 m

Radius = 3 m

Area of the linear surface of the well, S = $2\Pi rh = 2\Pi \times 3 \times 21m^2$

According to question, the cost per m^2 is Rs 9.5.

∴ Inner surface cost is Rs 396 x 9.50 = Rs 3762

Q12. A cylindrical vessel open at the top has diameter 20 cm and height 14 cm. Find the cost of tin-plating it on the inside at the rate of 50 paise per hundred square centimetres.

Soln:

Given: Diameter, d = 20 cm

Radius, r = 10 cm

Height, h = 14 cm

Area inside the cylindrical vessel that is to be tin- plated = SS =
$2\Pi rh + \Pi r^2 = 2\Pi \times 10 \times 14 + \Pi \times 10^2 = 280\Pi + 100\Pi = 380 \times \frac{22}{7} cm^2 = \frac{8360}{7} cm^2$

According to question: cost per 100 cm^2 = 50 paise

Cost per cm^2 = Rs 0.005

Cost of tin – plating the area inside the cylindrical vessel = $Rs\ 0.005 \times \frac{8360}{7} = \frac{41.8}{7} = Rs\ 5.97$

Q13. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find the cost of plastering its inner curved surface at Rs 4 per square meter.

Soln:

Given:Inner diameter of the circular well = 3.5m

∴ Inner radius of the circular well, r = 1.75 m

Deth of the circular well, h = 10 m

Inner curved surface area, S = $2\Pi rh$

$S = 2\Pi \times 1.75 \times 10 m^2 = 2 \times \frac{22}{7} \times 1.75 \times 10 m^2 = 110 m^2$

Cost of plastering 1 m^2 = Rs 4

Cost of plastering 110 m^2 area = Rs(110 x 4) = Rs 440

Q14. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions moving once over to level a playground. What is the area of the playground?

Soln:

Given: Diameter of the roller = 84 cm

∴ Radius, r = $\frac{Diameter}{2}$ = 42 cm

In 1 revolution, it covers the distance of its lateral surface area.

Roller is a cylinder of height; h = 120 cm

Radius = 42 cm

Lateral surface area of the cylinder = $2\Pi rh = 2 \times \frac{22}{7} \times 42 \times 120 = 31680 cm^2$

It takes 500 complete revolutions to level a playground.

∴ Area of the field = 31680 x 500 = 15840000 cm^2

= 1584 m^2.

Thus, the area of the in m^2 is 1584 m^2.

Q15. Twenty-one cylindrical pillars of the Parliament House are to be cleaned. If the diameter of each pillar is 0.50 m and height is 4 m, what will be the cost of cleaning them at the rate of Rs 2.50 per square meter?

Soln:

Given: Diameter of the pillars = 0.5 m

Radius of the pillars, r = 0.25 m

Height of the pillars, = h = 4 m

Number of pillars = 21

Rate of cleaning = Rs 2.5 per square meter

Curved surface area of one pillar = $2\Pi rh = 2 \times \frac{22}{7} \times 0.25 \times 4 = 2 \times \frac{22}{7} = \frac{44}{7} m^2$

∴ Curved surface area of one pillar = $\frac{44}{7} m^2$.

Cost of cleaning 21 pillars at the rate of Rs 2.50 per m^2 = Rs 330

Q16. The total surface area of a hollow cylinder which is open from both sides is 4620 sq. cm, the area of the base ring is 115.5 sq. cm and height 7 cm. Find the thickness of the cylinder.

Soln:

Given: Total surface area of the cylinder = 4620 cm^2

Area of the base ring = 115.5 cm^2

Height, h = 7 cm

Let R be the radius of the outer ring and r be the radius of the inner ring.

Area of the base ring = $\Pi R^2 - \Pi r^2 115.5 = \Pi(R^2 - r^2) R^2 - r^2 = 115.5 \times \frac{7}{22}$

$(R+r)(R-r) = 36.75$ ....(i)

Total surface area = inner curved surface area + outer curved surface area + Area of the bottom and top rings

4620 = $2\Pi rh + 2\Pi Rh + 2 \times 115.5$

$2\Pi h(R+r) = 4620 - 231$

$R+r = \frac{4389 \times 7}{2 \times 22 \times 7} R + r = \frac{399}{4}$
.............(ii)

Substitution the value of R + r from the equation (ii) in (i):

$\frac{399}{4}(R-r) = 36.75(R-r) = 36.75 \times \frac{4}{399} = 0.36cm$

∴ Thickness of the cylinder = (R −r)= 0.368 cm

Q17. The sum of the radius of the base and height of a solid cylinder is 37 m. If the total surface area of the solid cylinder is 1628 m^2, find the circumference of its base.

Soln: Let r and h be the radius and height of the solid cylinder.

Given: r + h = 37 m

Total surface area, S = $2\Pi r(r+h)$

$1628 = 2\Pi \times r \times 37$

$r = \frac{1628}{2\Pi \times 37} = \frac{1628}{232.477} = 7$

Circumference of its base, S_1 = $2\Pi r$

= $(2 \times \frac{22}{7} \times 7)m = 44m$

Q18. Find the ratio between the total surface area of a cylinder to its curved surface area, given that its height and radius are 7.5 cm and 3.5 cm.

Soln:

Let S_1 and S_2 be the total surface area and curved surface area, respectively.

Given: Height, h = 7.5cm

Radius, r = 3.5 cm

$S_1 = 2\Pi r(r+h)$

$S_2 = 2\Pi rh$

According to the question: $\frac{S_1}{S_2} = \frac{2\Pi r(r+h)}{2\Pi rh}$

$\frac{S_1}{S_2} = \frac{r+h}{h}$

$\frac{S_1}{S_2} = \frac{3.5+7.5}{7.5}$

$\frac{S_1}{S_2} = \frac{11}{7.5} = \frac{110}{75} = \frac{22}{15}$

Therefore, the ratio is 22 : 15

Q19. A cylindrical vessel, without lid, has to be tin-coated on its both sides. If the radius of the base is 70 cm and its height is 1.4 m, calculate the cost of tin-coating at the rate of Rs 3.50 per 1000 cm^2.

Soln:

Let r cm and h cm be the radius of the cylindrical vessel.

Given: Radius, r = 70 cm

Height, h = 1.4 m = 140 cm

Rate of tin- plating = Rs. 3.50 per 1000 cm^2

Cost of tin- plating the cylindrical vessel on both the surfaces (inner and outer): Total surface area of a vessel = Area of the outer side of the base + Area of the inner and the outer curved surface = $2(\Pi r^2 + 2\Pi rh) = 2\Pi r(r+2h) = 2 \times \frac{22}{7} \times 70 \times (70 + 2 \times 140)$

= $40 \times 10 \times 350 = 154000cm^2$

Cost of painting at the rate of Rs 3.50 per 1000 cm^2= $154000 \times \frac{3.50}{1000} = Rs\ 539$

Therefore, cost of painting is Rs 539.

Chapter 22: Mensuration III (Surface Area and Volume of a Right Circular Cylinder)

Exercise 22.2

Q1. Find the volume of a cylinder whose: (i) r = 3.5 cm, h = 40 cm (ii) r =2.81 m, h =15 m

Soln:

(i) Given : r = 3 5 cm, h = 40 cm

Volume of cylinder, V =$\pi r^2 h$

= $\frac{22}{7} \times (3.5)^2 \times 40$ =1540 cm^3

(ii) Given :

r = 2.8 in, h = 15 m

Volume of cylinder, V =$\pi r^2 h$

= $\frac{22}{7}\times(2.8)^2\times 15$ =369.6 m^3

Q2. Find the volume of a cylinder, if the diameter (d) of its base and its altitude (h) are: (i) d = 21 cm, h = 10 cm (ii) d =7 m, h = 24 m

Soln:

(i) Given :

d = 21 cm, radius, r = $\frac{d}{2} = 10.5cm$

height, h = 10 cm

Volume of the cylinder, V = $\pi r^2 h$

= $\frac{22}{7}x(10.5)^2x10$

= 3465 cm^3

(ii) Given :

d= 7 m, radius, r= $\frac{d}{2} = 3.5m$

height h = 24 m

Volume of the cylinder, V = $\pi r^2 h$

= $\frac{22}{7}x(3.5)^2x24$

=924 m^3

Q3. The area of the base of a right circular cylinder is 616 cm^2 and its height is 25 cm. Find the volume of the cylinder.

Soln:

Let the area of the base of a right circular cylinder be S cm^2.

Given :

S = 616 cm^2

Height, h = 25 cm

Let the radius of a right circular cylinder be r cm.

S = πr^2

616 = $\frac{22}{7}xr^2$

$r^2 = \frac{616\times 7}{22}$

r^2 = 196

r = 14 cm

Volume of the cylinder, V = $\pi r^2 h$

= $\pi\times(14)^2x25$

= 15400 cm^3

Q4. The circumference of the base of a cylinder is 88 cm and its height is 15 cm. Find the volume of the cylinder.

Soln:

Let r cm be the radius of a cylinder.

Circumference of the cylinder, S = $2\pi r$

Given :

Height, h = 15 cm

Circumference, S = 88 cm

S = $2\pi r$

88= $2\times\frac{22}{7}\times r$

r = 14 cm

Volume of cylinder, V = $\pi r^2 h$

= $\frac{22}{7}x14^2x15$

= 9240 cm^3

Q5. A hollow cylindrical pipe is 21 mm long. Its outer and inner diameters are 10 cm and 6 cm respectively. Find the volume of the copper used in making the pipe.

Soln:

Let the length of the cylinder pipe be h = 21, dm = 210 cm.

Let the outer and the inner radius of the pipe be R cm and r cm, respectively.

∴ 2R = 10 and 2r = 6

R = 5 cm and r = 3cm

Volume of the copper used in making the pipe, V = $\Pi(R^2 - r^2)h$

= $\frac{22}{7}x(5^2 - 3^2)x210$

=22x(25-9)x30 =22x 16x 30 =10560 cm^3.

Q6. Find the (i) curved surface area (ii) total surface area and (iii) volume of a right circular cylinder whose height is 15 cm and the radius of the base is 7 cm.

Soln:

Given:

Height, h= 15 cm

Radius, r = 7 cm

(i) Curved surface area, $S_1 = 2\pi rh$

= $2 \times \frac{22}{7} \times 7 \times 15$

= 660 cm^2

(ii) Total surface area, $S_2 = 2\pi r(r + h)$

= $2 \times \frac{22}{7} \times 7 \times (7 + 15)$

= 44 x 22

= 968 cm^2

(iii) Volume of the right circular cylinder, $V = \pi r^2 h$

= $\frac{22}{7} \times 7^2 \times 15$

= 2310 cm^3

Q7. The diameter of the base of a right circular cylinder is 42 cm and its height is 10 cm. Find the volume of the cylinder.

Soln:

Given:

Diameter, d = 42 cm

Radius, r = 21 cm

Height, h = 10 cm

Volume of the cylinder, $V = \pi r^2 h$

= $\frac{22}{7} \times 21^2 \times 10$ = 13860 cm^3

Q8. Find the volume of a cylinder, the diameter of whose base is 7 cm and height being 60 cm. Also, find the capacity of the cylinder in liters.

Soln:

Given:

Diameter, d = 7 cm

Radius, r = 3.5 cm

Height, h = 60 cm

Volume of the cylinder, $V = \pi r^2 h$

= $\frac{22}{7} \times (3.5)^2 \times 60$

= 2310 cm^3

Capacity of the cylinder in liters = $\frac{2310}{1000}$ (1 Liters = 1000 cubic cm)

= 2.31 L

Q9. A rectangular strip 25 cm x 7 cm is rotated on the longer side. Find the volume of the solid, thus generated.

Soln:

Given:

Rectangular strip has radius, r = 7cm

Height, h = 25 cm

Volume of the solid, $V = \pi r^2 h$

= $\frac{22}{7} \times 7^2 \times 25$

= 3850 cm^3

Q10. A rectangular sheet of paper, 44 cm x 20 cm, is rolled along its length to form a cylinder. Find the volume of the cylinder so formed.

Soln:

Given:

The length (l) and breadth (b) of the rectangular sheet are 44 cm and 20 cm

Now, the sheet is rolled along the length to form a cylinder.

Let the radius of the cylinder be r cm.

Height, h = b = 20 cm

Circumference, S = 44 cm

$2\pi r$

$\frac{22}{7} \times 2 \times r = 44cm$

r = 7 cm

Volume of the cylinder, $V = \pi r^2 h$

$\frac{22}{7} \times 7^2 \times 20$

3080 cm^3

Q11. The volume and the curved surface area of a cylinder are 1650 cm^3 and 660 cm^2 respectively. Find the radius and height of the cylinder.

Soln:

Curved surface area of the cylinder = $2\pi rhcm^2$........ (1)

Volume of the cylinder $V = \pi r^2 h = 1650cm^3$..........(2)

From (1) and (2), we can calculate the radius (r) and the height of cylinder (h).

We know the volume of the cylinder i.e. 1650 cm^3

∴ 1650 = $\pi r^2 h$

h = $\frac{1650}{\pi r^2}$

Substitution h into (1)

660 = $\pi r^2 h$

660 = $\pi r^2 \times \frac{1650}{\pi r^2}$

660r = 2(1650)

r = 5 cm

h = $\frac{1650}{\pi r^2}$

h = $\frac{1650}{\frac{22}{7} \times 5^2} = 21cm$

Hence, the radius and the height of the cylinder are 5 cm and 21 cm, respectively.

Q12. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Calculate the ratio of their volumes.

Soln:

Here,

ri = Radius of cylinder 1

hi = Height of cylinder 1

r2 = Radius of cylinder 2

h2 = Height of cylinder 2

V1 = Volume of cylinder 1

V2 = Volume of cylinder 2

Ratio of the radii of two cylinders = 2:3

Ratio of the heights of two cylinders = 5:3

Volume of the cylinder = $\pi r^2 h$

V_1 / V_2 = $(\pi r_1^2 h_1)/(\pi r_2^2 h_2)$

= $(\pi (2r)^2 5h)/(\pi (3r)^2 3h)$

= $(\pi 4r^2 5h)/(\pi 9r^2 3h)$

=20/ 27

Hence, the ratio of their volumes is 20:27

Q13. The ratio between the curved surface area and the total surface area of a right circular cylinder is 1:2. Find the volume of the cylinder, if its total surface area is 616 cm2.

Soln:

Let r cm be the radius and h cm be the length of the cylinder. The curved surface area and the total surface area is 1:2.

The total surface area is 616 cm^2.

The curved surface area is the half of 616 cm^2, i.e. 308 cm^2.

Curved area = $2\pi rh$

So, h = $\frac{308}{2\Pi r}$

Total surface area = Curved surface area + Top and bottom area

Top and bottom area = 616 − 308 = 308 cm^2 = $2\pi r^2$

$r^2 = \frac{308}{2\times\frac{22}{7}}$

r = 7 cm

h= $\frac{308}{2\Pi\times 7}$ = 7cm

Then, the volume of the cylinder can be calculated as follows:

V=$\frac{22}{7} \times 7^2 \times 7 = 1078$ = 1078

Hence, it is obtained that the volume of the cylinder is 1078 cm^3.

Q14. The curved surface area of a cylinder is 1320 cm^2 and its base has diameter 21 cm. Find the volume of the cylinder.

Soln:

r cm = Radius of the cylinder

h cm = Height of the cylinder

The diameter of the cylinder is 21 cm. Thus, the radius is 10.5 cm.

Since the curved surface area has been known, we can calculate h by the given below:

The curved surface area of the cylinder = $2\Pi rh$

1320 cm^2 = $2\Pi rh$

1320 cm^2 = $\frac{2\times 22}{7} \times (10.5cm) \times h$

h = 20 cm

∴ volume of the cylinder (V) = $\Pi r^2 h$

V = $\frac{22}{7}(10.5cm)^2(20cm)$

V = 6930 cm^3

Q15. The ratio between the radius of the base and the height of a cylinder is 2 : 3. Find the total surface area of the cylinder, if its volume is 1617 cm^3.

Soln:

Let r cm be the radius and h cm be the height of the cylinder. It is given that the ratio of r and h is 2:3,

So h = 1.5r

The volume of the cylinder (V) is 1617 cm^3

So, we can find the radius and the height of the cylinder from the equation given below:

V = $\Pi r^2 h$

1617 = $\Pi r^2 h$

1617 = $\Pi r^2 (1.5r)$

r^3 = 343

r = 7 cm and h = 10.5 cm

Total surface area = $2\Pi r^2 + 2\Pi rh$

= $2 \times \frac{22}{7} \times 7^2 + 2 \times \frac{22}{7} \times 7 \times 10.5 = 770\ cm^2$

Hence, the total surface area of the cylinder is 770 cm^2.

Q16. The curved surface area of a cylindrical pillar is 264 m^3 and its volume is 924 m^3. Find the diameter and the height of the pillar.

Soln:

Here, r = radius of the cylinder

h = height of the cylinder

Curved surface area of the cylinder = $2\Pi rh$........(1)

Volume of the cylinder = $\Pi r^2 h$

924 = $\Pi r^2 h$

h = $\frac{924}{\Pi r^2}$

Then, substitute h into equation (1)

264 = $2\Pi rh$

264 = $2\Pi r(\frac{924}{\Pi r^2})$

r = 7 m, so d = 14 m

h = $\frac{924}{\Pi r^2}$

h = $\frac{924}{\frac{22}{7} \times 7^2} = 6m$

Hence, the diameter and the height of the cylinder are 14 m and 6 m, respectively.

Q17. Two circular cylinders of equal volumes have their heights in the ratio 1 : 2. Find the ratio of their radii.

Sol:

Here, V_1 = Volume of cylinder 1

V_2 = Volume of cylinder 2

r_1 = Radius of cylinder 1

r_2 = Radius of cylinder 2

h_1 = Height of cylinder 1 h2

h_2 = Height of cylinder 2

Volumes of cylinder 1 and 2 are equal.

Height of cylinder 1 is half the height of cylinder 2

∴V_{1} = V_{2}

$(\Pi r_1^2 h_1) = (\Pi r_2^2 h_2)$

$(\Pi r_1^2 h) = (\Pi r_2^2 2h)$

$\frac{r_1^2}{r_2^2} = \frac{2}{1}$

$\frac{r_1}{r_2} = \sqrt{\frac{2}{1}}$

Thus, the ratio of their radii is $\sqrt{2} : 1$

Q18. The height of a right circular cylinder is 10.5 m. Three times the sum of the areas of its two circular faces is twice the area of the curved surface. Find the volume of the cylinder.

Soln:

It is known that three times the sum of the two circular faces, of the right circular cylinder, is twice the area of the curved surface.

Hence, it can be written using the following formula:

$3(2\Pi r^2) + 2(2\Pi rh)$

$3\Pi r^2 + 2\Pi rh$

3r = 2h

It is known that the height of the cylinder (h) is 10.5 m

Substituting this number in the equation:

3r = 2(10.5)

r = 7 m

Volume of the cylinder = $\Pi r^2 h$

= $\frac{22}{7}(7)^2(10.5)$

= 1617 m^3

Thus, the volume of the cylinder is 1617 m^3.

Q19. How many cubic meters of earth must be dug-out to sink a well 21 m deep and 6 m diameter?

Soln:

The volume of the earth that must be dug out is similar to the volume of the cylinder which is equal to $\pi r^2 h$.

Height of the well =21 m

Diameter of the well 6 m

∴ Volume of the earth that must be dug out = $(\pi(3^2)(21))$ m^3= 594 m3

Q20. The trunk of a tree is cylindrical and its circumference is 176 cm. if the length of the trunk is 3 m, find the volume of the timber that can be obtained from the trunk

Soln:

Circumference of the tree = 176 cm = $2\pi r$

Length of the trunk, h = 3 m = 300 cm

So, the radius (r) can be calculated by:

$r = \frac{176}{2\times\frac{22}{7}} = 28cm$

Thus, the volume (V) of the timber can be calculated using the following formula:

$V = \Pi r^2(h) = \frac{22}{7}(28)^2(300)cm^2 = 739200cm^3 = 0.74m^3$

Q21. A well is dug 20 m deep and it has a diameter of 7 m. The earth which is so dug out is spread out on a rectangular plot 22 m long and 14 m broad. What is the height of the platform so formed?

Soln:

Height of the well = h = 20 m

Diameter of the well = d = 7 m

Radius of the well = r = 3.5 m

Volume of the well = $\Pi r^2 h = \frac{22}{7}(3.5)^2(20)m^3 = 770m^3$

Volume of the well = volume of the rectangular plot

Length of the rectangle plot = 22 m

Breadth of the rectangular plot = 14 m

Volume of the rectangular plot = 770 m^3 = (Length x Breadth x Height) of the rectangle plot

Height = $\frac{770}{22\times14} = 2.5m$

Thus, the height of the platform is 2.5 m

Q22. A well with 14 m diametre is dug 8 m deep. The earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. Find the height of the embankment.

Soln:

Diameter of the well = d = 14 m

Height of the well = 8 m

Radius of the well = r = 7 m

Volume of the well = $\Pi r^2 h = \Pi(7m)^2(8m) = 1232m^3$

Volume of the well = volume of the embankment

An embankment is a hollow cylinder with thickness. Its inner radius would be equal to be the radius of the well, i.e. r = 7 m, and it s outer radius is R = 7 + 21 = 28 m

Volume of the embankment = $\pi h(R^2 - r^2)$

To find the height of the well (h), we use the fact that the volume of the well is equal to the volume of the embankment.

1232 = $\pi h((28)^2 – (7)^2)$

h = $\frac{1232}{\frac{22}{7}\times[(28)^2-(7)^2]}$ = 0.533 m

Hence, the height of the embankment is 0.533 m or 53.3 cm

Q23. A cylindrical container with the diameter of base 56 cm contains sufficient water to submerge a rectangular solid of iron with dimension 32 cm x 22 cm x 14 cm. Find the rise in the level of the water when the solid is completely submerged.

Soln:

Diameter of the cylinder container = d = 56 cm

Radius of the cylindrical container = r = 28 cm

Volume of cylindrical container = volume of the rectangle solid

Length of the rectangle solid = 32 cm

Breadth of the rectangle solid = 22 cm

Height of the rectangle solid = 14 cm

Volume of the rectangle solid = Length x Breadth x Height = 32 cm x 22 cm x 14 cm = 9856 cm^3

Volume of the cylindrical container = 9856 cm^3 = $\pi r^2 h$

9856 cm^3 = $\frac{22}{7}(28cm)^2 h$

h = 4 cm

Thus, when the solid is completely submerged, the water will rise up to 4 cm.

Q24. A rectangle sheet of paper 30 cm x 18 cm can be transformed into the curved surface of a right circular cylinder in two ways i.e. either by rolling the paper along its length or by rolling it along its breadth. Find the ratio of the volumes of the two cylinders thus formed.

Soln:

Case 1:

Height = h = 30 cm

Diameter = d = 18 cm

Radius = r = 9 cm

∴ Volume = $(\Pi r^2 h)$ or $(\Pi(9)^2(30)) = 2430\pi cm^3$

Case 2:

Height = h = 18 cm

Diameter = d = 30 cm

Radius = r = 15 cm

∴ Volume = $(\Pi r^2 h)$ or $(\Pi(15)^2(18)) = 4050\pi cm^3$

Hence, the ratio of the volumes of the volumes of the two cylinders formed is 3: 5

Q25. The rain which falls on a roof 18 m long and 16.5 m wide is allowed to be stored in a cylindrical tank 8 m in diameter. If it rains 10 cm on a day, what is the rise of water level in the tank due to it?

Soln:

Length of the water on a roof = 18 m

Breadth of the water on a roof = 16.5 m

Height of the water on a roof = 10 cm = 0.1 m

Volume of the water on a roof = Length x Breadth x Height = 18 m x 16.5 m x 0.1 m = 29.7 m^3

Since water is to be stored in the cylindrical tank, the volume of water on a roof is equal to the volume of a cylindrical tank.

Volume of cylindrical tank = $\Pi r^2 h = 29.7m^3$

h = $\frac{1232}{\frac{22}{7}\times(4)^2} = 0.5906m = 59.06cm$

Thus, the rise of water level in the tank is 59.06 cm.

Q26. A piece of ductile metal is in the form of a cylinder of diameter 1 cm and length 5 cm. It is drawn out into a wire of diameter 1 mm. What will be the length of the wire so formed?

Soln:

Diameter of the ductile metal = 1 cm

Radius of the ductile metal = 0.5 cm

Volume of the ductile metal = $\Pi r^2(length) = \Pi(0.5cm)^2(5cm) = 1.25\Pi cm^3$

Ductile metal is drawn into a wire of diameter 1 mm

Radius of the wire = 0.5 mm = 0.05 cm

Length of wire = $\frac{1.25\Pi cm^3}{\Pi(0.05cm)^2} = 500cm = 5m$

Thus, the length of wire is 5 m.

Q27. Find the length of 13.2 kg of copper wire of diameter 4 mm, when 1 cubic cm of copper weighs 8.4 gm.

Soln:

Density of copper = Weigth/ Volume = 8.4 gram/ 1 cm^3

Volume = Weight/ Density = 13.2 kg x 1000 gram/kg / 8.4 grams/cm^3 = 1571.43 cm^3

L = $\frac{V}{\Pi r^2} = \frac{1571.43cm^3}{\frac{22}{7}\times(0.2cm)^2} = 123500.01cm = 125m$

Thus, length of 13.2 kg of copper is 125 m.

Q28. 2.2 cubic dm of brass is to be drawn into a cylindrical wire 0.25 cm in diameter. Find the length of the wire.

Soln:

Diameter of the cylindrical wire = 0.25 cm

Radius of the cylindrical wire = 0.125 cm

Volume of the brass = 2.2 dm^3 = 2200 cm^3

Volume of the brass = Volume of the cylindrical wire

Length of the wire = $\frac{2200cm^3}{\frac{22}{7}\times(0.125cm)^2} = 44800cm = 488m$

Q29. The difference between inside and outside surfaces of a cylindrical tube 14 cm long is 88 sq. cm. If the volume of the tube is 176 cubic cm, find the inner and outer radii of the tube.

Soln:

r = Inner radii of the tube

R = Outer radii of the tube

h = lengthy of the tube

$2\Pi h(R-r) = 88$ ……….. (1)

$\Pi h(R^2-r^2) = 176$ …………. (2)

Substituting h = 14 cm in equation (1) and (2):

$\Pi(R-r) = \frac{88}{28}$ …………. (a)

$\Pi(R-r)(R+r) = \frac{176}{14}$ ………. (b)

Simplifying the second equation by substituting it with the first equation:

R + r = 4 cm or R = (4 – r) cm

Re- substituting R = 4 – r into equation (1):

$\frac{22}{7}(4-r-r) = \frac{88}{28}$

4 – 2r = 1

R = 4 – 1.5 = 2.5 cm

Hence, the inner and outer radii of the tube are 1.5 and 2.5 cm, respectively

Q30. Water flows out through a circular pipe whose internal diameter is 2 cm, at the rate of 6 meters per second into a cylindrical tank, the radius of whose base is 60 cm. Find the rise in the level of water in 30 minutes?

Soln:

Radius of the circular pipe = 0.01 m

Length of the water column in 1 sec = 6 m

Volume of the water flowing in 1 s = $\pi r^2 h = \pi(0.01)^2(6)m^3$

Volume of the water flowing in 30 mins = $\pi(0.01)^2(6)x30x60m^3$

Let h m be the rise in the level of water in the cylindrical tank.

Volume of the cylindrical tank in which water is being flown = $\pi(0.6)^2 xh$

Volume of water flowing in 30 mins = Volume of the cylindrical tank in which water is being flown

$$\pi(0.01)^2(6) \times 30 \times 60 = \pi(0.6)^2 \times h$$

$$h = \frac{6(0.01)^2 \times 30 \times 60}{0.6 \times 0.6}$$

h = 3 m

Q31. A cylindrical tube, open at both ends, is made of metal. The internal diameter of the tube is 10.4 cm and its length is 25 cm. The thickness of the metal is 8 mm everywhere. Calculate the volume of the metal.

Soln:

Here, r = inner radius = 5.2 cm

R = outer radius

t = thickness = 0.8 cm

h = length = 25 cm

R = r + t = 5.2 cm +0.8 cm = 6 cm

Volume of the metal = $\pi h(R^2 – r^2)$

$\frac{22}{7} \times (25) \times ((6)^2 – (5.2)^2)$= 704 cm^3

Thus, the volume of the metal is 704 cm^3

Q32. From a tap of inner radius 0.75 cm, water flows at the rate of 7 m per second. Find the volume in liters of water delivered by the pipe in one hour.

Soln:

Radius of the water tap = 0.75 cm = 0.0075 m

Length of the water flowing in 1 s = 7 m = 700 cm

Volume of water flowing in 1s = $\pi(0.0075)^2 \times 700$

Volume of the water flowing in 1 hour = $\pi(0.0075)^2 \times 700 \times 60 \times 60$

Volume of the water flowing in 1 hour = $\frac{22}{7} \times (0.0075)^2 \times 7 \times 60 \times 60 = 4.455m^3 = 4455l$

(1000 l = 1 m^3)

Q33. A cylindrical water tank of diameter 1.4 m and height 2.1 m is being fed by a pipe of diameter 3.5 cm through which water flows at the rate of 2 meters per second. In how much time the tank will be filled?

Soln:

Radius of the cylindrical tank = 0.7 m

Height of the cylindrical tank = 2.1 m

Volume of the cylindrical tank = $\pi(0.7)^2(2.1)m^3$

Length of the water column flown from the pipe in 1 s = 2 m

Let the time taken to completely fill the water tank be x sec.

Length of the water column flown from the pipe in x sec = 2x m

Radius of the pipe = 1.75cm = 0.0175 m

Volume of the water column flown from the pipe in x sec = $\pi(0.0175)^2(2x)m^3$

Volume of the cylindrical tank = volume of the water column flown from the pipe $\pi(0.7)^2(2.1) = \pi(0.0175)^2(2x)$

$$x = \frac{(0.7)^2(2.1)}{(0.0175)^2(2)} = 1680sec = 28min$$

Thus, the time required to fill the water tank is 28 min.

Q34. A rectangular sheet of paper 30 cm x 18 cm can be transformed into the curved surface of a right circular cylinder in two ways i.e., either by rolling the paper along its length or by rolling it along its breadth. Find the ratio of the volumes of the two cylinders thus formed.

Soln:

Let h cm be the length of the paper and r cm be the radius of the paper.

We know that the rectangular sheet of paper 30 cm x 18 cm can be transformed into two types of cylinder.

Type 1:

Length = 30 cm

Diameter = 18 cm

Volume = $(\pi r^2 h) = (\pi(9cm)^2(30cm)) = 2430\pi cm^3$

Type 2:

Length = 18 cm

Diameter = 30 cm

Volume = $(\pi r^2 h) or (\pi(15cm)^2(18cm)) = 4050\pi cm^3$

Hence, the ratio of the volumes of the two cylinders formed is 3 : 5

Q35. How many liters of water flow out of a pipe having an area of the cross-section of 5 cm^2 in one minute, if the speed of water in the pipe is 30 cm/sec?

Soln:

We know:

Area of cross section = 5 cm^2

Rate = 30 cm/s and

Time = 1 min

So, the volume of water flow is:

Volume = volumetric rate x Time = (30 cm/s)(5 cm^2)(60 s/min) = 9000 cm^3 = 9 litres

Thus, 9 litres of water flows out of the pipe

Q36. A solid cylinder has a total surface area of 231 cm^2. Its curved surface area is 3 — of the total surface area. Find the volume of the cylinder.

Soln:

We know that the total surface area of the cylinder is 231 cm^3 and the curved surface area of the cylinder is 2/3 of the total surface area.

So, the curved surface area is:

2/3 x (231 cm^3) = 154 cm^3

Then, the radius of the cylinder can be calculated in the following manner:

Curved surface area = $2\pi rh$

154 cm^2 = $2\pi rh$.............. (1)

Here, r cm is the radius of the cylinder and h cm is the length of the cylinder

$2\pi r^2$ = (231 − 154)cm^2 = 77 cm^2

77 cm^2 = $2\pi r^2$

From here, the radius (r) can be calculated in the following manner:

$$r = \sqrt{\frac{77}{2\times\frac{22}{7}}}$$

r = 3.5 cm

Substituting this result into equation (1)

154 cm^2 = $2\pi(3.5cm)h$

h = $h = \frac{154}{2\times\frac{22}{7}\times(3.5cm)}$

h = 7 cm

$$\therefore V = \pi r^2 h = \frac{22}{7} \times (3.5cm)^2 \times (7cm) = 269.5cm^3$$

Hence the volume of the cylinder is 269.5 cm^3

Q37. Find the cost of sinking a tubewell 280 m deep, having diameter 3 m at the rate of Rs 3.60 per cubic meter. Find also the cost of cementing its inner curved surface at Rs 2.50 per square meter.

Soln:

Cost of sinking a tube well = volume of the tube well x cost of sinking a tube well per cubic metre

= $\frac{22}{7} \times (1.5)^2 \times (280) \times Rs3.6/m^3 = Rs\ 7128$

Cost of cementing = Inner surface area of the tube well x Cost of cementing per square meter

= $2 \times \frac{22}{7} \times 1.5 \times (280) \times Rs2.5/m^2 = Rs\ 6600$

Q38. Find the length of 13.2 kg of copper wire of diameter 4 mm, when 1 cubic cm of copper weighs 8.4 gm.

Soln:

Since we know the weight and the volume of copper, we can calculate its density.

Density of copper = $\frac{weight}{volume} = \frac{8.4gram}{1cm^3} = 8.4\frac{gram}{cm^3}$

If the weight of copper wire is 13.2 kg and the density of copper is 8.4 g/cm^{3,} then:

Volume = weight / density = 13.2 kg and the density of copper is 8.4 g/ cm^3 = 1571.43 cm^3

The radius of copper wire is 2 mm or 0.2 cm. So, the length of the wire can be determined in the following way:

L = $\frac{V}{\pi r^2} = \frac{1571.43m^3}{\pi(0.2cm)^2} = 125050.01cm = 125m$

Thus, the length of 13.2 kg of copper is 125 m.

Q39. 2.2 cubic dm of brass is to be drawn into a cylindrical wire 0.25 cm in diameter. Find the length of the wire.

Soln:

Let r cm be the radius of the wire and h cm be the height of the wire.

Volume of brass = Volume of the wire

We know that the volume of brass = 2.2 dm^3 = 2200 cm^3

Volume of the wire = $\pi r^2 h = (0.125cm)^2(h)$

$h = \frac{2200cm^2}{\pi(0.125cm)^2} = 44800cm = 448m$

Thus, length of the wire is 448 m.

Q40. A well with 10 in inside diameter is dug 8.4 m deep. Earth taken out of it is spread all around it to a width of 7.5 m to form an embankment. Find the height of the embankment.

Soln:

Let r be the radius and d be the depth of the well that is dug.

Volume of the well = $\pi r^2 d = \pi(5m)^2(8.4m) = 660m^3$

An embankment has the shape of hollow cylinder with thickness. Its inner radii is equal to the well's radii, i.e. r = 5 m, and its outer radii is R = (5 + 7.5) = 12.5 cm

Then, the volume of the embankment = $\pi h(R – r^2)$

Volume of the well = volume of the embankment

$659.73m^3 = \Pi h((12.5m)^2 – (5m)^2)$

$h = \frac{660cm^3}{\frac{22}{7}\times[(12.5m)^2-(5m)^2]} = 1.6m$

Hence, the height of the embankment is 1.6 m

Q41. A hollow garden roller, 63 cm wide with a girth of 440 cm, is made of 4 cm thick iron. Find the volume of the iron.

Soln:

Here, R = outer radius

r = inner radius

t = thickness = 4 cm

w = width = 63 cm

Girth = 440 cm = $2\pi R$

$R = \frac{440}{\frac{22}{7}\times 2} = 70cm$

r = R − t = 70cm − 4cm = 66 cm

Volume of the iron = $\pi(R^2 – r^2)w = \frac{22}{7} – (70^2 - 66^2) - (63) = 107712cm^3$

Hence, volume of the iron is 107712 cm^3.

Q42. What length of a solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of length 16 cm, external diameter 20 cm and thickness 2.5 mm?

Soln:

Here, r = Internal radius

R = External radius = 10 cm

h = Length of the cylinder

t = Thickness = 0.25 cm

Volume of the hollow cylinder = $\pi h(R^2 – r^2) = \pi(16)(10^2 - (10 – 0.25)^2) = 79\pi cm^3$

Volume of the solid cylinder = Volume of the hollow cylinder

We know that the radius of the solid cylinder is 1 cm.

$\therefore \pi(1^2)h = 79\pi$

h = 79 cm

Hence, length of the solid cylinder that gives the same volume as the hollow cylinder is 79 cm.

Q43. In the middle of a rectangular field measuring 30m x 20m, a well of 7 m diameter and 10 m of depth is dug. The earth so removed is evenly spread over the remaining part the field. Find the height through which the level of the field is raised.

Soln:

Let r be the radius and h be the depth of the well that is dug.

Volume of the well = $\pi r^2 h = \frac{22}{7} \times (3.5m)^2 \times (10m) = 385m^3$

Area of the embankment field = $30 \times 20 – \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{1123}{2} m^2$

Volume of the well = Area of the spread out x height of the rectangular field

Volume of the rectangular field = 385 m^3 = $\frac{1123}{2}$ x height

Height = $\frac{385m^3 \times 2}{1123m^2} = 0.6856m = 68.56cm$

Hence, the height through which the level of the field is raised is 68.56 cm.